# *A unified approach to unit commitment and economic dispatch in power system control.*


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# A UNIFIED APPROACH TO UNIT COMMITMENT AND ECONOMIC DISPATCH IN POWER SYSTEM CONTROL

**A Thesis Submitted to**

**the *University of Durham***
**for the *Degree of Doctor of Philosophy***

*by*

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*November 1990*

*This thesis is dedicated to*

*my parents and to my family*

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University of Durham

# A UNIFIED APPROACH TO UNIT COMMITMENT AND ECONOMIC DISPATCH IN POWER SYSTEM CONTROL

*Chak H. Cheung*

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## ABSTRACT

*This thesis is concerned with the subjects of unit commitment and economic dispatch which are at the heart of economic and secure operation of a power system. A common thread running through a major part of the thesis is the application of a new dynamic programming (DP) recursive formula to solve these two problems. A composite cost model proposed allows the application of the new formula to efficiently decide the optimal on/off schedule of the available generating units. For the economic dispatch problem, an iterative DP procedure is presented to successively determining the optimal generation output to high accuracy comparable to existing approaches. A new loss formula is also described. This new loss formula is designed for computational simplicity and is capable of responding to the rapid changes in system topology, load distribution and generation pattern.*

*The (N-1) security constrained dispatch is dealt with using a linear programming approach. A new line outage simulation technique called Current Injection Method (CIM) is derived. The security constrained dispatch algorithm implemented utilizes the CIM technique to generate a list of critical line failures within the solution process. The resulting optimal generator outputs schedule will ensure that any unscheduled single line failure will not cause overloading in any of the remaining lines. Tests indicate that the new algorithm takes only a fraction more CPU time than a conventional dispatch which ignores line failure contingencies. The work on security constrained dispatch is extended to consider the post-contingency corrective capability of a system. Examples show that by allowing transmission lines to be loaded to their short time ratings during the short period while the generator outputs are being adjusted in response to a contingency, significant economic saving can be achieved.*

*All new algorithms proposed in the thesis are shown to be capable of dealing with large realistic networks and are potentially compatible with real time operation.*

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# CHAPTER 1

# INTRODUCTION

Power system control is required in order to maintain a continuous balance between the supply of electricity and a varying load demand. A power system is capital intensive and the annual operating costs can run into billions of pounds sterling for a large system. Inefficient operation will incur excessive energy losses and faults may cause serious damage to vital plants. The control of an electric system is therefore necessarily comprehensive in order to ensure the secure and economic operation of the system. In the OCEPS (Operational Control of Electric Power Systems) Research Group at the School of Engineering and Applied Science of the University of Durham, a suite of power system application software has been in development for the last 17 years. The structure of the software package with details on the inter-relationship and data flow of the major control elements are shown in Fig.1.1. The suite of software is equivalent to the monitoring and control functions of a typical Energy Management System (EMS) found in a modern control centre of an electric network It has the additional facility of a dynamic simulator of the physical system. A brief description of each of the control function in Figure 1.1 is provided in Appendix A. The advantages of having a real time simulation control package in a laboratory environment are numerous. From a research and development point of view, the dynamic models of generators, transmission network, transformers and other plants provide a realistic test bed for the study, analysis and verification of


Fig.1.1

Operational Control of Electric Power System - Overview of Major Functional Elements
( Extract from "Power System Monitoring and Control Software" manual, OCEPS, Durham University)

the performance of the existing and new monitoring/control algorithms. The effect of a new algorithm on the overall control strategies of a power system may also be tested under diverse scenarios without endangering a real system. Furthermore, the need for robustness of the software, data communication between one control module to another, the time scheduling of control tasks within the overall control scheme, the processor loading during normal and emergency system states and the problem of effective interaction between the operators and the control functions are highlighted. Another important role of a real time simulator is for system operator training. In real system operation, an emergency happens very occasionally. With a simulator available, different emergency situations can be simulated as often as required. Through regular training sessions, the quality and response speed of the operators to real emergencies therefore may be improved and maintained. Many modern control centres now have the dual functions of controlling the system operation as well as providing on-line data to trainees for system control practices.

This thesis is concerned with the problem of optimizing the operational cost of a power system. The areas of concern are unit commitment and economic dispatch whose relation to the rest of the control strategies in an EMS is shown in Fig.1.2. These two optimization problems represent a time decomposed approach to achieve the economic operation objective. Unit commitment deals with a longer time span problem, typically of 24 hours to one week period. It schedules the ON/OFF timing of generating units to achieve

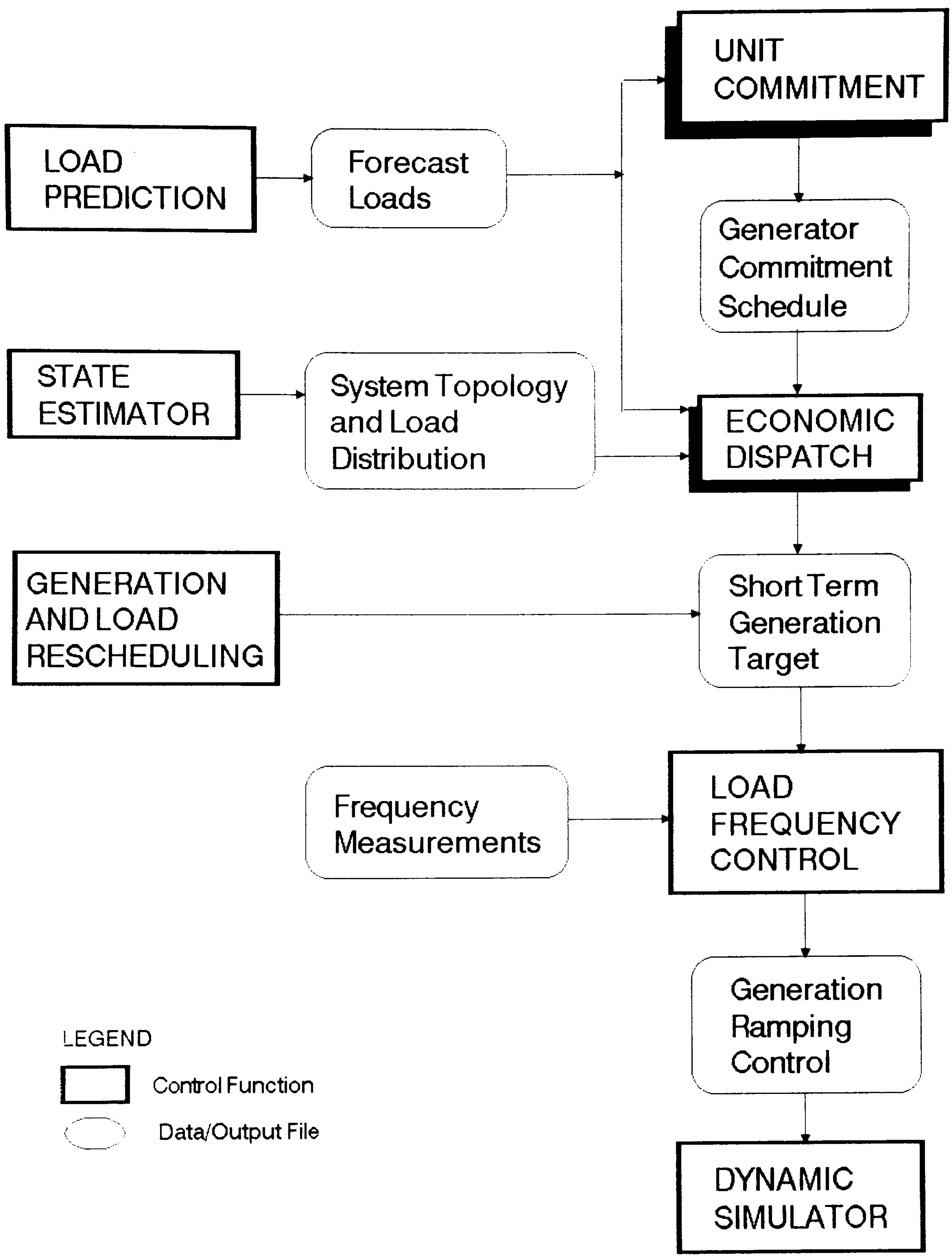



Fig.1.2 Generation Control Subsystem

minimum overall operating cost. Economic dispatch deals with the shorter term problem, typically of 5 to 30 minutes horizon. It allocates the optimal sharing of generation outputs among synchronized units to meet the forecast load. The complexity of the cost minimisation problem and the rapid response requirement in real time operation necessitate such a two step approach. Fortunately, the long lead time of starting up steam turbine generators and their relatively fast response to the power ramp up/down command after synchronization give a natural subdivision of the overall cost optimization problem.

In unit commitment, the load model used is an aggregated power demand of the whole system at any time instant. The total capacity available from the generators at any time instant are scheduled to meet such total demand with due regard to the possibility of loss of generation. Spinning reserve therefore is an important aspect a solution method needs to address. The objective of the unit commitment control function is to minimise the total operational cost to meet the predicted load within the study period of 24 hours or longer ahead by controlling the start up and shut down timing of the generating units.

In the economic dispatch problem, an increased degree of detail for the system models is used as the length of the planning horizon decreases to only a few minutes. The unit availability from the unit commitment solution is part of the input data for the economic dispatch solution algorithm. The best estimated topological details of the system from the on-line state estimator are also needed to provide the most up to date information on the system conditions such as locations of the generating sources, load distributions and transmission network topology. The power system is modelled sufficiently accurately in order to produce an optimal dispatch solution implementable in the physical world.

## 1.1 Organization of the Thesis

To address the unit commitment and economic dispatch problems, the thesis is organized into three major parts.

Part one: System component modelling

In Chapter 2, the essential operational characteristics of system components including generators, loads, and the transmission network which directly affect the economic and secure operation of the system are outlined. The models which may be used to incorporate these operational characteristics in determining the optimal generation costs are described. The important aspects of system security requirements in relation to unit commitment and economic dispatch problems are clarified.

Part two : Unit Commitment

The subject is divided into two chapters. Chapter 3 surveys the existing unit commitment techniques. These include heuristic approaches, mixed integer-linear programming, branch and bound, Lagrangian relaxation and dynamic programming methods. In Chapter 4, a new scheduling algorithm based on the dynamic programming (DP) optimality principle is proposed. The new DP methodology is designed to overcome the dimensionality and formidable storage requirements normally associated with DP techniques. A highly efficient computational procedure capable of handling large systems is described.

Part Three: Economic Dispatch

This is divided into four chapters.

Chapter 5 surveys the 'pure' economic dispatch solutions. The conventional merit order dispatch and Lagrangian multipliers methods are outlined first. Linear Programming (LP) and Quadratic Programming (QP) approaches are then reviewed in detail.

Chapter 6 proposes the use of dynamic programming for economic dispatch application. Derivation of the solution scheme is revealed. It is shown that the proposed technique has the unique capacity to handle non-linear, non-convex generation cost functions. Tests are carried out to compare the optimality and efficiency of the method to the popular LP and QP approaches.

The (N-1) security constrained economic dispatch problem is investigated in Chapter 7. The conventional methods to deal with post-contingency system conditions and the enormous possible number of constraints arising from such contingencies are described. An original approach to simulate line outages, called Current Injection Method (CIM), is proposed. The advantage of CIM lies in its simplicity. It is based on the fundamental current divider and superposition theorems, thus making it easily understandable. The matrix inversion lemma[87] or outage distribution factors[228] as used by many existing approaches are comparatively more complex. The CIM technique inherently requires less programming effort and as a result is more computationally efficient. A linear programming solution of a 'pure' economic dispatch problem available from the OCEPS research group is adopted and extended to incorporate the CIM concept to consider post-contingency system security. Tests demonstrate the superior computability of the method for large power systems.

Chapter 8 describes the notion of including post-contingency corrective actions in a security constrained economic dispatch. To date two approaches are published on the subject. These are the Benders decomposition[149] method proposed by Monticelli el al and a two step[183] approach proposed by Schnyder el al. They are reviewed and compared. The chapter extends the CIM technique introduced in chapter 7 to consider this most interesting and computationally demanding problem. A solution algorithm based on Sparse Matrix Dual Revised Simplex algorithm is proposed. Numerical examples including a test data set from the CEGB are included to demonstrate that the proposed methodology is computationally efficient and potentially applicable to large scale networks for real time operation.

Finally, Chapter 9 presents the main conclusions of the thesis and suggests areas of further research.

## 1.2 Contributions of the Thesis

The major contributions of this thesis can be enumerated as follows:

1. Introduction of a new DP technique for scheduling thermal generators in a power system.
2. Demonstrating that by formulating the DP recursive formula in a different state space framework, the well known "curse of dimensionality" problem associated with dynamic programming approaches can be overcome.
3. Introducing and testing that DP is applicable and computationally efficient to solve economic dispatch problems.
4. Demonstrating that with the use of the successive dynamic programming (SDP) technique, the storage requirement of the proposed DP algorithm can be further economized. Solution speed and precision of the SDP approach to the economic dispatch problem is demonstrated to be comparable to the popular LP and QP techniques.
5. Deriving a Current Injection technique for post-contingency real power flow estimation.
6. Introduction of a novel technique for the estimation of generator fuel cost penalty factors which is responsive to real time system topological changes, forecast load distribution and generation distribution.

7. Introduces and demonstrates that the LP approach for security constrained dispatch is computationally efficient and is capable of dealing with large realistic system.

8. Demonstrates that the short-term rating of transmission lines can be utilized for economic savings.

9. Production of a set of software routines for on-line, interactive power system operational cost optimization; consisting of:

   - Unit Commitment using DP and priority list approaches.
   - Economic dispatch using DP
   - Security constrained dispatch using LP
   - Security constrained dispatch considering the post contingency generation rescheduling capability of a system.

# CHAPTER 2

## CONSTRAINT MODELLING IN POWER SYSTEM OPERATIONAL COST MINIMISATION

The objective of the thesis is to investigate and to develop new solution algorithms for thermal power plant optimum commitment scheduling and economic dispatch. Surveys show that constrained mathematical optimization methods are employed extensively in the solutions of the two problems. In concise form, the two problems can be stated as follows:

Objective: Minimize { Total Generation Cost }

Subject to constraints :

- o Component limitations:
  - Generator operational characteristics
  - Transmission Network Capabilities
  - Load Prediction Accuracy
- o System requirements:
  - Generation/Load Balance
  - Security requirements

The constraints are due to the inherent technical limitations either of the components which make up the power system or the operational policies such as security requirements set down by the management according to long term economic strategy. They affect directly the production cost minimisation objectives. In this chapter, the essential elements of these constraints and the ways they can be modelled for incorporating into a solution scheme are reviewed.

## 2.1 Generator Operational Characteristics

Within a power system, many types of generating units using different prime mover designs and burning different fuel types exist. The efficiencies and relative fuel costs in terms of BTU/KWh of these different generator types dominate the loading merit order of these units in an electric power system. However, the cost and speed of starting up and shutting down of a unit, and the ramping up and ramping down capability of a unit also play an important part to economize the overall operating cost. In the following sections, the operational characteristics of thermal plants most frequently found in a power system are outlined.

### 2.1.1 Conventional Thermal Turbine Generators

A conventional steam turbine generator is shown schematically in Fig.2.1. The electrical output of the power plant is not only connected to the electric power system but also to the auxiliary power supply of the generating set. A turbine generator set requires typically 2 to 6%[229] of its gross output to power its auxiliary equipment such as fans, air heater, coal pulverizing mill, starting and stand-by boiler feed pumps, condenser circulating water pumps etc. In defining the input-output characteristic of a generating unit, the net output versus gross input is used since the power consumption of the auxiliary equipment is a necessary overhead to obtain the net output from the set. The net output is the electrical power measured in Megawatts available to the utility for sale. Fuel input is measured in Btu/hour. Given

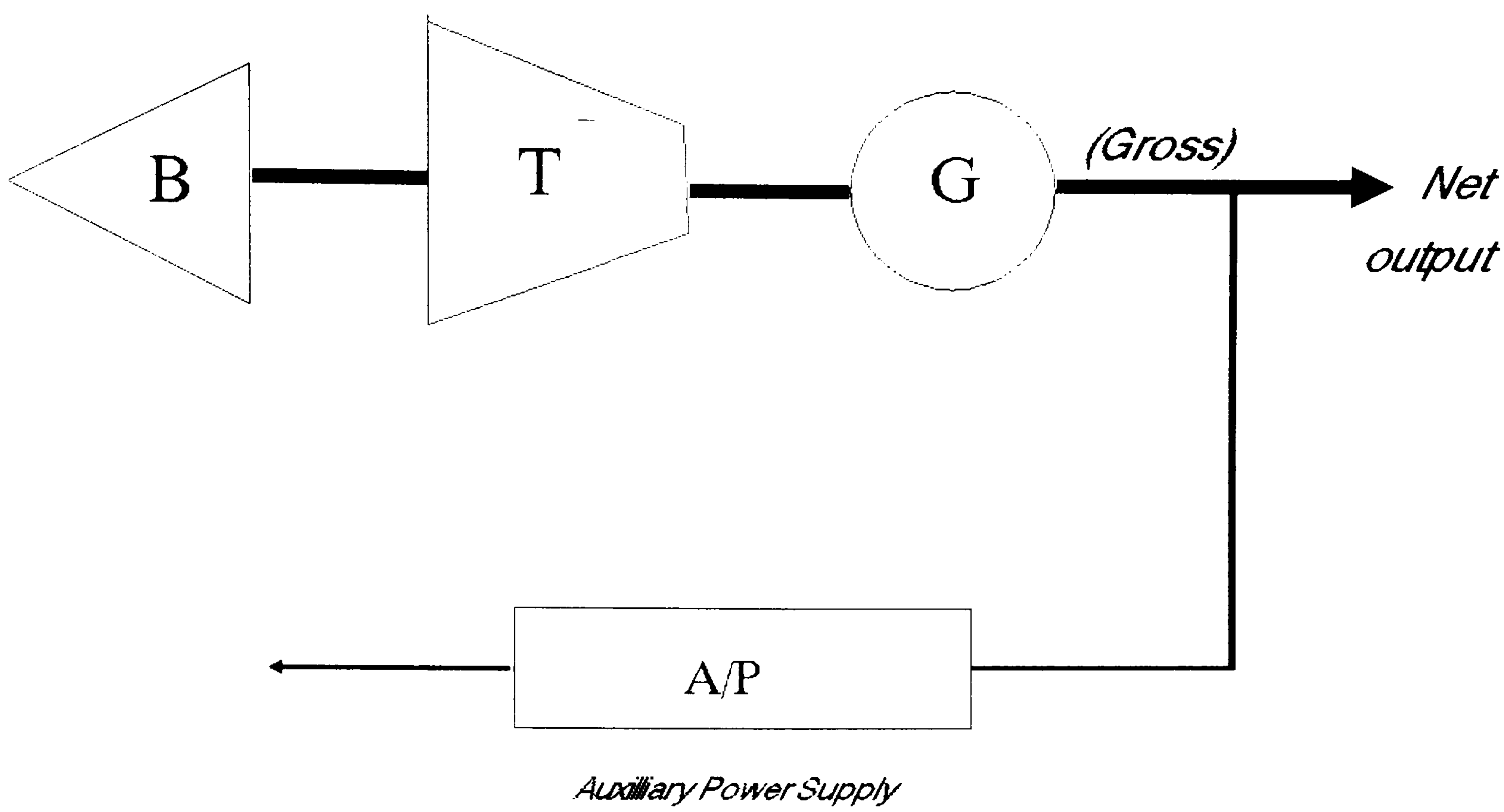



Fig. 2.1 Typical Steam Turbine Generator

the low heat calorific value of a fuel, cost per ton and plant thermal efficiency, a graph of the input fuel cost/hour against the output MW may be plotted over the operating region for each generating set. The operating cost of a unit may include the pro rata transport cost, fuel and ash handling costs, repair and maintenance costs if they can be expressed as a function of the power output of a unit. Fig.2.2(a) shows a typical cost curve of a multi-valve steam turbine generator. The input-output characteristic may be obtained from design calculations, from heat rate tests or simply from the manufacturers. However, as time goes on different parts of the generating set become aged, components and auxiliary equipment changed or modified, fuel quality may also be different[78]. The unit should therefore be re-tested from time to time to obtain a more faithful representation of its fuel efficiency. With the dramatic increase in fuel prices and advance in monitoring technology, on-line plant performance calculations are increasing popular[220]. This coupled with ever improving design the large coal fired generating plants now have a thermal efficiency reaching 39%[44]. For smaller and older sets, the thermal efficiency can be as low as 19%. Generally the large new and economic generating sets are base loaded to provide as much energy as possible to the system. The expensive old sets are used for peak lopping and load following duties.

The cost curve in Fig.2.2(a) is often approximated by various forms for easy analysis. A number of approximations have been suggested. Navarro[157] had the opinion that the approximating polynomials should have no more than third order

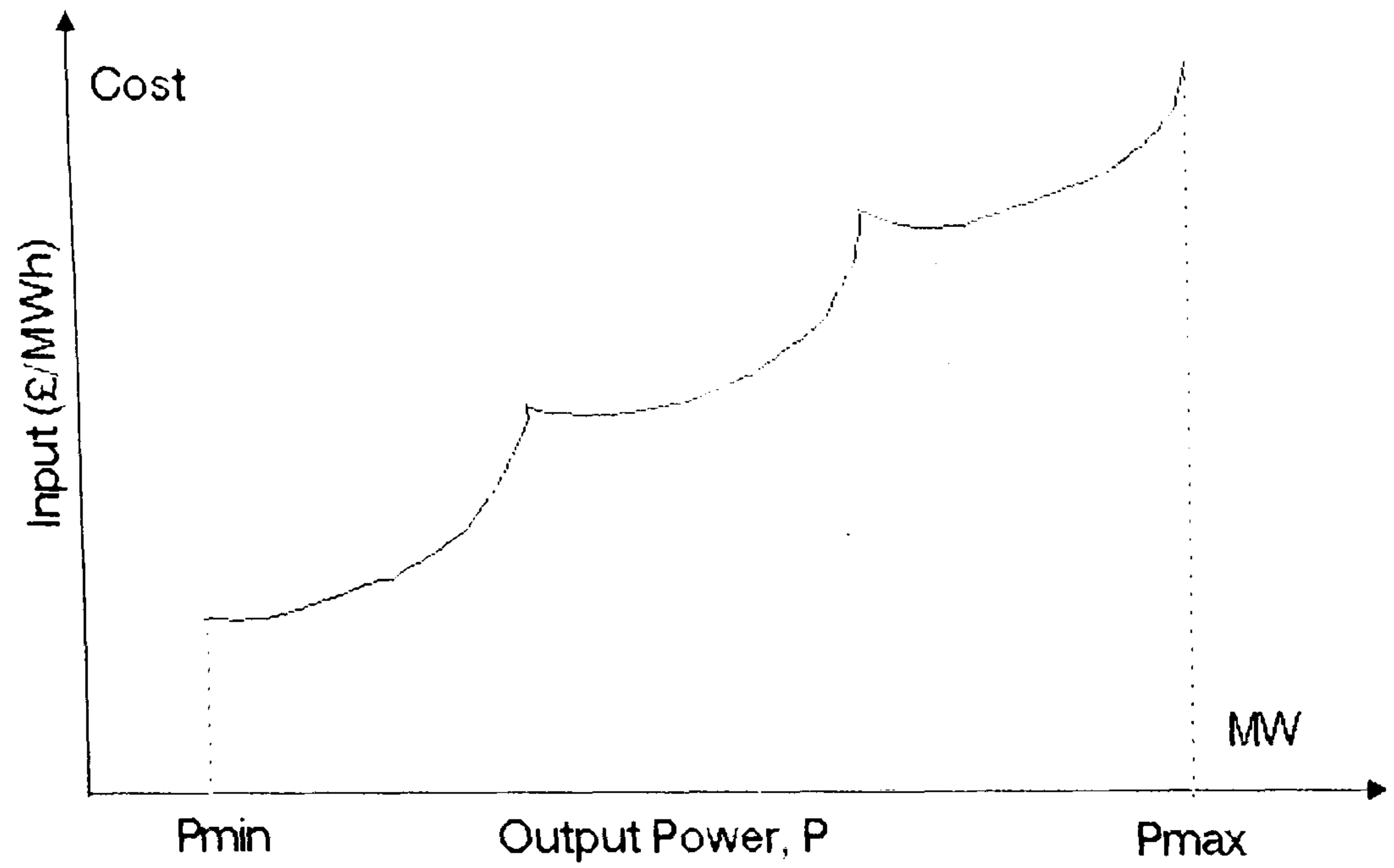



Fig. 2.2(a) Typical Input-Output Characteristic of a Multi-valve Steam Turbine Generator

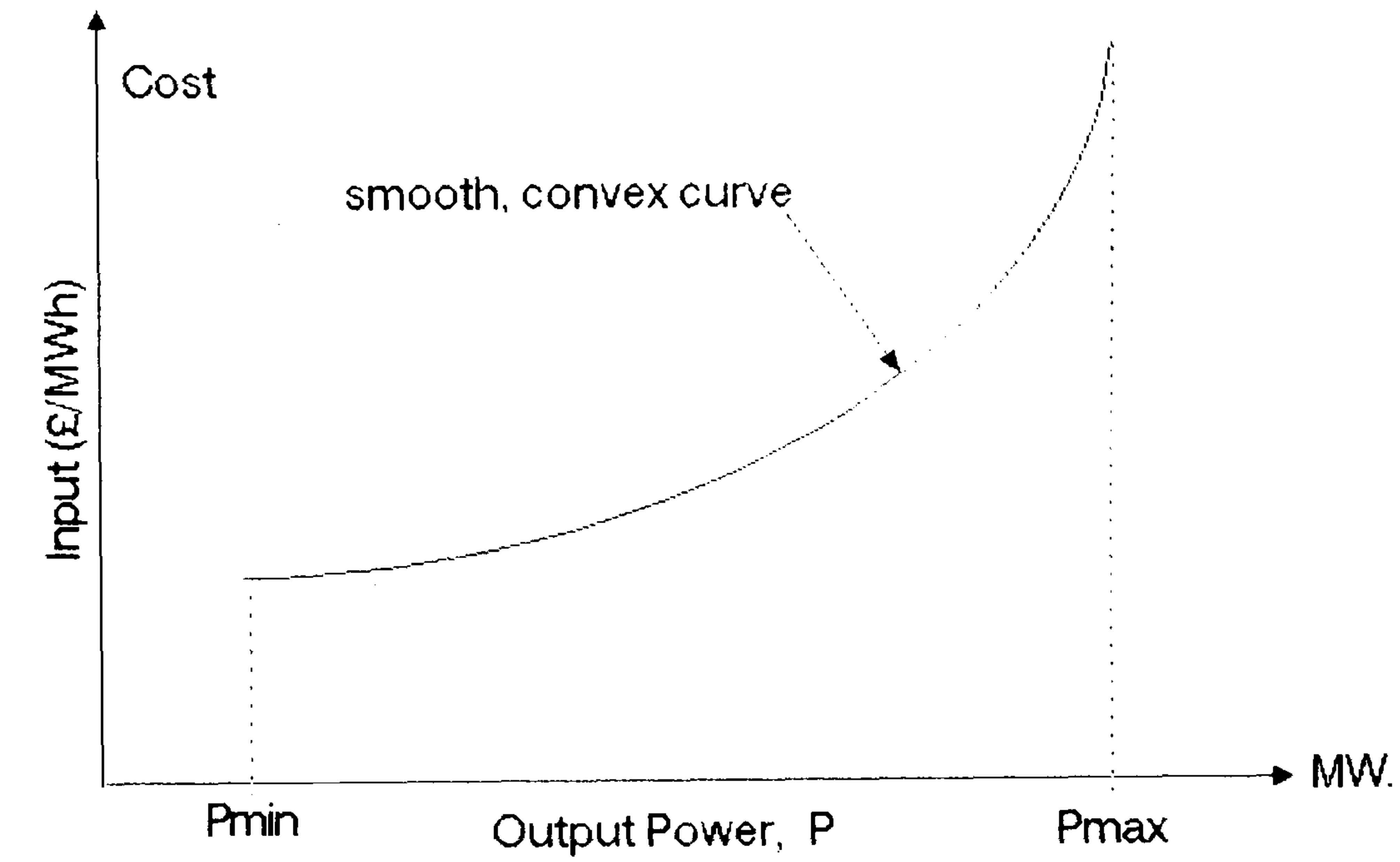



Fig. 2.2(b) Quadratic Cost Representationr

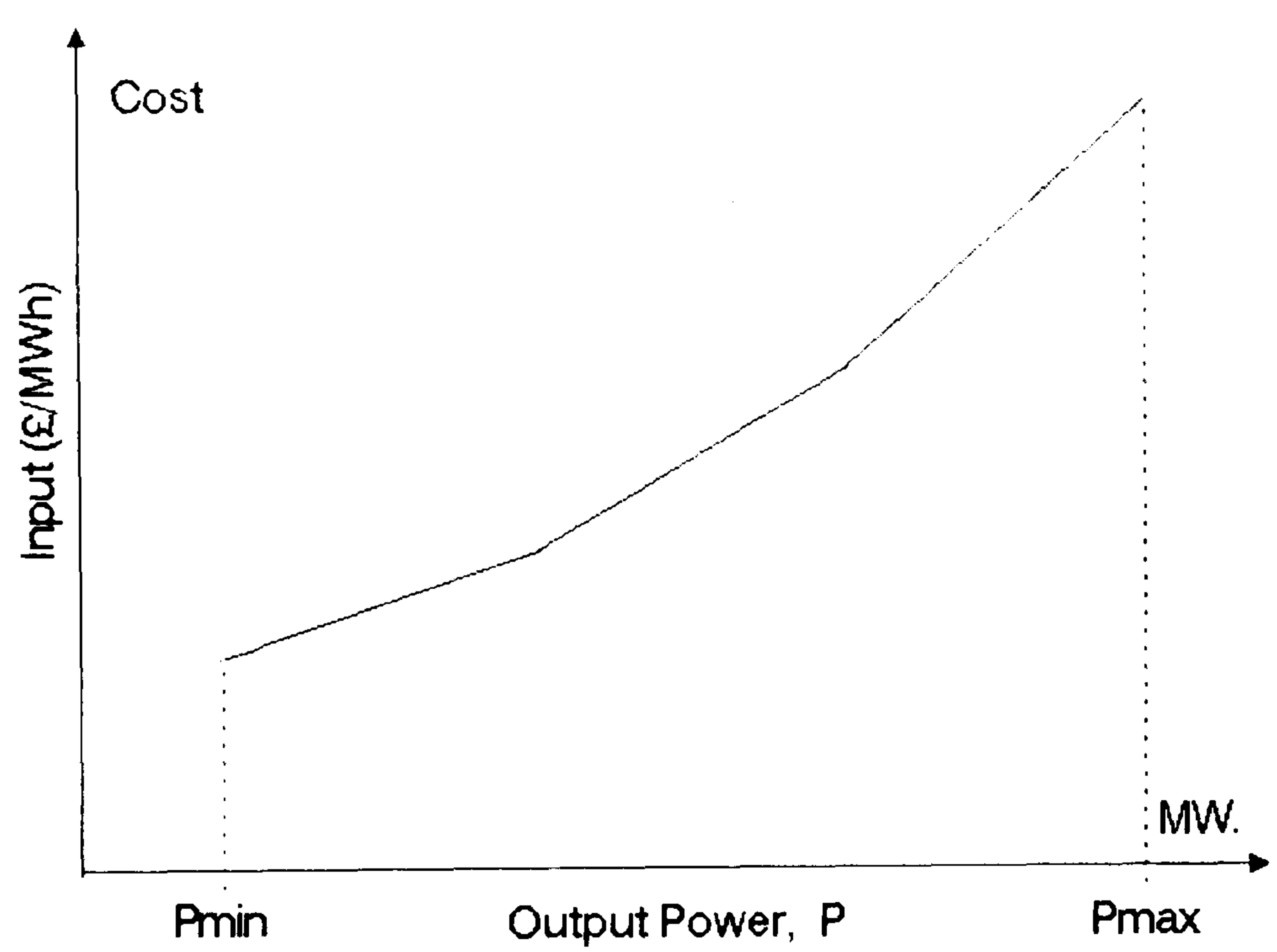



Fig. 2.2(c) Piece-wise Linear Cost Representation

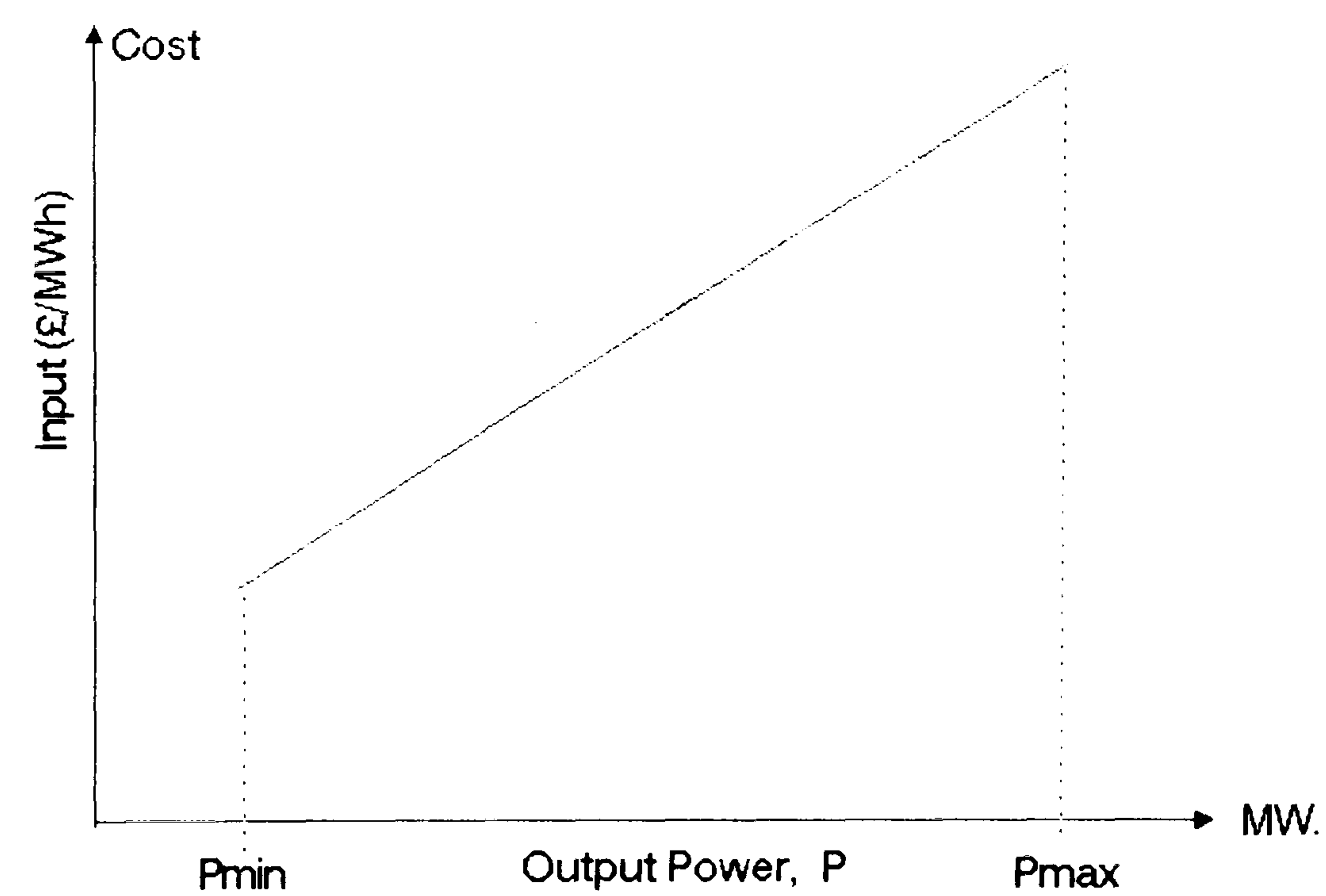



Fig. 2.2(d) Linear Cost Approximation

Fig.2.2 Typical Operation Cost Characteristics of a Turbine Generator terms. The popular approximations are a smooth convex curve as shown in Fig.2.2(b), piece-wise linear as in Fig.2.2(c) or simply a linear function as in Fig.2.2(d). The incremental fuel cost characteristic of a generating unit is of great importance in the operating cost minimisation problem. This incremental fuel cost is the slope or the derivative of the cost function. The incremental cost characteristic of the units in Figs.2.2(a),(b),(c),(d) are shown in Figs.2.3(a),(b),(c),(d) respectively. In the United Kingdom, the turbine generators are mainly single throttle valve units. The cost curve is only slightly non-linear. The representation in Fig.2.2(d) and Fig.2.3(d) are found to be adequate to describe the cost and output relationship of the units.

Another consequential input-output characteristic of a turbine generator is its efficiency. Fig.2.4 shows the efficiency curve of a typical turbine generating set. Notice that the best efficiency is achieved near the rated power of the unit. In the U.S.A. this best efficiency point is designed to be at approximately 80% to 90% of the unit's maximum capacity. This operating characteristic has a significant effect on the operation of the system. When a unit is operating at its maximum efficiency and below its maximum output, there will be spinning reserve available from this unit. As a result, the system may be more secure. A steam turbine generating set has a typical overall efficiency of 20% to 40% depending on design and age of the set. A synchronous alternator normally has an efficiency over 98%. The main loss of efficiency is therefore in the process of converting the chemical energy in the fuel into heat energy in

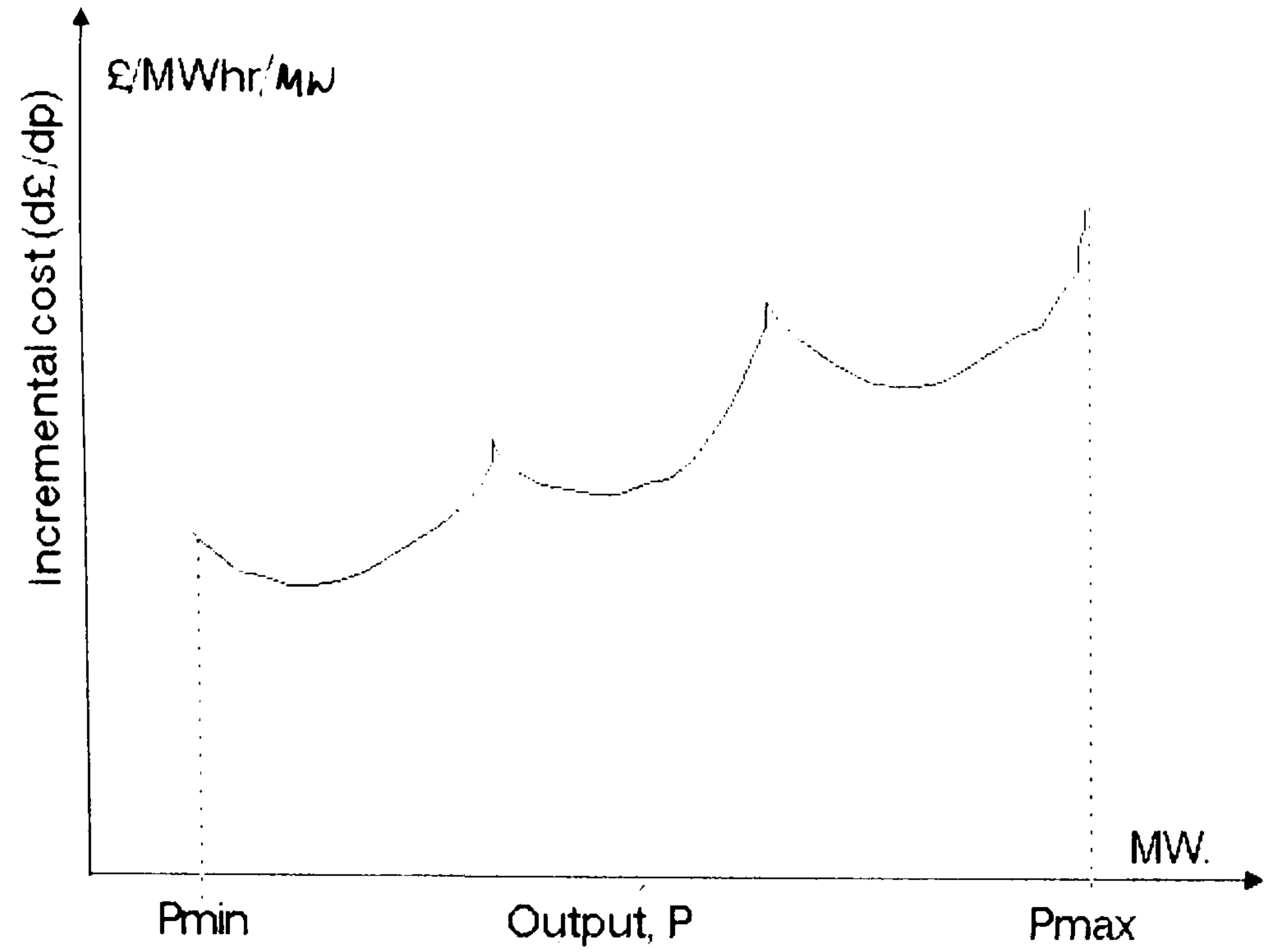



Fig. 2.3(a) Incremental Cost Function for a Multi-valve Turbine Generator

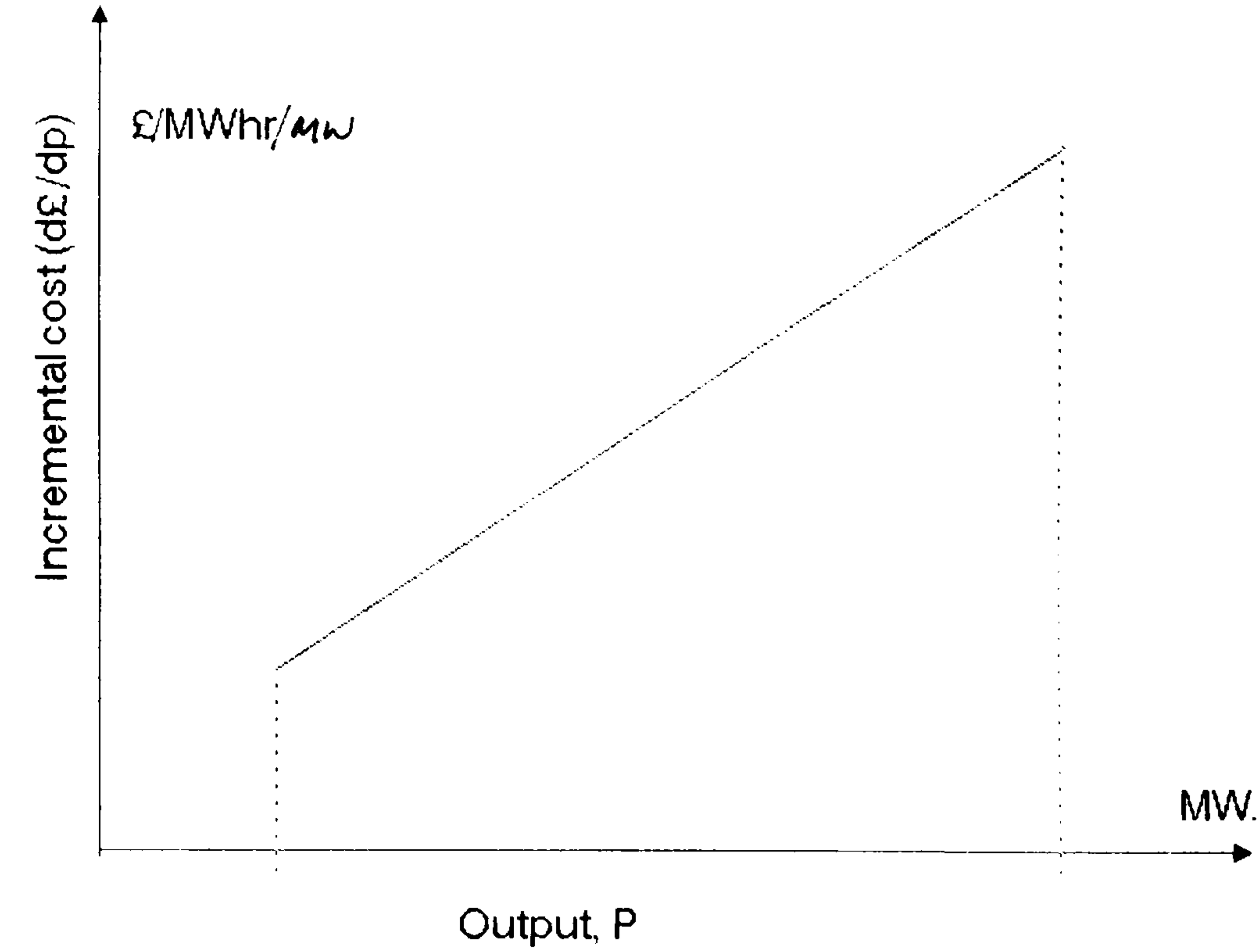



Fig. 2.3(b) Incremental Cost Function of a Quadratic Cost Function for a Typical Turbine Generator.

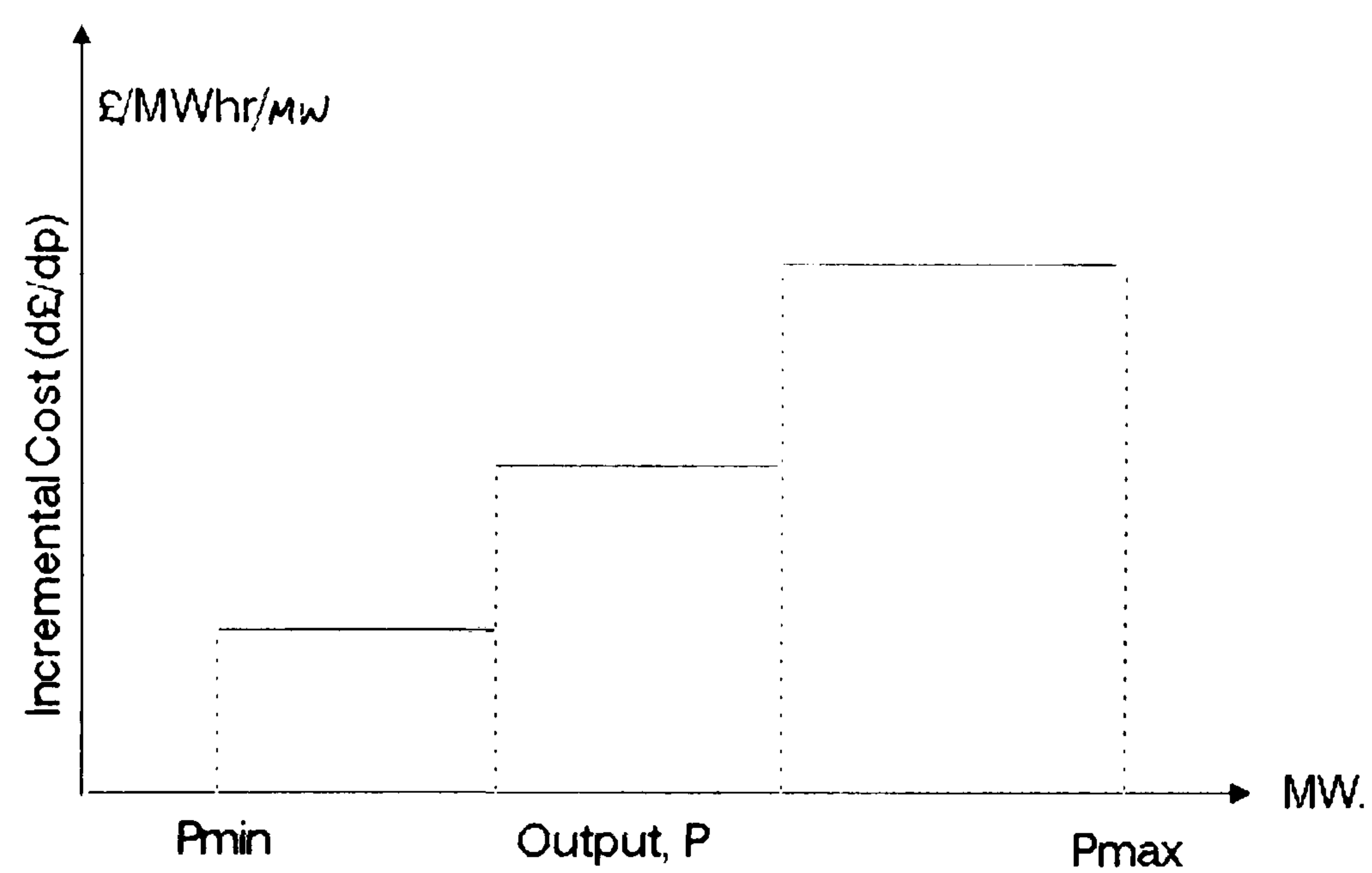



Fig. 2.3(c) Incremental Cost for a Piece-wise Linear Cost Function

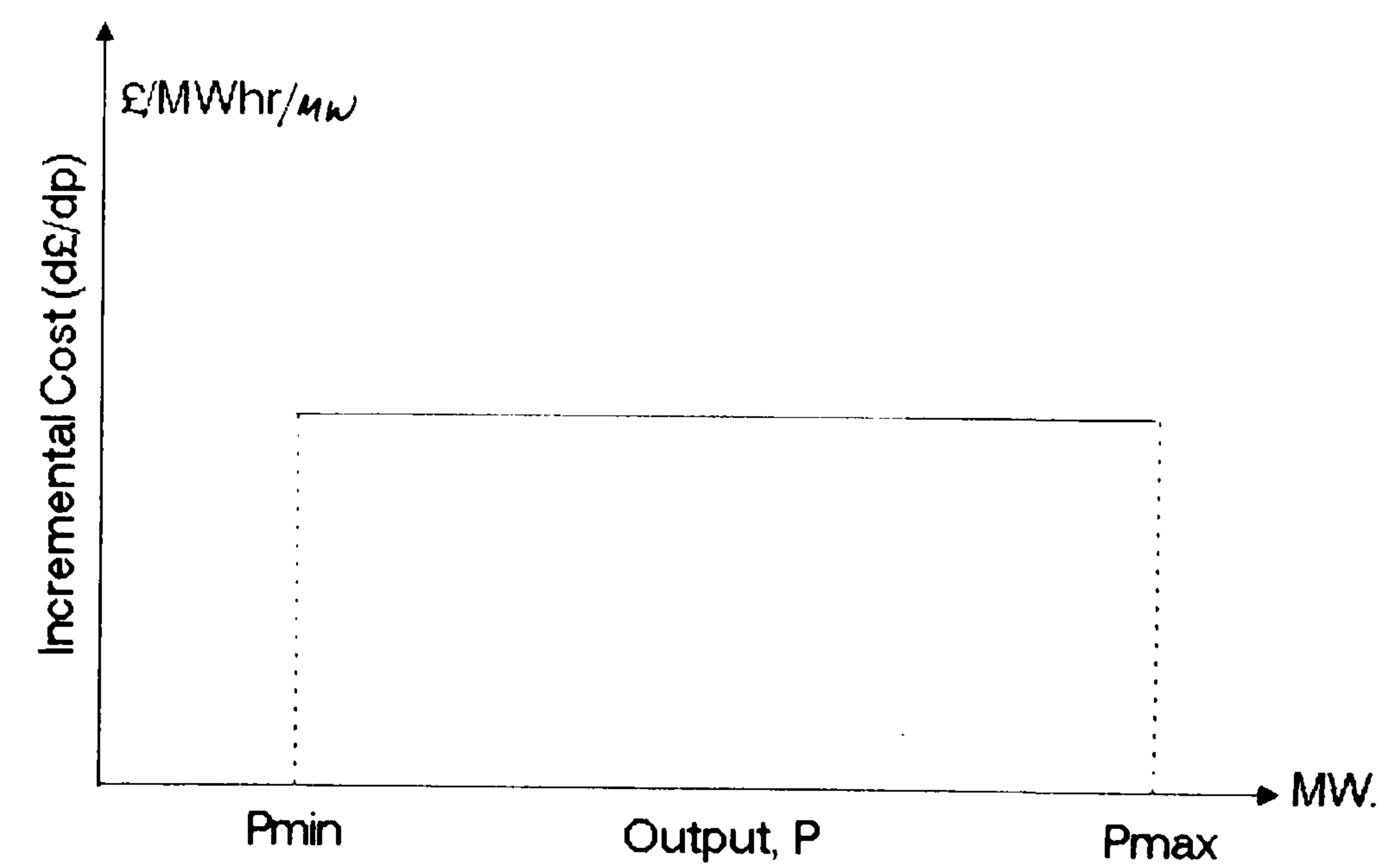



Fig. 2.3(d) Incremental Cost Function for a Linear Cost Function

Fig.2.3 Typical Incremental Cost Representations

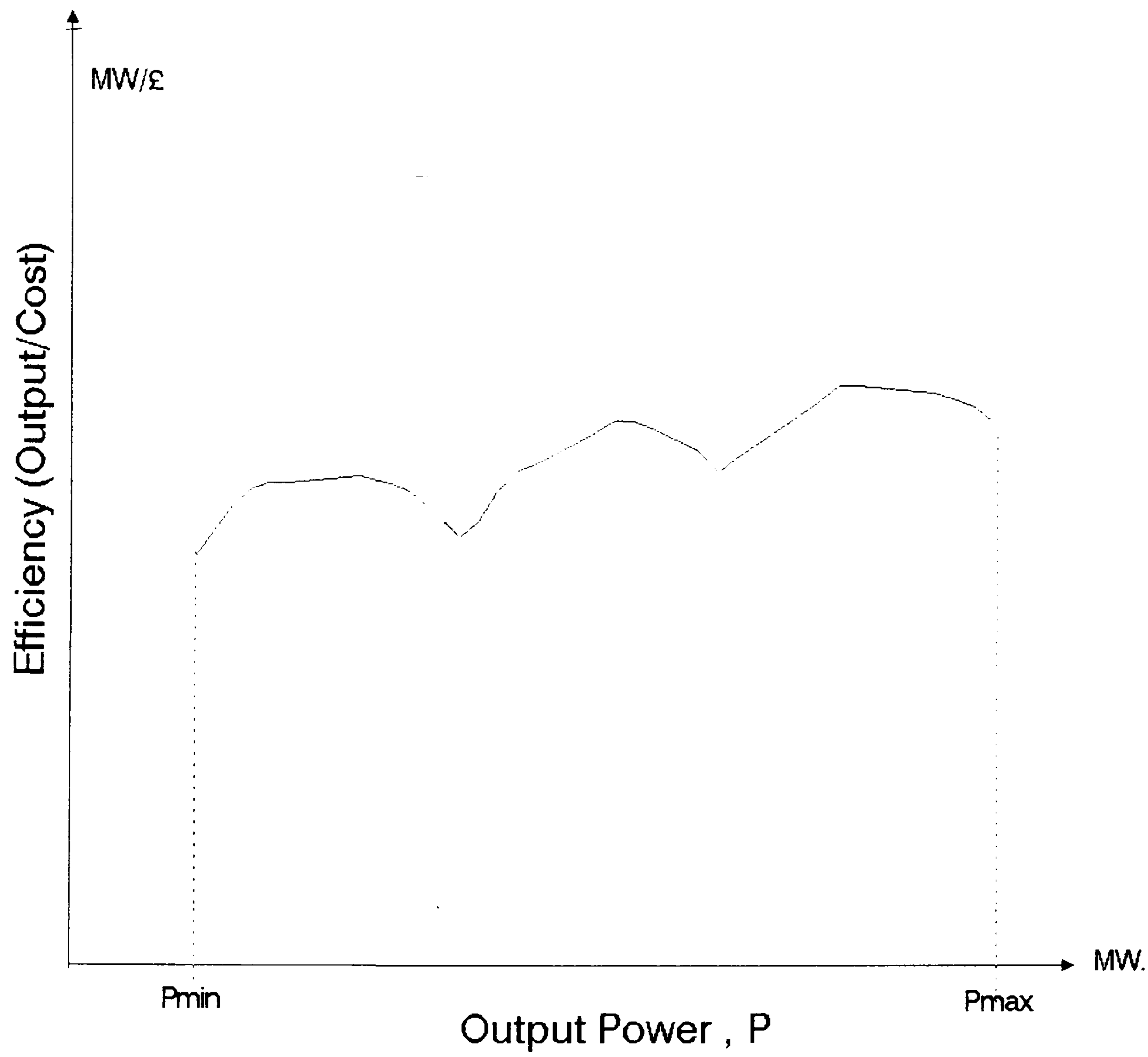



Fig. 2.4 Efficiency of a Typical Turbine Generator.

the steam and then into mechanical rotating energy in the shaft of the turbine. When a steam turbine generator is synchronized on to the system, there are several critical constraints governing its operating regime:

**(a) Minimum Power Output**

Generally, a synchronous alternator does not have a minimum output limitation. Its terminal can be opened and not supplying any load at all. The turbine also inherently does not have a minimum loading requirement. The only critical parameters for the turbine are its shell and rotor metal differential temperatures, exhaust hood temperature, rotor and shell expansion and thermal stress. These are not related to the absolute output of the turbine but rather to the rate of output changes. The minimum loading limitations are basically caused by the fuel combustion stability and inherent boiler design constraints. For example, most superheated units cannot operate below 30% of their rated capacities. A minimum flow of 30% is required to cool the tubes in the boiler adequately. The minimum real power limitation may be expressed as:

$$[P^{min}] \leq [P] \tag{2.1}$$

where $[P]$ represents a vector of the generator real power outputs and $[P^{min}]$ the corresponding minimum MW limits.

**(b) Maximum Power Output**

When a synchronous alternator is connected to a power system, it has two upper operating limits, namely a stability limit and a power limit. To ensure the safe operation of a power system, it is vital to realise the difference between them. For the power limit if not exceeded by a large amount, the synchronous generator will continue to function for an appreciable time although the unit might be overheated and cause its winding insulation to deteriorate. Overloading will shorten the expected life of a synchronous alternator but may not cause an immediate problem to the system. The stability limit, however, if exceeded even for a short time will cause loss of synchronism. As a result, there will be a large surge of power whereupon the protective relay will disconnect the generator from the system. The loss of a generator from the system is extremely undesirable. This may cause unbalance of generation to load demand. The load flow of the system also will be affected and possibly causing one or more transmission lines to overload and be tripped out.

Unit commitment and economic dispatch studies the production cost of the system for a time horizon of 5 minutes to 24 hours. Transient stability is not considered. The steady state stability limit is outlined in the following.

A generator may be modelled by a simple network in Fig.2.5[73], where $E_q$ is the generator internal voltage, $x_d$ is the synchronous reactance with resistance considered negligible and $\delta$ is the generator voltage phase angle. From the generator terminal, the power system is a vast network of interconnected transformers, lines and other generators. The external network can be treated as an infinite bus with an ideal voltage source. Its Thevenin equivalent is also shown in Fig 2.5 where $E_t$ is Thevenin equivalent system voltage and

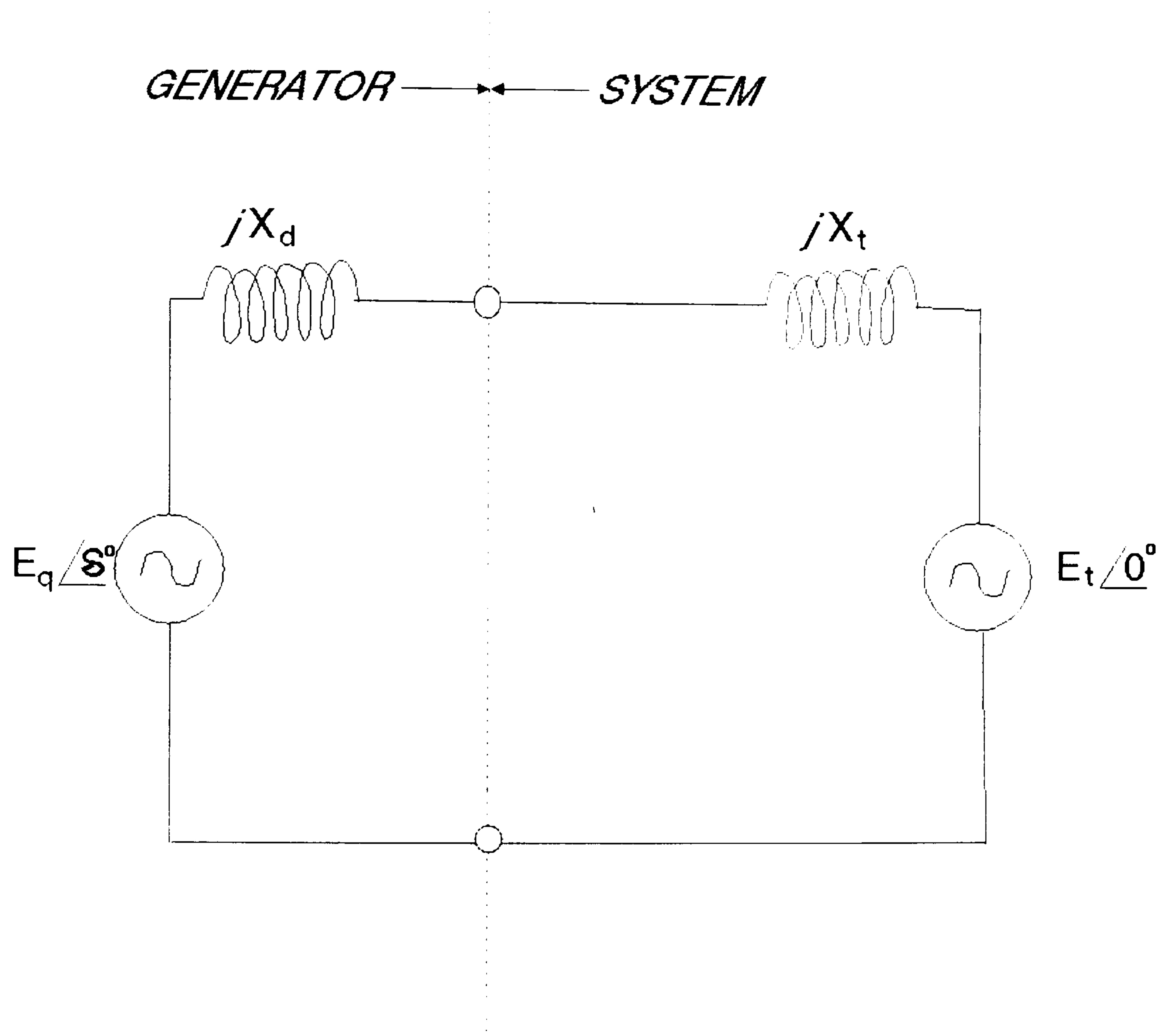



Fig. 2.5 Ideal Generator Connected to a Simplified Network

$x_t$ is Thevenin equivalent system reactance. Resistance is considered negligible. The electrical power, $P_e$, transferred from the generator to the power system is

$$P_e = \frac{E_q\ E_t}{x_d + x_t} \sin \delta \qquad (2.2)$$

The rotating speed of the turbine-generator mass, neglecting damping, is related to electrical demand by the following equation.

$$P_m - P_e = M \frac{d^2\ \delta}{d\ t^2} \qquad (2.3)$$

where $P_m$ is the turbine mechanical power and M relates to the rotating inertia of the turbine-generator. When $P_m$=$P_e$, $d^2\delta/dt^2$=0; and the system is in equilibrium. $P_m$ is a constant for steady state. The functions of $P_e$ and $P_m$ are plotted in Fig.2.6. Observe that for a given value of $P_m$, there are two equilibrium values of $P_e$ corresponding to $\delta_1$ and $\delta_2$. When the generating set operates at $\delta_1$, a sudden increase of $\delta$ and hence $P_e$ due to some system disturbances will cause $d^2\delta/dt^2$ in Eq.(2.3) to become negative. Therefore, the generating unit will respond by decreasing $\delta$ and returning to $\delta_1$. However, if the unit is originally operating at $\delta_2$. A small increase in $\delta$ will make $P_m > P_e$ and $d^2\delta/dt^2$ is positive. The unit responses by further increases in $\delta$ and moves further from $\delta_2$. It is therefore clear that the power output of the turbine generating set should not be greater than $P_\delta^{max}$ indicated in Fig 2.6. From Eq.(2.3), it can seen that $P_\delta^{max}$ depends on the generator internal voltage which is a function of the

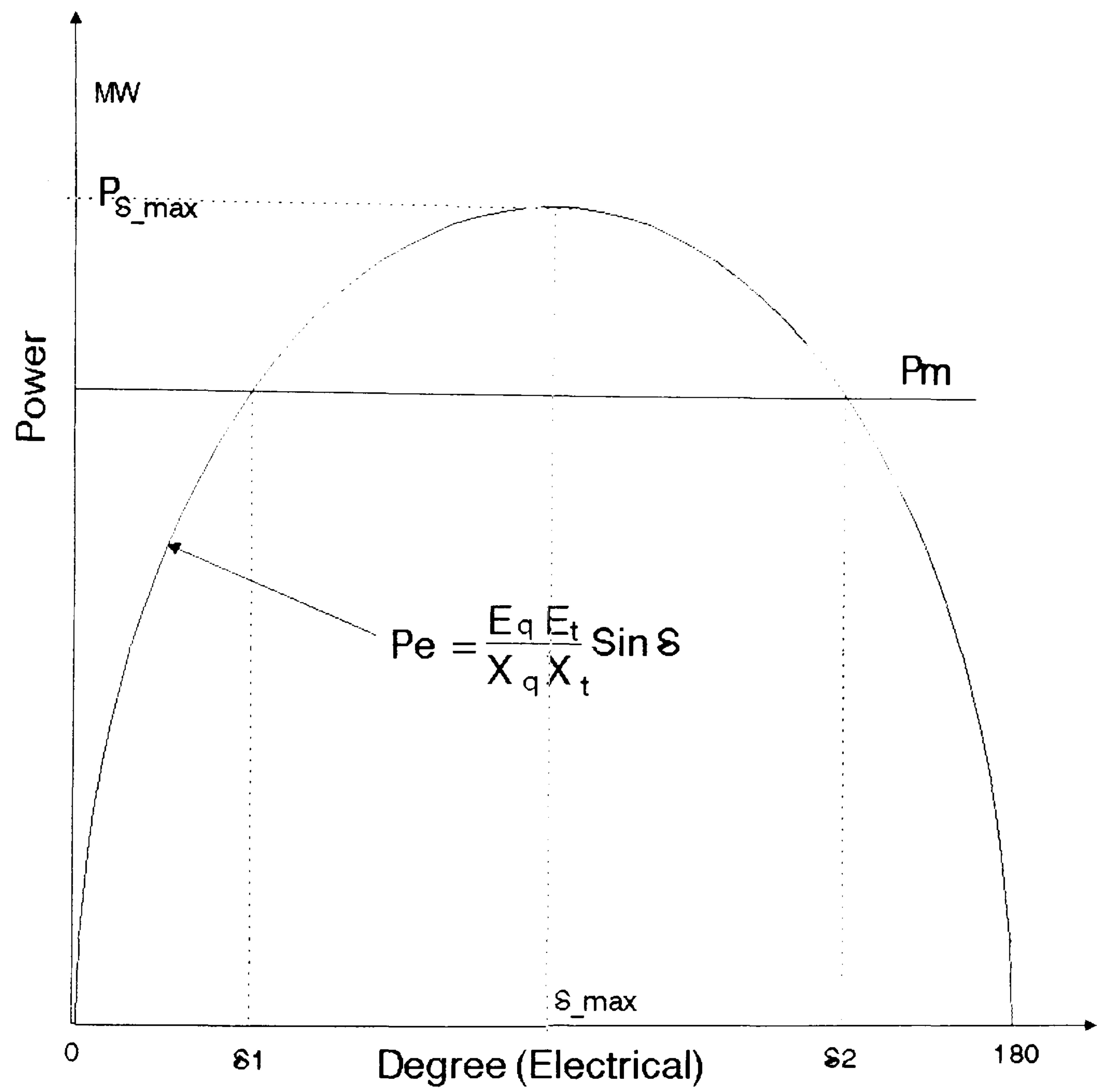



Fig. 2.6 Power Transfer Between a Generator and a Network

generator reactive power output and the external system Thevenin equivalent voltage and reactance which in turn are functions of system configuration and state. For a given system condition, the allowable real and reactive power output of a boiler-turbine-generator is limited by both the synchronous generator and the turbine. Fig.2.7 shows the operating regime of a typical steam turbine generating set. The copper loss in the stator winding limits the maximum armature current. This limits the MVA rating of a generator to semicircle a-b-c-d-e-f-g in the figure. The copper loss in the rotor winding determines the maximum field current. This in turn fixes the excitation limit and hence the reactive power output. This constraint is indicated by arc i-j-b. The turbine has a rated power output. The real power output of the alternator is therefore limited by the capacity of the turbine shown by arc c-n-e. Stability consideration described above form the boundary f-k-h. These together with minimum stable real power output constraint reduce the operating regime of a power plant to the area enclosed by arc j-b-c-n-e-f-k-m-j. The operating regime of a generator plotted in Fig.2.7 is known as a 'capability chart'[197]. For simplicity the maximum real power limits are generally expressed as:

$$[P] \leq [P^{max}] \tag{2.4}$$

where $[P^{max}]$ represents a vector of generator maximum MW outputs.

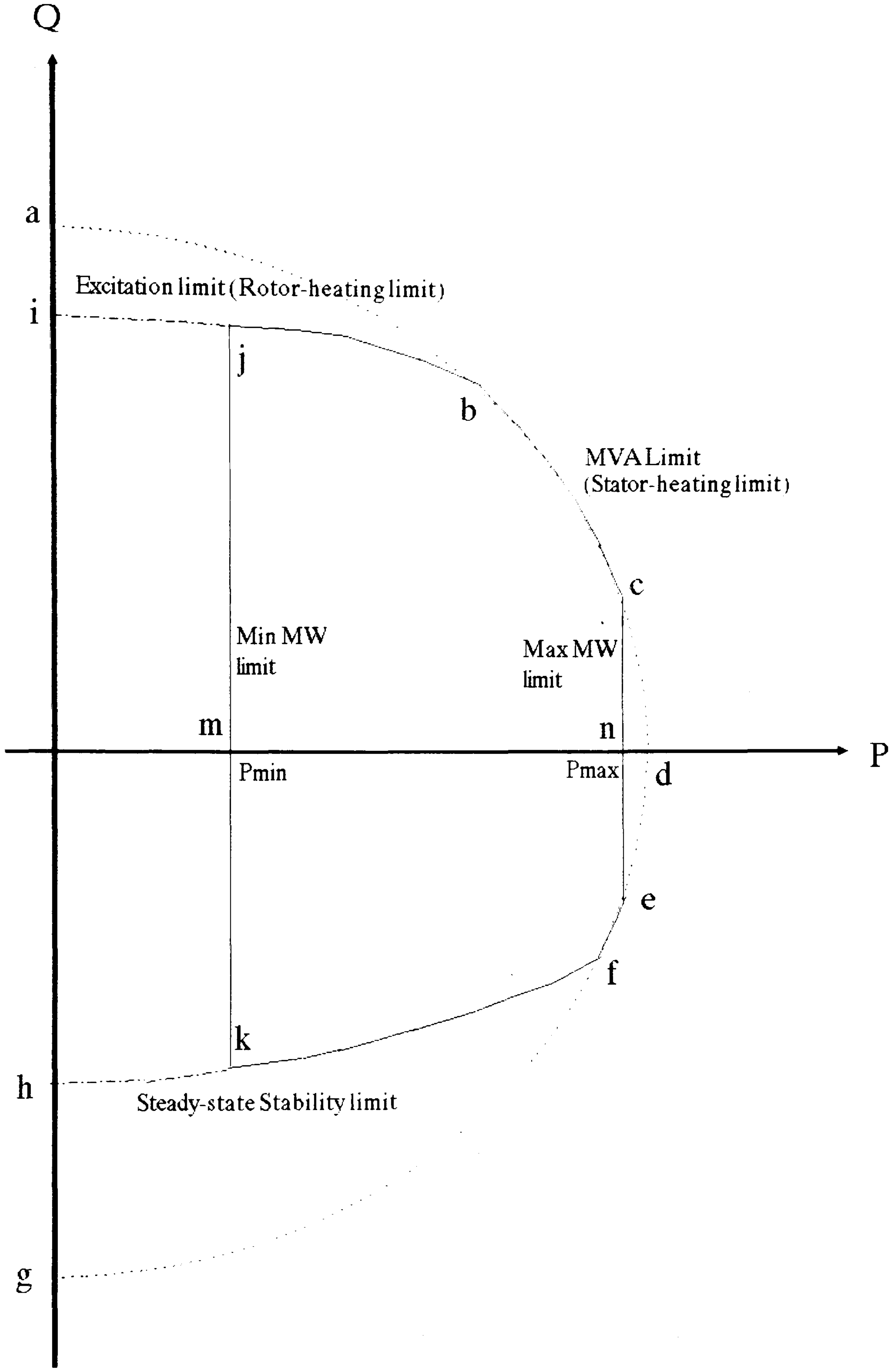



Fig.2.7 Turbine Generator Operating Limits

### (c) Maximum Rate of Change

The loading of the generating units is carried out by adjustment of the turbine governor set point. The rate at which a turbine generator can change its power output is determined by the characteristics of the generating plant and the ability of the boiler to supply steam at the temperature required to match the thermal condition of the turbine at its inlet. The transient thermal stresses in the turbine and the boiler must be limited to prevent thermal fatigue (cracking) of the major components [38,139]. In the boiler fuel control, there is the limiting factor of increasing the rate of fuel infeed from the mills and the time lag to convert the chemical energy of coal to heat energy. In the United Kingdom, the rate of generator output increase ranges from 0.1% to 3% of manufacturer declared capacity per minute for light loaded units and 5% to 10% per minute for heavier loaded units [78,99]. The boiler tends to be the limiting factor at the higher initial loads and the turbine at the lower initial loads. The rate of change to increase generally is not the same as for decreasing. The rate of decrease can be 30% higher than the rate of increase [194]. These ramp rate limits may be specified by:

$$[P_{decrease}^{max}] \leq [P_{ramp}] \leq [P_{increase}^{max}] \qquad (2.5)$$

where $[P_{ramp}]=[dP/dt]$ and $[P_{increase}^{max}]$, $[P_{decrease}^{max}]$ are vectors representing the maximum increasing and decreasing rates respectively. $[P_{decrease}^{max}]$ are negative values.

### (d) Start Up and Shut Down of Steam Turbine Generators

For economic reasons, efficient units, usually new and larger, generally will be base loaded so that these units will provide as much as possible of the energy requirement of the system. The less efficient units are used to satisfy the peak load demand and perform load following duty. With the increased amount of nuclear generation in the United Kingdom which is primarily for base loading, it is necessary for all conventional generating plant to be designed to be suitable for two-shift operation, also known as load cycling. Two shift operation implies that the plant is required to be started up during periods of peak system demand followed by overnight shut-down periods of about 6 hours[39,78,194]. During this brief overnight shut-down, the boiler tube system cools down much faster than the well-insulated turbine casings. Consequently, during start-up, the boiler is slow in building up matching superheat and reheat system steam temperature. After overnight shut-down, the cool steam produced after lighting-off the boiler cannot be accepted by the turbine because of the temperature mismatch between the cool steam and turbine metal. Such temperature mismatch will produce thermal stresses severely shortening turbine life. In older units, it may take several hours from igniting the boiler to start rolling of the turbine. The minimum time requirement to start up a steam turbine generator depends on how long the units have been shut down. Cold starts can take over 9 hours from first firing the boiler to attain full output. For hot start, even equipped with advance computer control and thermal stress prediction technique[138,139], it will still take over one hour to bring the unit from shut down to synchronism. Table 2.1 shown some typical start up times for various sizes of generating sets[46].

**Table 2.1** Typical Turbine Generator Start Up Time

| Approx. Shut Down Period | Time from first burner ignition to start of run-up | Run-up time to Synch. | Time from Synch. to 100% load |
|---|---|---|---|
| UNITS 250-400MW | | | |
| >1 week | 3 hours | 90 min | 190 min |
| 72 hours | 3 hours | 45 min | 145 min |
| 36 hours | 1.25 hour | 20 min | 90 min |
| 8 hours(2 shift) | 1 hour | 5 min | 25 min |
| 2 hours* | 1 hour | 5 min | 10 min |
| UNITS 500-660MW | | | |
| >1 week | 3.5 hours | 120 min | 200 min |
| 72 hours | 3.5 hours | 60 min | 165 min |
| 36 hours | 1.5 hours | 30 min | 110 min |
| 8 hours(2 shift) | 1.25 hour | 5 min | 25 min |
| 2 hours* | 1.25 hour | 5 min | 10 min |

* Fossil units cannot normally restart in less than 1.5hr unless by-pass[39] provided.

The start up process of a steam turbine typically can be divided into four phases.

1) Boiler ignition phase.
2) Turbine speed rise phase from rolling off to the synchronizing speed via heat soak operations at several predetermined speed levels.
3) Synchronization to the grid and succeeding initial load hold for further heat soak operations.

4) Load rise phase from the initial to the desired output levels.

The four phases of starting up a steam turbine generator is depicted in Fig.2.8.

Associated with the long start up time of a steam turbine generator, there is the start up cost which also depends on the length of time a unit has been shut down. In addition to the labour cost of the operation crew, fuel consumed by the time the alternator was synchronised can be over 60 tonnes of coal, representing £10000 at 1989/89 fuel price[78], for cold starting a 660MW coal plant. When a boiler shuts down, its temperature can be approximated by an exponential drop with time. A common representation[194] of the start-up cost of a unit is therefore given by

$$C_{su} = C_{csu}(1-e^{-\alpha(t-1)}) + C_{tsu} \tag{2.6}$$

where

$C_{csu}$ = cold start up cost;

$C_{tsu}$ = cost of start up of the turbine alone including operation crew labour, maintenance cost;

$\alpha$ = cooling time constant of the boiler;

$t$ = number of hours since the unit was shut down

Instead of shutting down the generating set completely, the boiler can also be banked. That is fuel is continued to be supplied to the boiler to maintain its pressure and temperature. This takes advantage of the different cooling speeds of the boiler and turbine. When the boiler is banked, the unit may be brought on line after very short notice as indicated in Table 2.1. The fuel required to maintain the

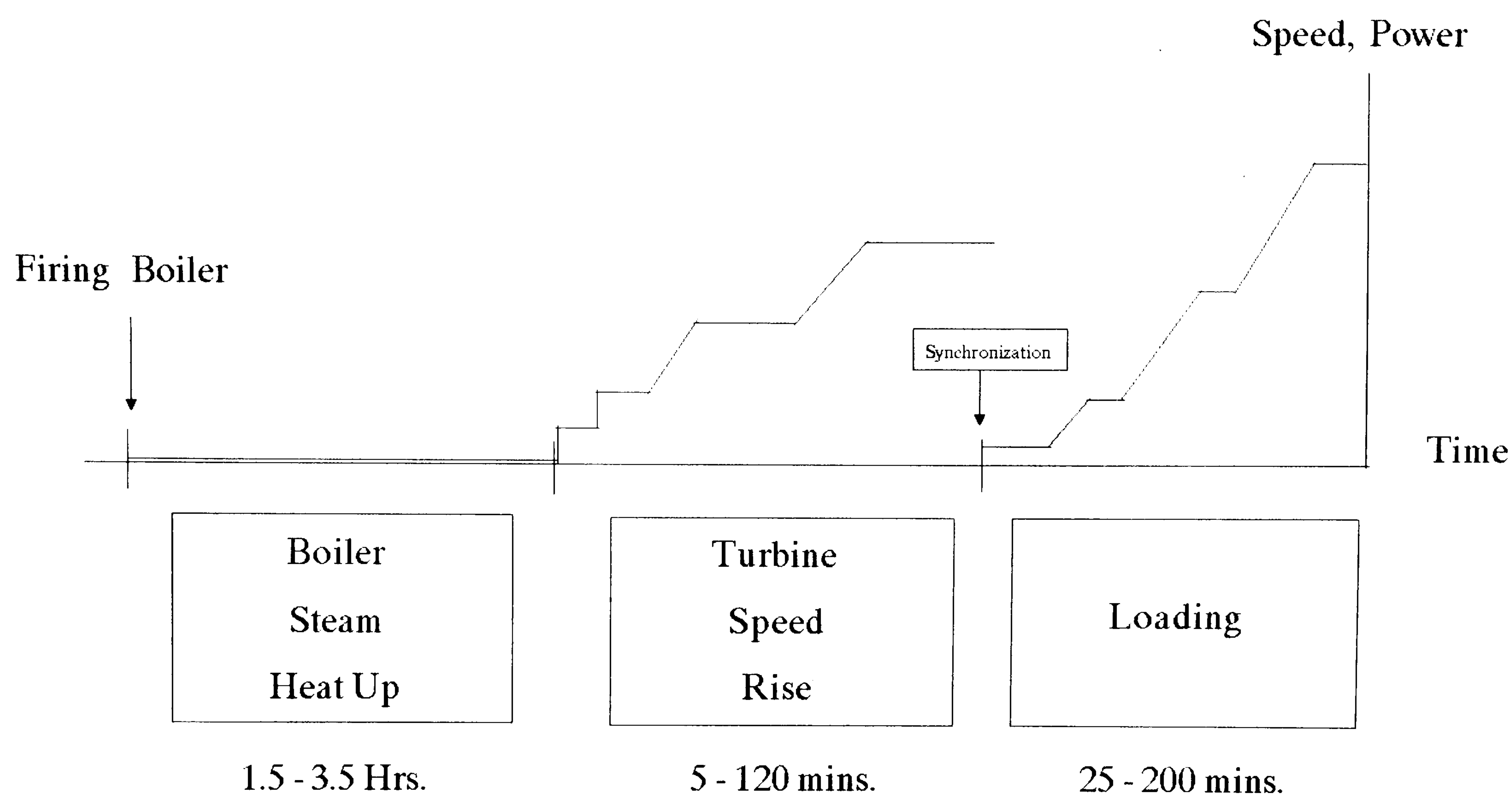



Fig. 2.8 Typical Start Up Process of a 2-shift Steam Turbine Generator

boilers pressure and temperature are constants, $C_b$, and consequently the cost contributable to the next start up is given by

$$C_{su} = C_b(t-1) + C_{tsu} \tag{2.7}$$

The decision whether to shut down or bank a boiler is determined by the length of the shut down period as can be seen from Fig.2.9. Shutting down a turbine generator in comparison to start up is quicker and costs much less. The typical time requirement from 100% load to off load is 20-30 minutes. The shut down involves the loss of the residual fuel in the boiler, and work crew labour cost. Shut down cost is much less than that of starting up and can be considered as a constant, $C_{sd}$.

### 2.1.2 Nuclear Plants

The fuel cost of a nuclear plant does not directly relate to combustion efficiency, but rather to economic and accounting considerations on the investment to produce the fuel rod assembly. This investment includes the cost of mining the uranium, milling the uranium core, converting it into a gaseous product that may be enriched, fabricating fuel assemblies, delivering them to the reactor plus the cost of removing the fuel assemblies after they have been irradiated and storing them. Each fuel assembly generates a given amount of electrical energy. A pseudo fuel cost is then obtained by dividing the total investment by the expected amount of electrical energy that can produce from this fuel assembly.

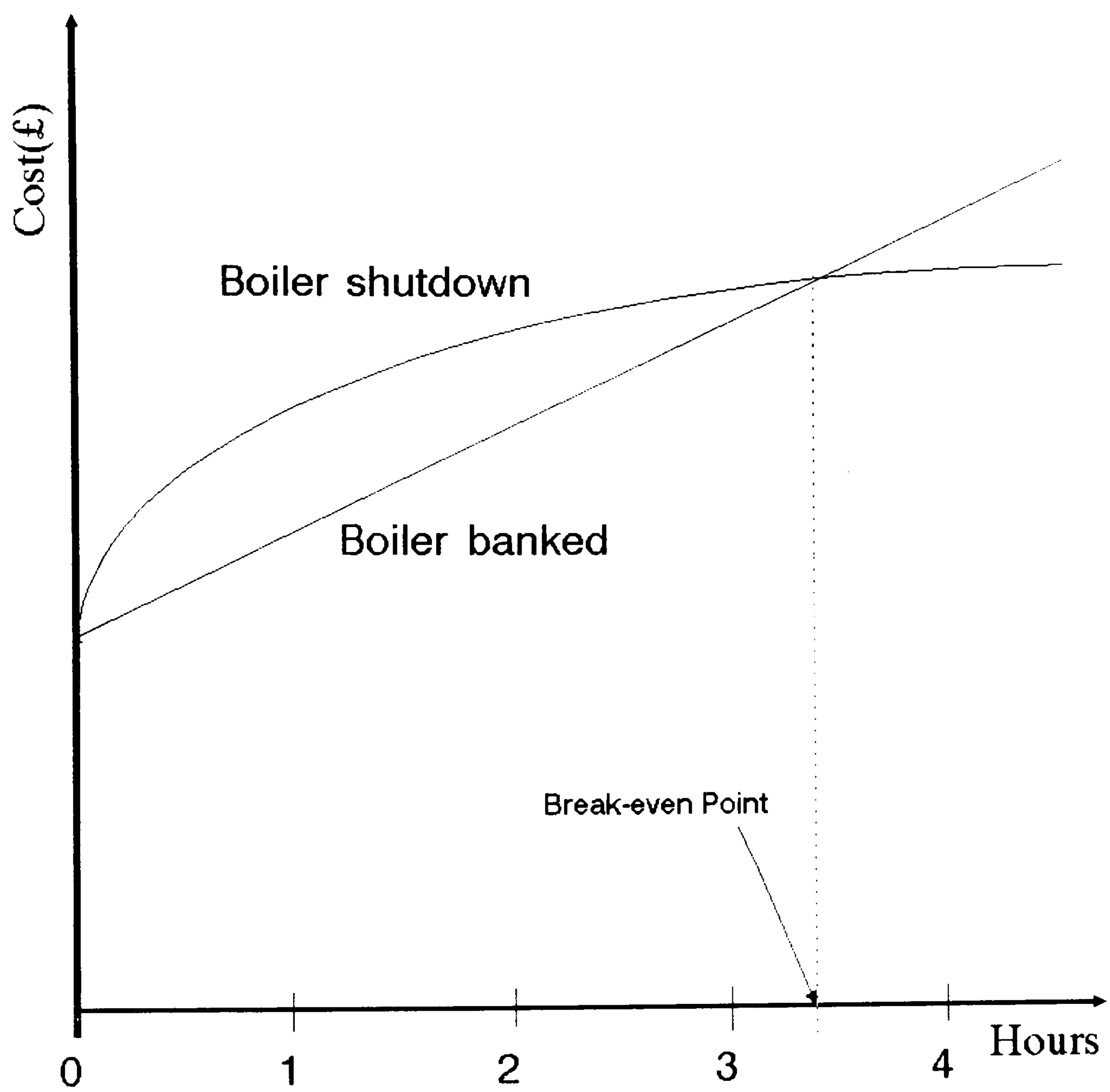



Fig. 2.9 Start Up Cost of a Steam Turbine Generator

The fuel cost curve of a nuclear plant is therefore a linear function.

Historically, nuclear power plants have been utilized in a base loaded mode. This role was prompted by fuel economics and not by any inherent design limitations. Nuclear plants produce electricity for an operating cost of about 1.5p/KWh as compared to 2.2p/KWh for coal and 3.7p/KWh for oil fired units based on 1988 CEGB statistics[45]. This favourable generation cost will continue to bias the current nuclear plants toward base load operations until more efficient nuclear units are installed. However, the widely held notion that the nuclear power plants are not well suited for a dynamic role in following the load is basically a misconception. Studies[153] indicated that it is within the design and practical capability of the nuclear stations to participate actively in meeting the varying generation need of the system. In Swedish State Power Board, nuclear units are allowed to produce power at two different levels: a day-time level and a night-time level[25] as shown in Fig.2.10. Mueller[153] showed that pressurized water reactor responds favourably to transients including load rejection, step change, day/night load following cycle and gradual power increase/decrease, all within design safety margins.

In the United Kingdom, nuclear units (AGR 660MW) use the same turbines as for the fossil fired units. The inherent limitation of a steam turbine applies to nuclear generators with the additional limitation of the reactor. The run up and loading of a nuclear power plant vary widely from station to station. Typically nuclear stations take 50-100% longer than

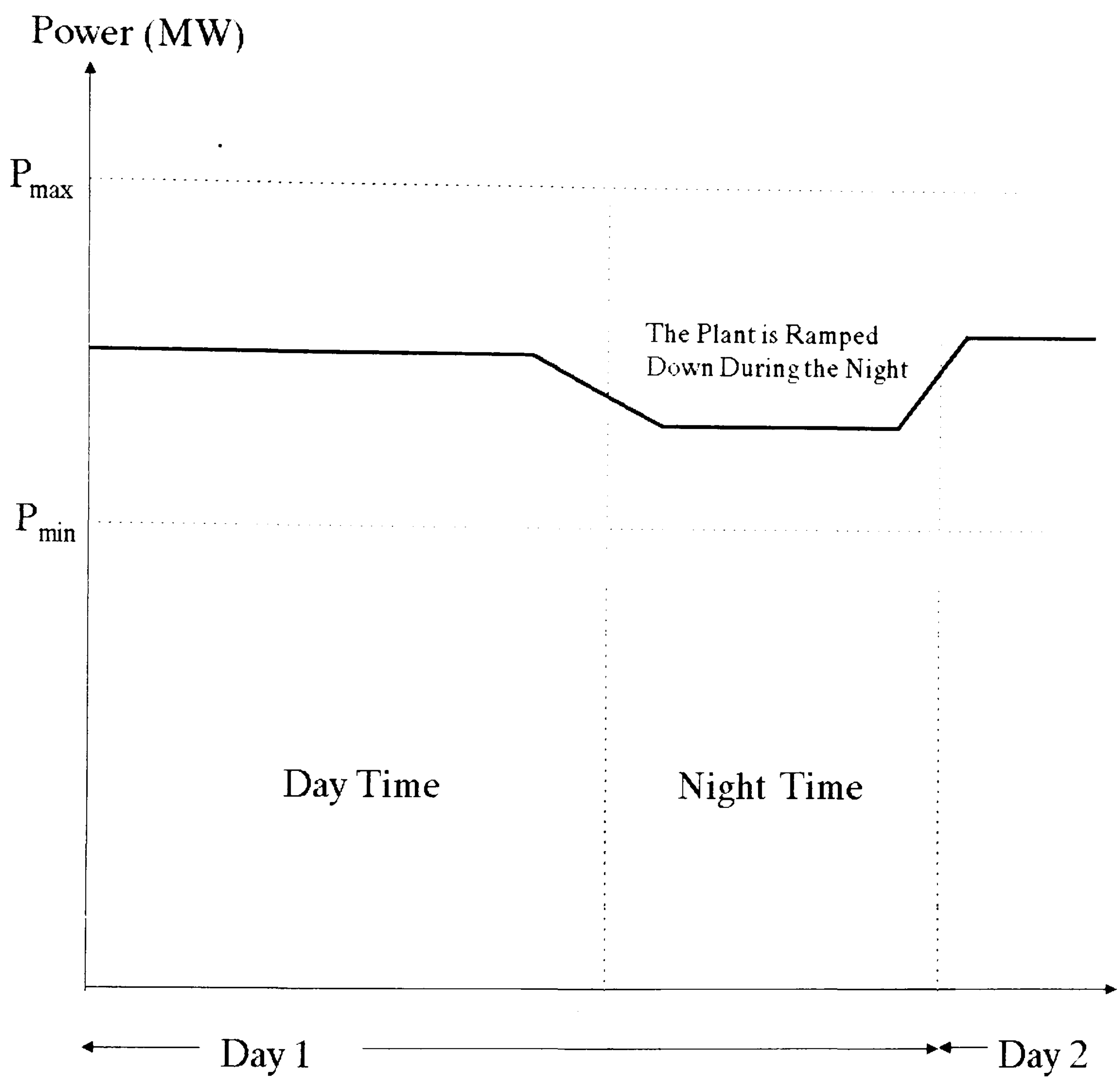



Fig. 2.10 Daily Regulation Strategy for a Nuclear Plant

when operated with a fossil fired boiler after short shut down (e.g. 8 hours). But for starts following a long shut down, the start time could be up to 30% less than for a fossil unit[40]. From operating cost point of view, a nuclear station can be modelled exactly as conventional fossil fuel steam turbine generator. In this thesis, the nuclear generators are treated as conventional turbine generators.

### 2.1.3 Gas Turbine Plants

Gas turbine (GT) generators have the most expensive running costs compared to the nuclear and conventional turbine generators. The construction of a GT is very different from nuclear or conventional turbine generators. In essential it uses air as the working fluid instead of steam. Air is compressed in the compressor, heated up in combustion chambers and then expanded in the turbine which drives the synchronous generator for electricity generation. Natural gas is the nominal fuel for base load units; distillate oil and heavy oil are mainly for peaking and standby duties. Modern GT's with high firing temperature in the 1100-1200$^{o}$C range have an efficient of about 34%. The largest unit size available at present is about 120MW depending on site conditions and specific designs. There is news that prototype 200MW units are being built and will be available in the near future. The operation cost models of the conventional turbine generators can be applied to GTs. But GTs have one major advantage; they are cheap to build and have fast pick up/shut down capabilities making them ideal for load cycling to provide the extra power needed during peak load demand periods and for emergency generation. In an optimal mix generation system, GTs provide a valuable option to reduce the overall economic/financial costs to an electricity supply utility. In general a gas turbine can produce full load in about 5-10 minutes after start up is initiated.

## 2.2 Transmission Network

The transmission network serves three essential purposes. (a) It pools generation sources at all levels and integrates large generating units and nuclear stations on to the system. (b) It provides bulk transmission of energy from the power stations to the load distribution centres and electrical whole sale points and (c) it interconnects systems for economic and security reasons. In the United Kingdom, the transmission network comprises a total of over 7,000 route kilometres of 400/275/132/66KV circuits, integrating over 90 major power stations together and serving over 22 million industrial and domestic consumers. A transmission line has a maximum load carrying capacity. It is vital that such a limit shall not be exceeded otherwise the protective relay will trip out the line and may cause further line overloadings. The MVA carrying capacity of a transmission line is primarily proportional to the cross sectional area of the conductor. Other factors which help to cool down a transmission line, such as higher wind speed, lower ambient temperature, can increase the normal ampacity rating of a transmission line by as much as 25% [194]. Furthermore, a transmission line also has short time ratings. Depending on the initial condition of a line, 100% overload can be sustained for 5 minutes. All these characteristics of the transmission lines are important for the economic allocation of generator outputs because for a given system topology the power flow in the transmission network is a function of the generator outputs and the load demand distribution.

The calculation of power flow and nodal voltages for a given generation and load demand distribution of a power system is generally known as load flow analysis. It is a non-linear problem because while the electrical transmission network is a linear system, the power generation and power demand which are regarded as known priori are non-linear quantities. In the last 30 years, an enormous amount of effort has been spent in research and development on load flow solutions. There are at least four important solution techniques, namely, Gauss-Seidel, Newton-Raphson, Fast Decoupled and Irving's[102] 2x2 submatrix methods. These methods are generally known as AC load flow since they determine an exact solution (within tolerance) with respect to real and imaginary parts of all system values including power flow and nodal voltages. Due to the limitation of computing resources and fast response requirements, a more suitable method for use in economic generation scheduling is a DC load flow. This load flow technique is briefly outlined in the following paragraphs.

Consider an electric network, the power balance at each node can be represented by :

$$P_i = V_i \sum_{k\,i}^{N_n} V_k(G_{ik}\mathrm{Cos}\theta_{ik} + B_{ik}\mathrm{Sin}\theta_{ik}) \qquad (2.8)$$

$$Q_i = V_i \sum_{k \epsilon i}^{N_n} V_k (G_{ik} \mathrm{Sin}\theta_{ik} - B_{ik} \mathrm{Cos}\theta_{ik}) \tag{2.9}$$

where

$P_i$ = net real power injection into the $i^{th}$ node by the generators, loads and tie lines connected directly to it;

$Q_i$ = net reactive power injection into the $i^{th}$ node by the generators, loads and tie lines connected directly to it;

$V_i \angle \theta_i$ = complex voltage at the $i^{th}$ node;

$G_{ik}, B_{ik}$ = real and imaginary parts of $Y_{ik}$, the element in the $i^{th}$ row and $k^{th}$ column of the network's admittance matrix;

$\theta_{ik} = \theta_i - \theta_k$

In an electrical transmission network, $B_{ik}$ is much greater than $G_{ik}$ and $\theta_{ik}$ is generally small. We can approximate $G_{ik}$=0, $\mathrm{Sin}\theta_{ik}=\theta_{ik}$, and $\mathrm{Cos}\theta_{ik}$=1 and $V_i$=1. Equations (2.8) and (2.9) become:

$$P_i = \sum_{k\ i}^{N_n} B_{ik} (\theta_i - \theta_k) \tag{2.10}$$

$$Q_i = - V_i \sum_{k\ i}^{N_n} B_{ik} V_k \tag{2.11}$$

Equations (2.10) and (2.11) have three significant implications:

(a) P and Q can be decoupled;

(b) Phase angles are closely related to the real power injections;

(c) Voltage magnitudes are closely related to the reactive power injections

In matrix form, Eq.(2.10) is:

$$[P] = [B][\theta] \tag{2.12a}$$

$$\Rightarrow \quad [\theta] = [B]^{-1} [P] \tag{2.12b}$$

Equation (2.12) is known as the DC load flow equation. The real power flow in line ik, $F_{ik}$, is approximately

$$F_{ik} = B_{ik} (\theta_i - \theta_k) \tag{2.13}$$

In matrix form,

$$[F] = [H][\theta] \tag{2.14}$$

Substitute Eq.(2.12b) in above,

$$[F] = [H][B]^{-1}[P]$$

$$= [S][P] \tag{2.15}$$

where

[F] = vector of line real power flow from sending end;

[H] = a $N_L$ x $N_n$ matrix,

= $B_{ik}$ at column i for line (row) *ik*

= $-B_{ik}$ at column k for line (row) *ik*

[S] = $[H][B]^{-1}$ = sensitivity matrix

The accuracy of the DC load flow is in the region of 5%. In a range of power system studies including planning, generation allocation and rescheduling, the speed of the solution is very often more important than accuracy. The DC load flow, which is about 50 times faster than an AC load flow, is widely adopted. DC load flow has another major advantage and that is its capability to link directly the power flow of a line to power injections at all nodes of the system as shown in Eq.(2.15). With this equation, the power flow in a line for any suggested generation allocation solution can therefore be checked against its rated capacity conveniently.

The limitation of the transmission network on optimal generation scheduling is that the real power carrying capacity of all lines in the system must not be exceeded. This limitation can be expressed as:

$$[F] \leq [F^{max}] \tag{2.16}$$

The reactive power flow in the system is assumed to be small, constant and independent of the real power generation schedule. This decoupling of real and reactive power is generally accepted as an adequate approximation for most real time operational control functions.

## 2.3 Load Prediction

Accurate electricity demand forecasts[1,24,61,75,140,178,208] are required for the secure and efficient operation of a power system. With estimates low, there is a risk of inadequate on line generation to meet the demand; if estimates are high, there is a risk of costly over-provision of resources. In unit commitment and economic dispatch problems, forecast loads typically from 5 minutes to 24 hours ahead are required as input data so that an optimal cost schedule may be obtained to meet the load. Load forecasting takes into account factors such as the past history, load growth, time of year, day of the week, time of the day, holidays, special events, extra demand at the end of a favourable television program and meteorological data from the meteorological centre. In general, forecast accuracy decreases with the length of time the prediction is aiming at. Methods of prediction fall into two basic categories: those which use weather forecasts and meteorological information, and those which model the demand variations by a time series analysis[140] of past load data.

A weather weighted regression algorithm[114] can produce a root mean square (RMS) error about 1.8% for lead times of three to four hours, 3% for lead times of 24 hours. In the United Kingdom, the CEGB stores and updates the meteorological data every half hour.

When only the past data is used, Laing and Metcalfe[114] suggested that a weighted-moving-average method is very accurate and is within 1% RMS error over timescales of 30 minutes. Gann[67] uses an adaptive time series analysis and achieves an accuracy of RMS error of about 2.6% for lead times up to about 24 hours. Spectral decomposition[196] methods are more accurate for longer lead times such a week ahead. Laing and Metcalfe[114] also explored the special features of both the meteorological methods and past load data only technique and obtained a prediction better than using either class of approaches.

## 2.4 Generation/Load Balance

Since electrical energy cannot be stored conveniently and economically, a continuous balance between electrical generation and the varying load demand must be maintained. The generation and load balance constraint applies:

$$\sum_{g=1}^{Ng} P_g - \sum_{n=1}^{Nn} D_n = 0 \qquad (2.17)$$

where $\Sigma P_g$ is the total generation of all units on the system, and $\Sigma D_n$ is the total load at all nodes of the system. If such balance is not maintained, the system frequency will deviate from the design frequency and may cause major break down of the system components as well as expensive damage to the consumer appliances connected to the system. In the above generation/load balance equation, transmission losses are not included. This can be dealt with by estimating the total system losses and treating this as further load to the system. Then equation (2.17) becomes:

$$\sum_{g=1}^{Ng} P_g - \sum_{n=1}^{Nn} D_n - \text{Loss} = 0 \qquad (2.18)$$

Methods to accurately determine the system losses will be reviewed in Chapter 5.

## 2.5 System Security Requirements - Operating Reserve

Operating reserve is one of the major consideration in scheduling the daily unit commitment and economic generation allocations. Operating reserve may be defined as the extra generation on demand from the available generators within a time period short enough to maintain acceptable frequency under possible operating contingencies. It is made up of components designed for each type of contingency. In practice, two types of operating reserves are generally provided: spinning reserve and non-spinning reserve. The reserve capacities and response time requirements for each reserve type vary widely among the utilities. The following description serves to identify the principles.

### 2.5.1 Spinning Reserve

For each day's operation, the load for the next 24 hour period is forecasted using various techniques. However, load exhibits variation patterns which cannot be predicted exactly. Studies show that while the average RMS error over a short period is in the region of 2 to 3% , instantaneous error can be over 10%. The speed of load increase also may be much greater than the rate at which units can be started, synchronized and then loaded. Units therefore must be started well in advance of load requirements. This extra capacity from the synchronized generator above the load is called 'spinning reserve'. Spinning reserve is costly as it implies that some units will be partially loaded at which points the units are less efficient than when they are at or near their full capacities. Hence it is desirable from an economic point of view to commit the minimum amount of spinning reserve subject to acceptable security risk. The risk to cover includes load forecast error, frequency regulation and dynamic pick-up as described in the following paragraphs[19,137].

#### (a) Load forecast error

The errors in prediction can be either due to error inherent in the prediction methods or from a variation in the consumer behaviour. The discrepancy in forecast load and the actual load is reflected in frequency deviation and will be detected by generator governors and frequency regulation loops; which then take up (or increase) the spinning reserve by increasing (or decreasing) the outputs from the generators. If the prediction error is big and detected early, such as 1 or 2 hours before actual happening, the commitment programme will be modified taking into account the latest load data. Hot standby units will be started to meet the increasing load. With a reliable load forecasting technique and an efficient unit commitment computer program, it is possible to reduce the required spinning reserve allowance for forecast load error contingency.

**(b) Frequency regulation**

Even if the total capacity on line were equal to the maximum load, there still may be a requirement for excess capacity for rapid response to load variation in order to maintain the system frequency within acceptable limits. The governors and frequency control loops on those generators participating in frequency regulation duties are again used to change their outputs to bring the deviated system frequency back to the reference value. It must be available within 2 to 5 seconds and therefore can only be provided by the spare capacity of the synchronized units.

**(c) Dynamic pick up capability**

Spinning reserve must also be provided for dynamic pick up of load in the event of the loss of a loaded generator.

The required amount of dynamic pick up capacity is ordinarily equal to the output of the largest unit; and this might represent 10% or more of the load for a small isolated system. For a large system, with proper planning of its unit size, or adequate interconnection, the requirement might be reduced to 2% to 4% of the total effective interconnected capacity. In order to avoid excess frequency swings and the risk of instability, the response time for the remaining on-line generators to pick up the lost generation is in the order of 5 to 20 seconds. For units loaded below their maximum capacities, the stored energy in the boilers will enable these units to increase their outputs, by governor action, by 15% of their rated capacities within 20 seconds and further increases within 5 minutes following increases in boiler firing rates. A wide spread of running spare between the generators will result in improved response; but part loading will decrease the units' efficiencies and increase the total fuel bill. Assuming sufficient spinning spares, the system will stabilise at a lower frequency value upon failure of a generating plant.

### 2.5.2 Non-spinning Reserve

Non-spinning reserve is the provision to cover those requirements which are considered not necessary to support with actual synchronized spinning reserve, based on either technical or economic reasons. The prime objective of non-spinning reserve is to ensure the capability to restore the system to the appropriate nominal frequency or to a secure state after sudden loss of a generator. Non-spinning reserve is composed of hot standby units, rapid start up units and interruptable loads. These requirements are described as follows.

**(a) Thermal back up**

After a sudden loss of a generator, the load may be made up by the spare running capacity and frequency subsequently stabilised within the acceptable limits. The system, however, is distorted from its scheduled conditions and possibly becomes insecure. The output of the generators are shifted from their scheduled loading following the loss of a generator. There is also the possibility that some lines in the system become overloaded. It is therefore considered necessary to have additional generation in the generation deficient area in perhaps 5 to 10 minutes to restore the lines to their normal operating limits before they are damaged.

**b) Contingency back up**

Following the loss of a generator, the spinning reserve of the system is drastically reduced. It may be therefore desirable to re-establish the nominal reserve level in perhaps 30 minutes to 2 hours ready for a possible second generator loss.

**(c) Load shedding**

This is the last resort to alleviate generation deficiency and electric utilities tend to avoid this practice as it means loss of revenue and dissatisfies customers. This option however provides the valuable flexibility to reduce the spinning reserve requirements. For smaller system, this is a particular important means to control the system frequency fluctuation during a major system disturbance. The utility generally has agreements with certain industrial users to allow it to interrupt service after a specified time of notification in exchange for lower tariff.

### 2.5.3 Typical Operation Reserve Requirement

Table 2.2 gives a typical operation reserve[137] requirements of a power system.

**Table 2.2** Typical Operation Reserve Requirement of a Power System

| Response Time | Capacity Requirement | Provided by |
|---|---|---|
| 0-20 sec | 1-10% of Load | On-line Generators, Pumped Storage(Trip Pump) |
| 1-5 min. | Largest on-line Unit | On-line Generators, Pumped-Storage, Hydro, Gas Turbine, Diesel |
| 1-2 hours | 10-20% of Load | Hot Standby Conventional Plants, Gas Turbine, Diesel |

The unit commitment and economic dispatch are parts of the complex control strategies in an EMS designed to schedule the generating units in such a way that the total available operating reserve from these units is able to meet the system response requirements set down by the management.

### 2.5.4 Reserve Margin of a Generator

In scheduling the generating units, the total amount for each type of reserve available from the system must be determined. This is equal to the sum of reserve margin from all units in the system. Stadlin[192] defined the reserve margin of a generator as the amount of available generation change of the unit within a specified time, referred to as margin time or response time taken into consideration the restriction imposed by its operating limits. It is apparent that if the margin time specified is long enough, a unit may have reserve capacity even if it is shut down. The following example serves to clarify the concept.

Let a power system whose operating reserves are specified to have three margin time requirements of 20 seconds, 5 minutes and 2 hours. The system has two generators, one conventional steam turbine and one gas turbine with operation characteristics as follows. The gas turbine needs 2 minutes to start up.

| | Steam Turbine | Gas turbine |
|---|---|---|
| Present Status | 40 MW output | Shut-down |
| Min Output | 20 MW | 0.0 MW |
| Max Output | 100 MW | 60.0 MW |
| Emergency Pick Up Rate | 5 MW/sec for 10 Sec | 5.0 MW/sec |
| Sustained Pick Up Rate | 10 MW/Min | 50.0 MW/min |
| Start Up Time | 3 Hrs. | 2.0 Minute |

Then, regulating reserve for 20 second margin time is:

| | | |
|---|---|---|
| Steam turbine | = 5 MW/sec x 10 Sec | = 50 MW |
| Gas Turbine | = | 0 MW |
| | Total | = 50 MW |

Note that the emergency pick up rate of the steam turbine lasts only 10 seconds and the gas turbine is shut down and therefore has no dynamic pick up capability.

Operating reserve for 5 minute response time is:

| | | |
|---|---|---|
| Steam Turbine | = 10 MW/min x 5 minute = | 50 MW |
| Gas Turbine | = 50 MW/min x 3 minute = | 60 MW |
| | Total | = 110 MW |

The gas turbine needs two minutes to start up. There are 3 minutes left to pick up the load but its reserve margin is limited by its maximum output limit.

The 2 hour reserve requirement is provided by :

| | | |
|---|---|---|
| Steam Turbine | = 100 - 40 MW | = 60 MW |
| Gas Turbine | = | 60 MW |
| | Total | = 120 MW |

The 2 hours reserve available is restricted by the maximum capacities of the two units.

From the above example, it can be seen that operating reserve available from a unit depends on several factors:

(a) Present output level of the unit;

(b) Margin time which relates to various reserve requirements as illustrated in the example above;

(c) Maximum capacity;

(d) Present operating state i.e. on-line or off-line.

The spare capacity of an on-line conventional steam turbine generator is typically as shown in Fig.2.11. The different sections of the reserve curve for increasing output power can be expressed mathematically as:

$$R \leq R^{max} \tag{2.19}$$

$$R \leq C_1 P \tag{2.20}$$

$$R \leq C_2 (P_{max} - P) \tag{2.21}$$

where

$R^{max}$ = maximum amount of spare from this unit for a given margin time

$C_1$ = reflects a lower ramping rate at lower generating output

$C_2$ = reflects the limitation of the generator max capacity

The margin to decrease is normally much larger than the rate to increase as shown in the negative region of Fig.2.11. Rapid decreasing of the generator output is normally not a problem. There is also the option of opening the generator breaker in emergency cases assuming steam by-pass available. Similar limits on a group or station spare may also apply.

$$R_s \leq R_s{}^{max} \tag{2.22}$$

$$R_s \leq [K_1]\ [P] \tag{2.23}$$

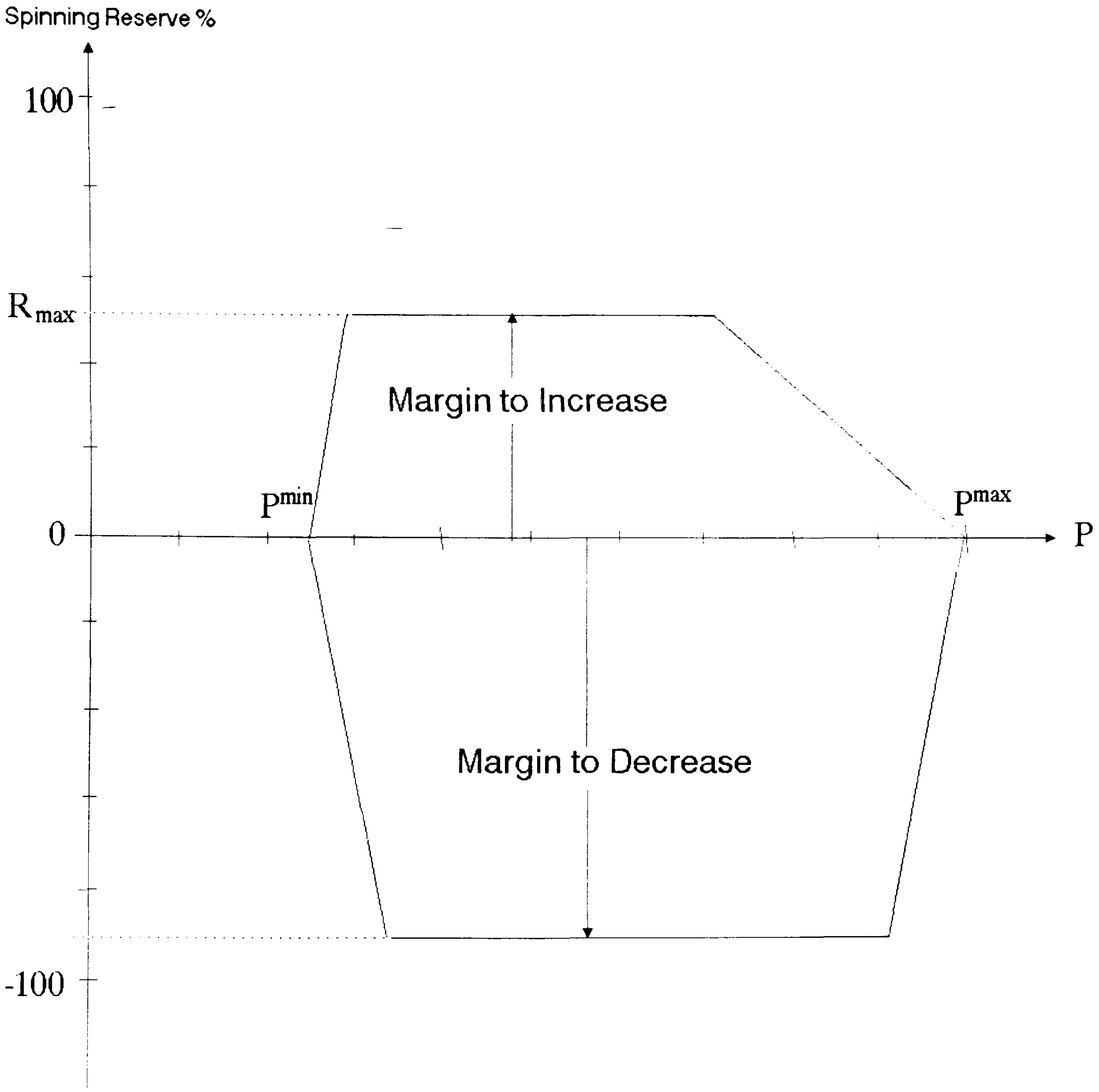
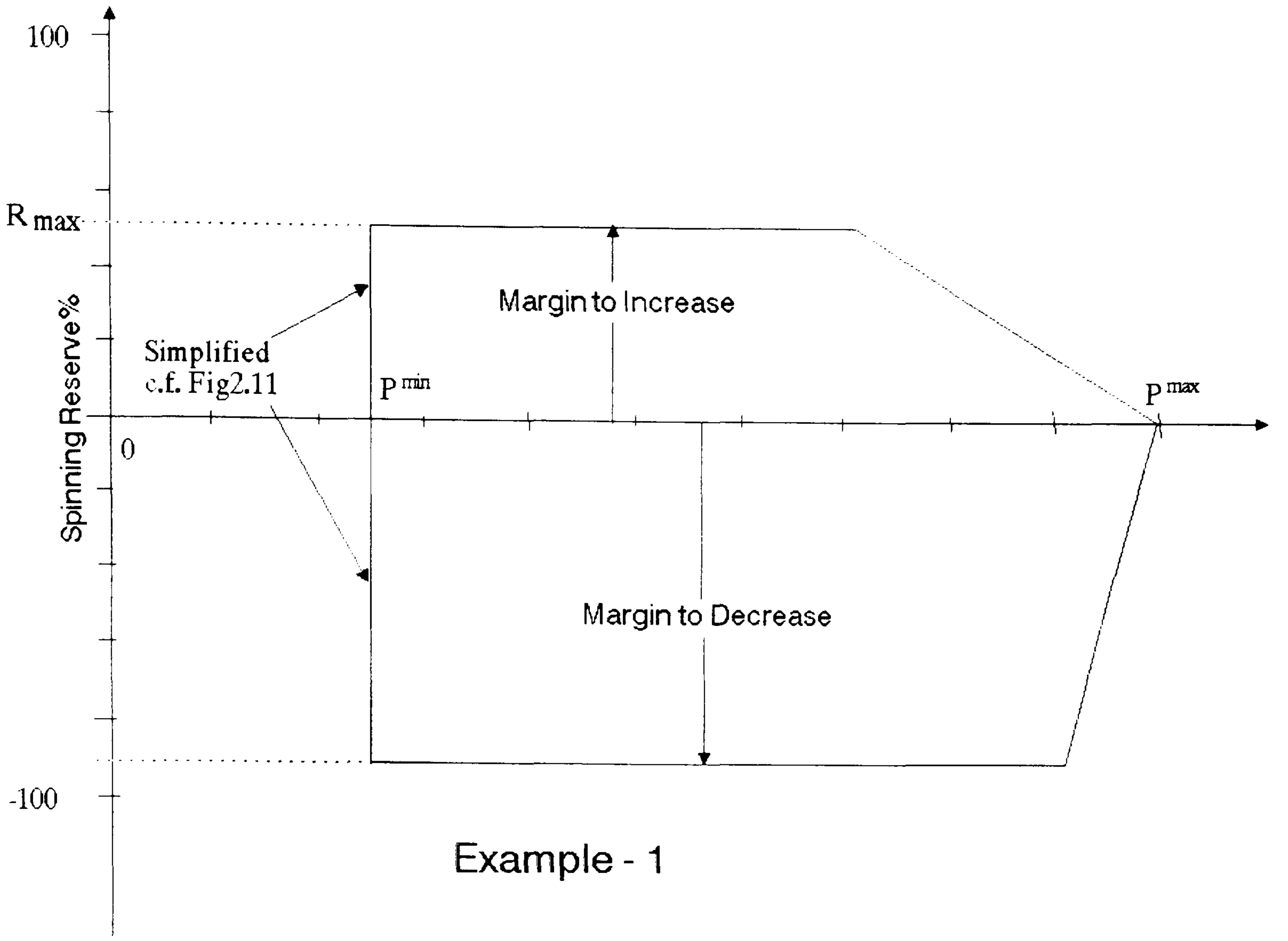



Fig.2.11 Spinning Reserve of a Conventional Turbine Generaor with Short Margin Time



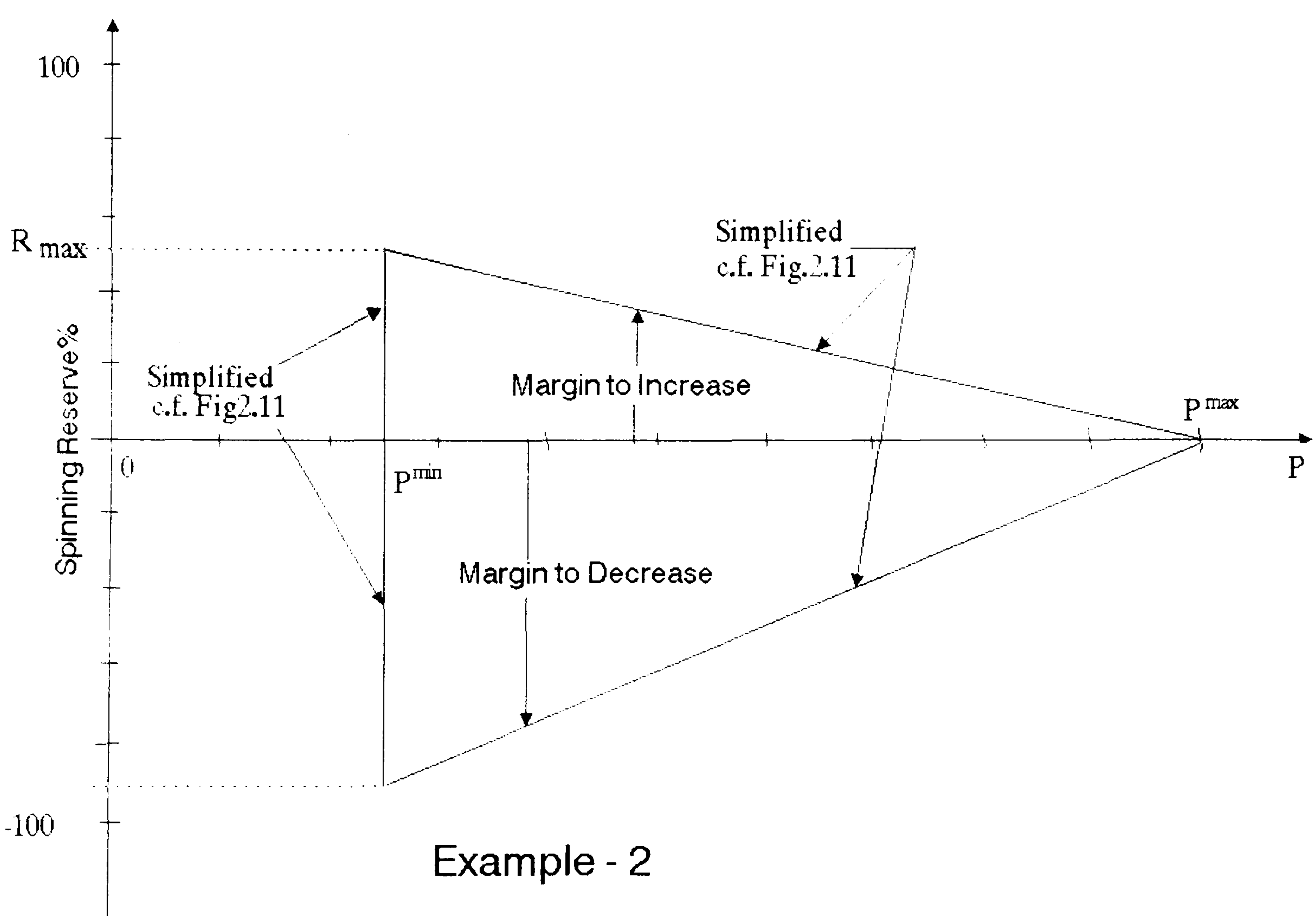




Fig.2.12 Simplified Spinning Reserve of a Conventional Turbine Generator with Short Margin Time

$$R_s \leq [K_2]\ [P^{max}-P] \tag{2.24}$$

where

$R_s^{max}$ = max group or station spare capacity

$R_s$ = group or station spare capacity available

$K_1$ = portion of the spare from the generator for the lower output region contributable to station spare

$K_2$ = portion of the spare from the generator for the higher output region.

Various approximations of spare capacity from generators are proposed. Figs.2.12(a) and 2.12(b) show two popular simplifications.

## 2.6 Summary

The essential operational constraints for the components and system requirements have been described in the preceding sections. Some constraints are more complex and more difficult to model than others. The relative importance and the degree of approximations needed to represent constraints should be considered in the context of the complexity of the generation scheduling problems, solution methods available and response requirements of real time applications. In the following chapters the solution methods to incorporate these constraints for the unit commitment and economic dispatch problems are presented.

# CHAPTER 3

## SURVEY OF TECHNIQUES FOR UNIT COMMITMENT

The daily load of a power system in general varies widely between a minimum and a maximum despite load management effort which attempts to produce a more uniform profile. In order to save cost, electric utilities usually have fewer generating units running at lighter load periods. Unit commitment, also known as plant ordering, with the task of scheduling the on/off of appropriate generators to meet the predicted demand at various times for a period of one day, or sometimes up to one week, so that minimum accumulated operational cost will incur. It is a complex combinatorial problem. As a mental exercise, consider a system with 20 units and a forecast daily load curve subdivided into 24 hour intervals. Let us assume that the commitment schedule will be established by an enumeration approach. The total number of possible combinations required to be examined in order to obtain the optimal generator on/off combinations for the 24 hour study period will be approximately $(2^{20})^{24}$, or $3.12\mathrm{x}10^{144}$. A "super-computer" having processing capability of 1000 million mathematical operations per second including cost comparison etc, which probably has more processing capability than the mightiest computer of present day technology, will require 128 years to complete the task. Although physical and operational constraints of the plants and of the system reduce the number of feasible combinations, the necessary constraint considerations other than pure combinatorial such as fuel and start up/shut down costs calculations, minimum on/off time limitations and spinning reserve requirements etc. add substantial complication to the problem. Throughout the years, the electricity supply industries and other research institutes have developed various algorithmic approaches to solve this cost minimisation problem. This chapter reviews the key methodologies employed.

The unit commitment problem can be described in a mathematical optimization format which has the general form as follows:

$$\text{Minimize} \quad C = \sum_{t=1}^{T} \sum_{g=1}^{N_g} \{F_g(P_g^t) + S_g^t\} \tag{3.1}$$

where

$T$ = number of subdivided intervals in the study period,

$N_g$ = number of generators in the system,

$F_g$ = output dependent generation cost function of generator $g$,

$P_g^t$ = active power output of generator $g$,

$S_g^t$ = start up/shut down cost of generator $g$ at interval $t$.

The primary objective of the unit commitment is to determine the on/off schedule of the generators so that the total operational cost C is minimum. However, as can be seen from Eq.(3.1), the operational cost of a generator relates directly to its output level. In the course of selecting the on/off unit combinations for any subdivided interval, the power output of each selected ON unit must also be determined in order to compare the costs of alternative schedules.

The objective function is subject to the physical and operational limitations of the components and of the system. The essential operational characteristics of the equipment and of the system causing some of these constraints have been outlined in Chapter 2. These constraints are in the main non-linear with some which cannot be modelled conveniently. The discrete unit on/off decision in particular is one of the sources of complication. Due to the computational complexity of the problem, development to date frequently employs a mixture of optimization techniques in a single solution scheme. In order to clarify the frameworks engaged by many workers and to provide an overall picture of the main conceptual approaches to the solution, this thesis classifies the existing methods reported in the literature into five broad categories.

1. Merit-order or Heuristic Methods
2. Mixed Integer-Linear Programming
3. Branch and Bound Technique
4. Lagrangian Relaxation
5. Dynamic Programming

Of these, the merit-order schemes are the most popular because of their simplicity and partly also because of the wide experience gained through manual calculation before the digital computer was generally used as a standard tool in power system engineering applications. With the advent of the digital computer, this class of methods has still proved to be most practical for large electric power systems. Mixed integer-linear programming and branch-and-bound approaches are not widely regarded as efficient for large scale problems. Recently the use of Lagrangian relaxation methods has shown signs of a rigourous and efficient approach. However, dynamic programming (DP) based unit commitment algorithms are probably by far the most widely reported approaches. Because of their inherent flexibility in dealing with nonlinear and discrete variables, DP based methods are perceived to be promising alternatives to merit-order schemes. Indeed, a number of DP schemes are implemented in commercially available Energy Management System software packages[64] targeted for real system operation. In the following sections, a closer examination of these five solution categories is offered.

### 3.1 Merit Order Methods

The simplest unit commitment solution methods are the merit order[9,19,83,172] or heuristic schemes. They are characterized by the use of some form of priority list. The list is generally obtained by ranking the average full load production cost (AFLC) of each unit in the system. AFLC of a unit, depicted in Fig.3.1, is obtained by dividing the generation cost at the full load by its megawatt output at full load. A start up/shut down schedule of the units is then constructed with the assistance of this priority list. The underlying principle of the approach is based on the operational experience that the on-line units are primarily loaded to their maximum capacities and that the total operational cost in Eq.(3.1) is dominated by the consumption of fuel supply to the generating units to meet the predicted load. A merit-order scheme might operate as follows.

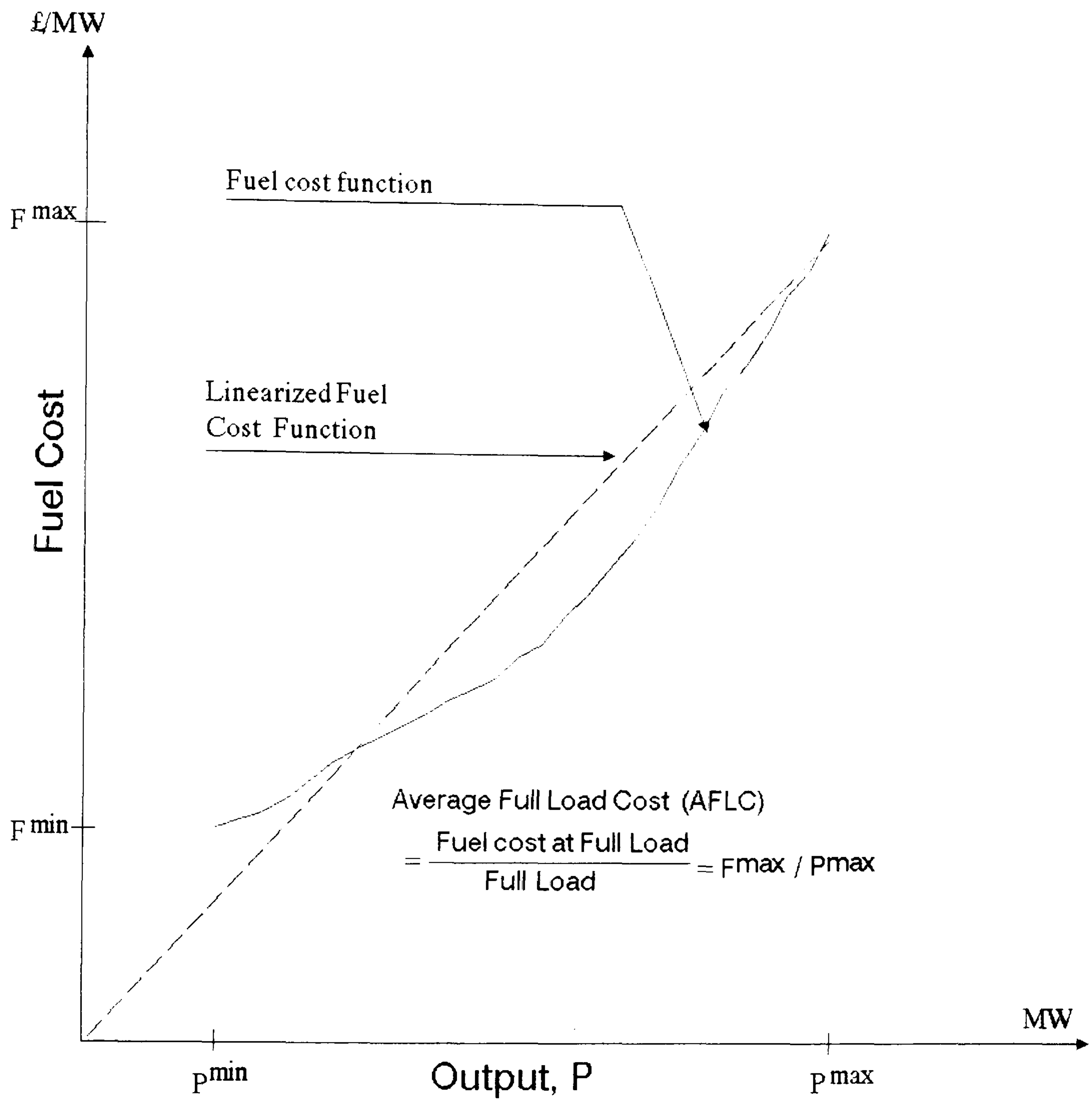



Fig. 3.1 Average Full Load Cost of a Thermal Generating Unit

The forecast load is firstly discretized into a number of time intervals each of which may or may not be of the same duration with the load within an interval assumed to have a constant demand level. Starting with the first interval, the method systematically scans through the whole study period and decides the on/off of the units in the system for each interval consecutively.

**Step 1:** For each interval I, the load is compared with load in the previous interval. If the loads are the same, keep the same commitment combination as the previous interval and repeat this comparison process for the next interval until load change is detected.

**Step 2:** If the load decreases, determine the number of intervals M (including the interval under consideration) before the load increases to a level equal to or exceeding the load of the previous interval. M is the possible number of intervals any unit will be required to be shut down assuming that the unit will be needed to generate again when the load picks up. Select the most expensive committed unit in the priority list satisfying minimum on time and other unit constraints. If M is greater than or equal to its minimum down time limitation then this unit is a possible candidate to shut down for the low load intervals. Check whether shutting down the unit will leave enough generation to supply the load, satisfying spinning reserve, regional requirements etc. If there is no such unit in the already committed unit list, keep the same commitment list as the previous interval and go back to step 1 to consider the next interval. Otherwise calculate and compare the operational cost for the next M intervals with and without the unit in the commitment list. For the case of taking the unit out of the commitment list, in addition to the fuel cost for the M intervals, shut down/banking and start up costs of the unit under consideration should be added to that cost. If there is saving in shutting down the unit, decommit the unit. Repeat this process for the next most expensive committed unit until no further saving may be achieved. Then go to step 1 to consider the next interval.

**Step 3:** If the load increases, check whether the committed units have sufficient capacity for the load and satisfying spinning reserve. If yes, keep the same commitment list as the last interval and go to step 1 to consider the next interval. Otherwise determine the cheapest uncommitted unit in the priority list that satisfies the minimum off time requirement. Commit the unit for interval I. With the added generation capacity, check whether it is sufficient for the load and spinning reserve. If not, determine the next cheapest uncommitted unit from the priority list and commit the unit. Repeat the process until sufficient generation is committed. When there is sufficient generation capacity committed, there might be a chance that the system's operational cost can be reduced by committing a further unit. Determine the number of intervals M, including the interval under consideration, before the load goes down to a level equal to or less than the load of the previous interval. Find the next cheapest uncommitted unit in the priority list that satisfies its minimum off time requirement and such that M is equal to or greater than the minimum up time of the unit. It is assumed that this unit will be shut down when load falls to a level equal to or less than the level of the previous interval. Calculate and compare the operational cost for the next M intervals with and without the unit in the commitment list. The operational cost for the M intervals with the unit committed should include the start up and shut down cost of the unit. If there is saving in starting up the unit, commit the unit. Repeat this step for the next cheapest unit in the uncommitted list. Otherwise go to step 1 to consider the next interval.

When all intervals have been considered using steps 1 to 3 described above, unit commitment scheduling for the forecast load period is considered completed. Various enhancement and modifications to the scheme outlined above were proposed. Happ et al[83] suggested that after such a systematic approach to produce a feasible solution which is generally reasonably close to the optimal, further optimization processes should be taken to examine if further cost improvements can be achieved. There may be savings to eliminate some start ups or to replace the energy generated by the expensive units by starting up some uncommitted units. Happ et al reported that this refinement has the benefit of improving the overall operational cost by a further 10% of the cost saving achievable in the initial optimization step .

In the basic merit order approach, the electrical network is ignored. The electric power system is modelled as multiple generators with a single lumped load interconnected by a transmission system of infinite capacity as depicted in

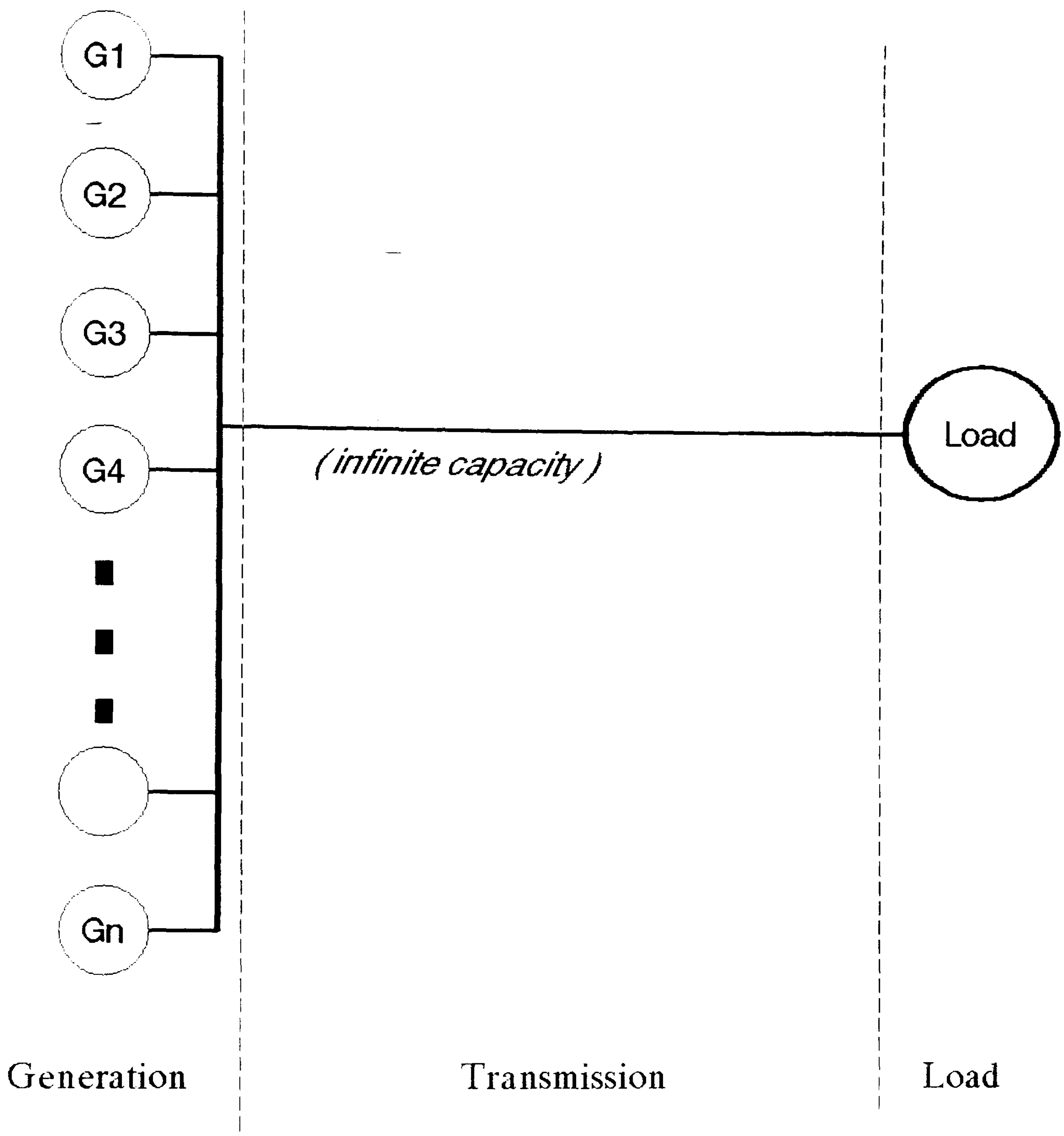



Fig.3.2 Electric Power System Model I - Multi-generator, Single Lumped Load, Infinite Transmission Capacity

Fig.3.2. Transmission losses, assumed to be negligible or estimated as a percentage of the forecast load, are added to the total system demand. A popular enhancement to this basic model is to approximate the limitations of the transmission network by transmission limitations between areas of a power system. This model is further simplified by restricting the output of groups of generators to certain limits, i.e. generator group import/export constraints. The electric supply network in this case in effect is modelled as shown in Fig.3.3. Both models are not sufficiently accurate to represent the dynamic nature of the transmission network topology and possible power flow violations. To overcome this deficiency, Piekntowski and Rose[172] utilized the merit order scheme in conjunction with a linear programming (LP) based economic dispatch program. The merit-order scheme nominates the commitment list for each time interval and LP is used to dispatch the generator outputs satisfying various constraints including the varying system topology and load distributions. A simplified flow chart of this method is summarized in Fig.3.4. The advantages of Piekntowski and Rose's method are:

1. It makes use of the well established and fast computational characteristics of both the merit order technique for unit selection and of LP technique for constraint checking.
2. In the dispatch phase, the real time topology of the system, generator group constraints, individual line flow limitation, distribution of the forecast demand can all be modelled accurately.

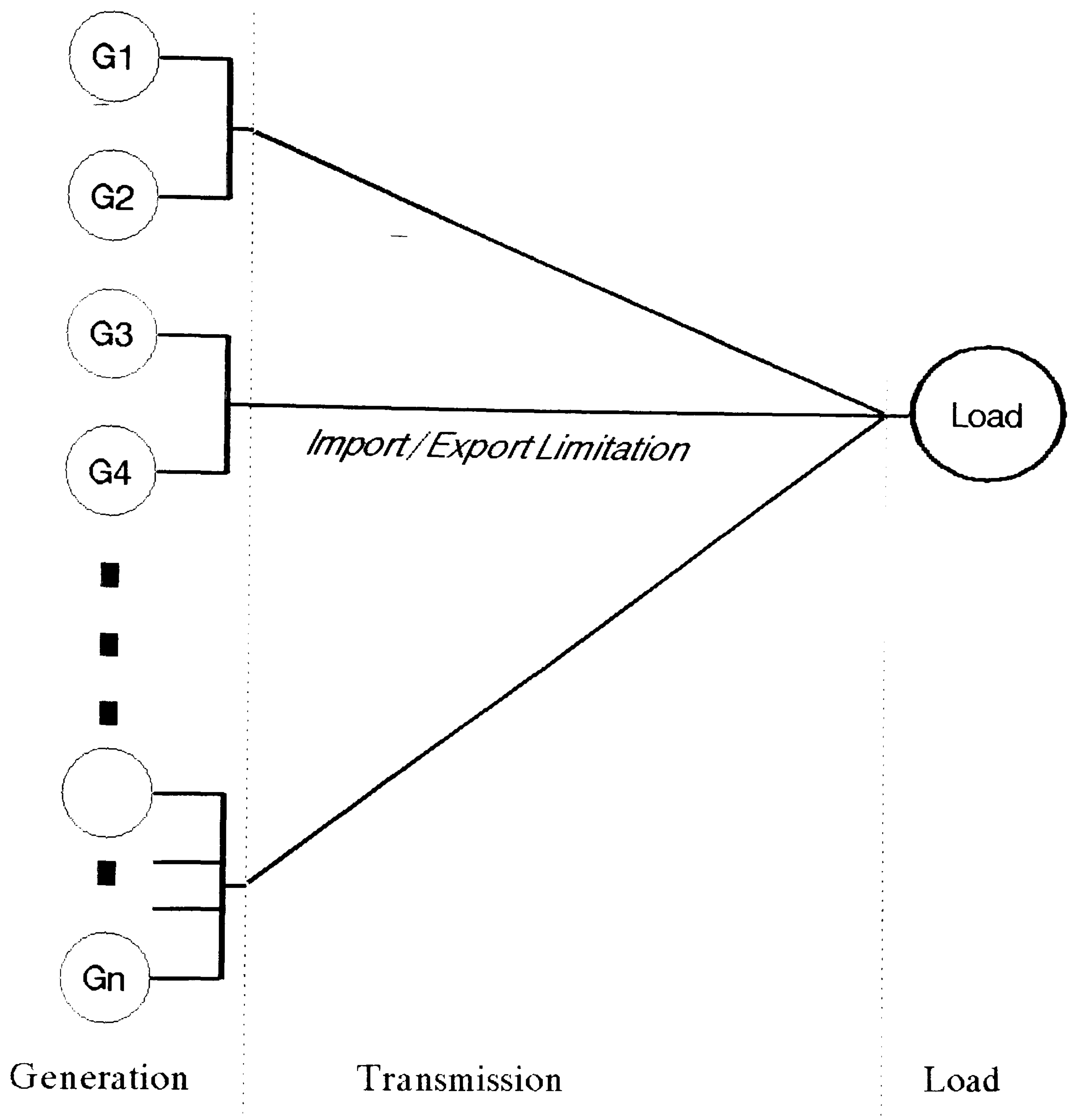



Fig.3.3 Electric Power System Model II - Multi-generator
Single Lumped Load, Import/Export Transmission Limitations

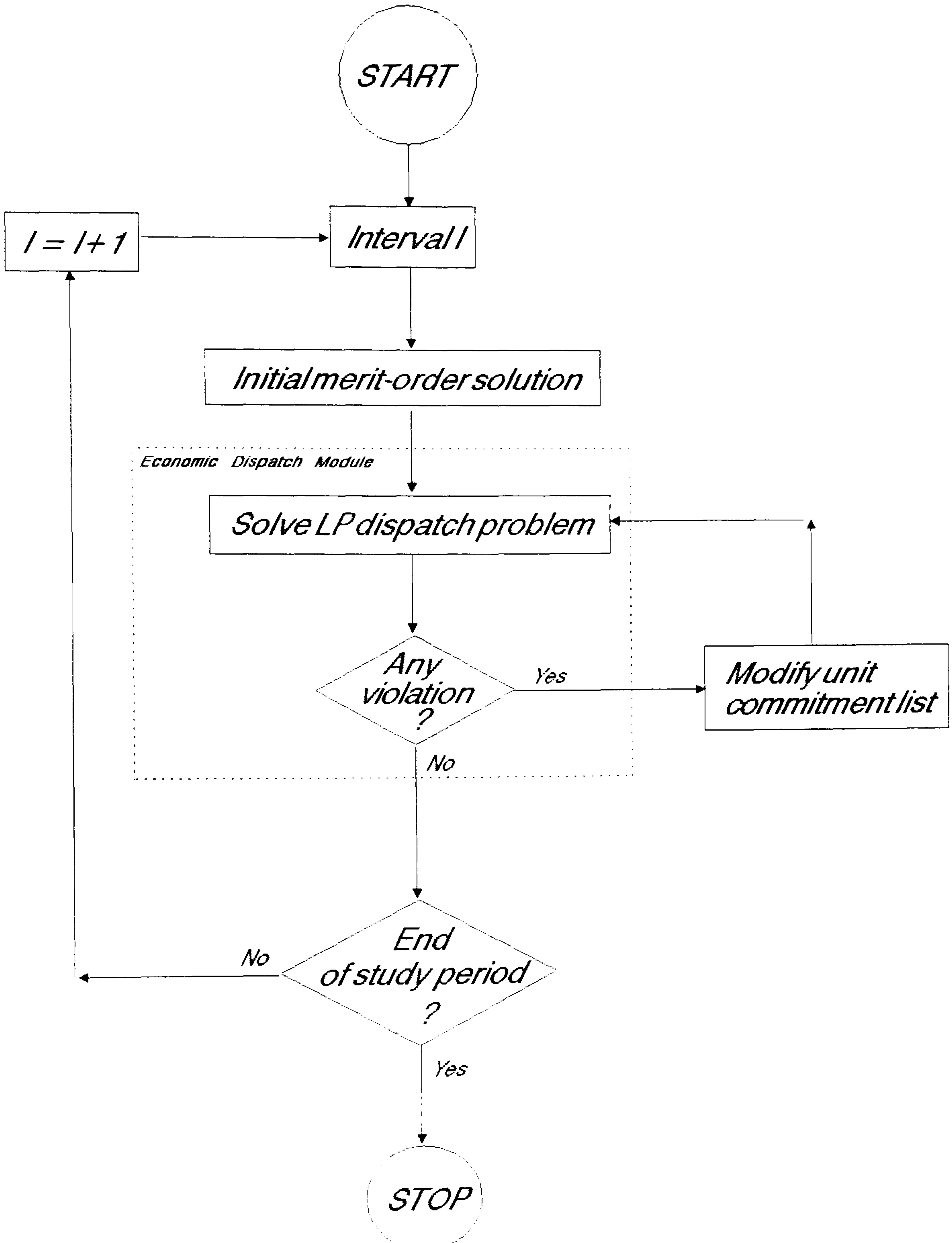



Fig. 3.4 Piekntowski and Rose's Method for Unit Commitment

One of the important advantages about priority order approaches is that the general principle of loading the cheapest generator first is embedded in the solution scheme. More significantly, due to their inherent simplicity, most of the essential operational constraints can be incorporated in the solution process without the limitations frequently encountered in a sophisticated mathematical optimization technique. The disadvantages of the methods are that any form of priority list will not easily deal with the diverse operating characteristics of various generator types. For example, gas turbine generators are frequently used for peaking duties and are frequently partially loaded depending on the level of load demand and relative costs and outputs of other units on the system. These units generally exhibit highly non-linear thermal efficiencies varying according to their loading. For example, at 40% loading a GT has a heat rate which exceeds that at full load by as much as 40% and at 80% loading its heat rate is only 5% above that at full load. AFLC does not reflect the thermal efficiency variations over the possible operating range of a unit. Again multi-fuel generators have distinct fuel cost characteristics varying with fuel mixture. Average full load cost therefore may not offer a realistic model for all types of generating units which may be installed in a power system. While enhancements such as those employed by Happ et al and Piekntowski et al overcome some obvious shortcomings of merit order approaches, the inaccuracy in generation modelling and the rigidity of loading and unloading of the units according to their positions in a merit order are some of the fundamental weakness.

### 3.2 Mixed Integer-Linear Programming

The unit commitment problem formulated by Garver[68] uses a mixed integer-linear programming technique. The objective function has a mix of whole numbers and continuous variables formulated as follows:

$$\text{Minimize} \quad C = \sum_{t=1}^{T} \sum_{g=1}^{N_g} \{CU_g \mu_g{}^t + CD_g \alpha_g{}^t + CL_g \beta_g{}^t + CI_g P_g{}^t\} \qquad (3.2)$$

This is subject to constraints:

$$D^t \leq \sum_{g=1}^{N_g} (P_g{}^{min}\ \beta_g{}^t + P_g{}^t)$$

$$D^t + R^t \leq \sum_{g=1}^{N_g} P_g{}^{max}\ \beta_g{}^t$$

$$P_g{}^{max} \geq (\ P_g{}^{min}\ \beta_g{}^t + P_g{}^t\ )$$

$$\mu_g{}^t - \beta_g{}^{(t-1)} = \mu_g{}^t - \alpha_g{}^t$$

where

$\mu_g{}^t$ = integer variable, normally equal zero, equals 1 for generator $g$ start up at interval $t$

$\alpha_g{}^t$ = integer variable, normally equal zero, equals 1 for generator $g$ shut down at interval $t$

$\beta_g{}^t$ = integer variable, normally equal zero, equals 1 for generator $g$ committed at interval $t$

$P_g{}^t$ = generator $g$ output above its minimum stable output $P_g{}^{min}$

$D^t, R^t$ = Power demand and reserve requirements for interval $t$

CU, CD, CL and CI are start up, shut down, minimum output and incremental costs of a generator

In the above equations, the generating units are assumed to have a constant incremental cost. Non-linearity of the cost curve can be accounted for by subdivision of the total output range into several segments each having a constant incremental cost, i.e. piece-wise linear representation. The most interesting aspects of Garver's approach is the ability to capture the essential on/off decision and individual generator output levels in every interval of the study period in a concise formulation. Furthermore, a Simplex type tableau computation algorithm can be applied directly to the above equations. Priority list and heuristic reasoning are not employed at all. Unfortunately, the method tends to generate a large Simplex type matrix. For example, based on the above formulation, a system with 100 units and for 24 hours study period with 1 hour interval will have roughly 9600 (4x100x24) independent variables and 19200 (8x100x24) constraints. Experience shown that Simplex type basic matrix exchange mechanism is generally efficient and stable only when the number of variables and constraints are reasonably small[101,203], i.e. to a maximum of about a few thousand variables. Assuming a powerful and well implemented basic matrix exchange algorithm is available and is capable of handling variables and constraints up to three thousands, then the number of generators the solution scheme can deal with will be about 11 units (approx = 3000/24/12). This is less than the number of units in a medium size power system. If more accurate modelling such as time dependent start up cost, piece-wise linear fuel cost representations, minimum up/down time etc. are considered in the solution process, the maximum number of generating units this approach can handle will be further reduced.

Muskstadt and Wilson[152] described a model which attempts to improve Garver's formulation by decomposing the problem into a hierarchical structure with a "restricted integer sub-problem" at a higher level deciding the values of the discrete variables, i.e. start up/shut down of the units, and a "inside sub-problem" at a lower level to determine the generation output of the units. Within the inside sub-problem, the concept of considering the load demand as a discrete random variable with a known probability distribution is also introduced. Given the start up/shut down schedule of the units, a linear programming technique is used to determine the optimal generator outputs for all possible loading. The minimum cost obtained is weighted by the probability of the respective loading occurrence to give the expected minimum operation cost for the planning horizon. The "inside subproblem" solution satisfies constraints normally considered in an economic dispatch formulation. It was shown that by proper scaling the integer coefficients indicating starting up or shutting down of a unit, the generation output of a unit may be restricted to integer values and hence the "inside subproblem" may be solved by a transportation formulation. At the higher hierarchical level of the "restricted integer problem", a branch-and-bound technique is used in deciding the on/off of the generating units. The contribution of the proposal is in providing a theoretical basis for tearing the mixed integer-linear formulation into pieces which may be treated separately for possible computational efficiency. The solution scheme, however, was not implemented and subsequently no example as to its efficiency or practicality is known. Again in the transportation subproblem solution phase of the scheme, a linear cost function is assumed, which is a limitation of the method. In general, a rigourous mixed integer-linear formulation is yet to be shown to be practical for a realistic size systems.

### 3.3 Branch and Bound Technique

One of the major deficiencies of the merit order approach is the rigid loading or deloading order of the generators. In order to overcome this problem and also because of the inherent discrete generator on/off decision in unit commitment scheduling, it seems natural that the branch-and-bound technique[41,51,52,118,163] (BBT) is applicable to solve the unit commitment problem. Branch and bound is a powerful and flexible optimization technique. It has found the widest application in those problems with a mixture of continuous and discrete variables. The method essentially subdivides the possible solution space into mutually exclusive groups by assigning fixed values to some of the discrete variables with the remaining discrete variables relaxed and treated as continuous. The group dividing process is called branching and each exclusive group is like a node of a tree structure. The objective function for each group is evaluated to give an underestimation of the best possible optimal solution obtainable by following the subdivision represented by the respective group. This is called bounding. The branching and bounding process are repeated many times until all the discrete variables have been assigned a value. The global optimum is the solution with the best objective cost function with all the discrete variable taking some fixed values. Appendix B gives a brief review of BBT. The efficiency of the approach revolves around the three stages of the solution process.

(1) Branching - an efficient method for partitioning the solution space must be devised. This is crucial because it will help, particularly in the early stage of the solution process, to reduce the number of long tree branches which may have to be fathomed.

(2) A rapid solution method for the sub-problem corresponding to each node. In the solution process, many sub-problems solutions will be needed before the optimal solution is acquired. The efficiency of the sub-problem solution process affects directly the overall computational effort required.

(3) The lower bound solution of the sub-problem obtained must be tight. The tighter the lower bound to the optimal of this branch, the better indication it will give as to the likelihood of finding the global solution of the original problem by following this path. This will again help to eliminate the unpromising nodes.

In many ways, the three requirements are conflicting and inter-locked. For example, to improve the computational efficiency of solving the subproblems, more constraints may have to be relaxed. This means that the lower bound obtained will be looser and more branching and hence more sub-problems are needed to be solved before a new upper bound of the global solution may be obtained. A delicate balance to satisfy the above three solution criteria and clever exploitation of the special structure of the unit commitment problem are essential in order that an efficient branch-and-bound based algorithm may be realised.

Cohen and Yoshimur[41] proposed that if the generating units can be assumed to have a maximum of one start up and/or one shut down in a 24 hour study horizon, the allowable start up and shut down intervals can be an effective partition criterion. In their approach, the units are separated one at a time with the same allowable start up and shut down interval. When all units have been separated, branching is continued by reducing the time span of start-stop intervals of each unit starting again with the first unit. The branching process continues until the allowable start-stop intervals consist of a single time slot. Fig.3.5 illustrates the branching process for a two unit system with 4 scheduling subperiods. It is easy to see that some of the non-linear constraints such as minimum up/down time, crew availability can be built into these permissible start-stop intervals. Many infeasible unit combinations can also be eliminated in this branching procedure.

The solution for each node of the tree is formulated as follows. Let the allowable start up interval for unit g be $SU_g=[SU_g^0,\ SU_g']$, and the allowable shut down interval be

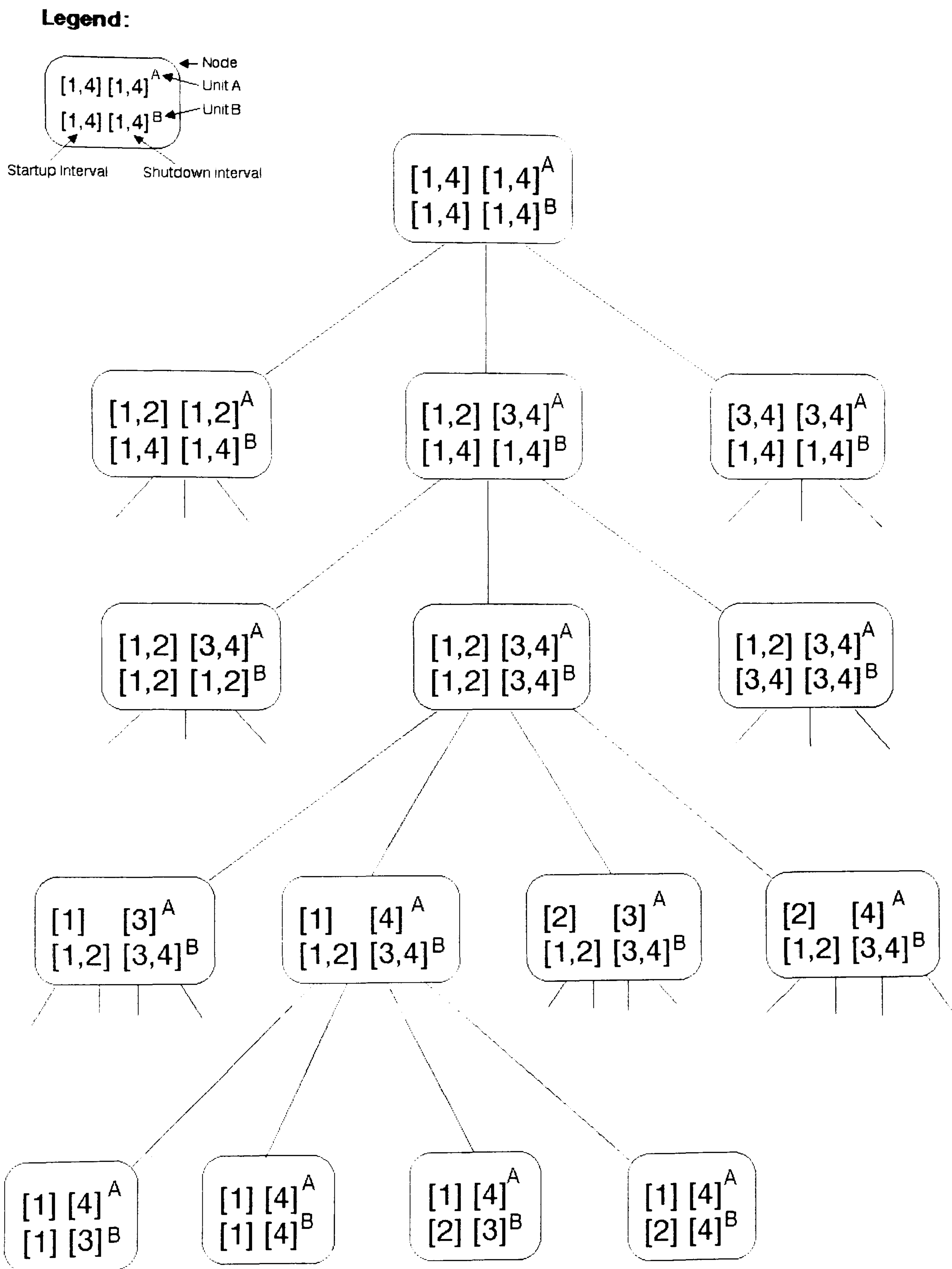



Fig.3.5 Branching Example - Start/Stop Interval Partitioning Criterion

$SD_g=[SD_g^0, SD_g']$, the lower bound for the given start-stop interval for any node is then:

$$\text{Minimize} \quad C = \sum_{t=1}^{T} \{ \sum_{g=1}^{N_g} F_g(P_g^t) \} + SC_g(SU_g, SD_g) \tag{3.3}$$

Subject to

a) Load balance constraint:

$$\sum_{g=1}^{N_g} P_g^t = D^t$$

b) The generation output of each unit is relaxed to:

$$P_g^t = 0 \qquad \text{for:} \quad t < SU_g^0 \quad \text{or} \quad t \geq SD_g'$$

$$P_g^{min} \leq P_g^t \leq P_g^{max} \qquad \text{for:} \quad SU_g' \leq t < SD_g^0$$

$$0 \leq P_g^t \leq P_g^{max} \qquad \text{for:} \quad SU_g^0 \leq t < SU_g' \quad \text{or} \quad SD_g^0 \leq t < SD_g'$$

c) Start up cost, $SC_g$ :

Start up cost increases with the time that a unit has been off. The earlier to start up the unit, the less will be the cost. Also, if the contribution to start up cost of the next day is included, then the start up cost decreases as time to shut down increases. Therefore minimum start up cost is incurred if a unit is started up at first allowable start up instant, $SU_i^0$, and shut down at latest allowable instant, $SD_i'$. Given the start up/shut down intervals by the branching, a minimum start up cost can be computed and the additional cost of delaying the start up or bring forward the shut down time can be evaluated.

$$SC_g(SU_g, SD_g) = SC_g^0 + SC_g^A \tag{3.4}$$

where

$SC_g$ = total start up cost of unit $g$ for the study period

$SC_g^0$ = minimum start up cost i.e start up at $SU_g^0$ and shut down at $SD_g'$

$SC_g^A$ = additional cost for start up/shut down at an instant other than $SU_g^0$ and $SD_g'$. For lower bound calculation, $SC_g^A$ can be taken as zero.

The solution of Eq.(3.3) is solved by a Maximum Margin Return algorithm described by Fox[62]. The disadvantages of Cohen and Yoshimur's solution method are that the branching mechanism can be very complex when various discrete constraints are included in the branching procedures. Furthermore, the amount of branching can be enormous when the number of units in the system is large and the study horizon is long. Even though there might be many infeasible start-shut intervals, feasibility considerations will impose further complexity to the branching process. Neither are the solutions to the subproblems very tight, since the generation outputs are allowed to vary between zero and minimum stable output within the allowable start up/shut down intervals. Cohen and Yoshimura also suggested that reserve constraints can be included in the objective function of the subproblem using Lagrangian multipliers. The methodology was shown to be effective for a relatively small system having 19 units only.

Ohuchi, Kaji[163], Dillon and Egan[51] explored the obvious branching rule of permissible on/off of a unit at each consecutive sub-period. For any node, there will be two branches: one with the unit under consideration assigned to be on and other with the unit assigned to be off. This branching rule is carried on for the same unit for each consecutive subperiod. When all subperiods are accounted for, the branching process is continued with consideration to the next unit. They proposed a compact way of describing the operating state of all units at any node of the branch-and-bound tree. If values 1 represents unit on, 2 represents unit off, 0 represents operating state unknown and if a system has $N_g$ generating units and a study horizon is subdivided into T subperiods, then a $N_g$xT matrix with 1, 2 or 0 as its components will completely describe the commitment schedule of all units at any node. Fig.3.6 illustrates the branching process of a 3-unit system with study period subdivided into 3 intervals. Dillon[41] suggested that a priority list can be incorporating in the branching process by branching the unit at the highest merit order first. Indeed the use of such a list allows the problem solver to enter his knowledge of the nature of the solution in aid of the solution algorithm. The list indicates the searching direction and should this be erroneous the BBT algorithm would reject it and offer a better alternative. However, even with the assistance of a priority list, the approach was demonstrated to be computationally viable for very small systems only.

Lauer, Bertsekas, Sandell and Posbergh[118] imbedded the Lagrangian relaxation approach under the framework of the Branch and Bound algorithm. They argued that for large scale problems, the possible ways to constrain the discrete generator on/off decision variables are astronomical. It is not computationally practicable to obtain the optimal solution

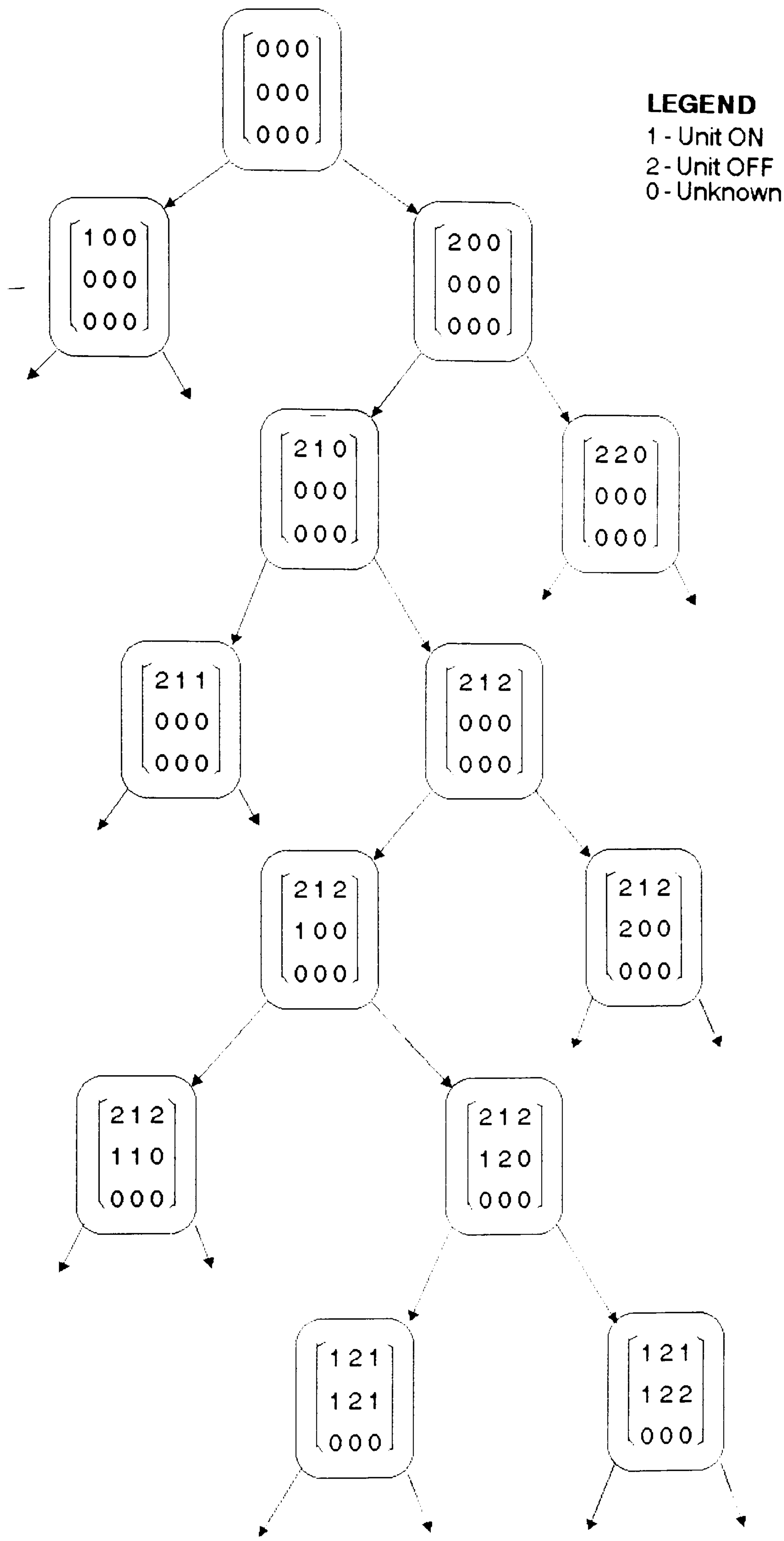



Fig.3.6 Branching Example - Unit On/Off Partitioning Criterion

by gradual constraining of all the decision variables as in most BBT approaches. In their method only a subset of the generating units which are least accurately determined will be decided using the branch-and-bound approach. Others variables are decided within the lower and upper bound computations of the subproblems. The subproblem is solved utilizing a mixture of Lagrangian multipliers, dynamic programming, Newton iterations and duality problem formulation as originally suggested by Muckstadt and Koenig[151]. To efficiently eliminate as many unnecessary tree branches as possible, a tolerance factor $\epsilon$ is introduced. For any node whose lower bound is within the $\epsilon$ boundary of the latest best upper bound, the node is eliminated from further consideration. It recognizes that for a system with a large number of generating units, the number of schedules near to the optimum can be large. The tolerance factor therefore has a second important function. When there is only one remaining node left and its upper and lower bound are within the $\epsilon$ tolerance, this will signify that the optimal schedule is obtained within the specified tolerance. In Lauer's method, besides determining the lower bound solution of a node as in most BBT approaches, an upper bound solution for the best lower bound node is also calculated. This is designed to help eliminating sets of unpromising possible solutions from consideration and hence improve the solution speed. Lauer reported that in many studies a few branchings or no branching are needed to achieve a solution near to 0.25% of the optimal. Tests on large systems ranged up to 250 units in 2-hour steps were said to complete in 30 minutes of VAX 11/780 CPU time. The main computational effort of the scheme is spent on the solution of the subproblem utilizing the Lagrangian relaxation approach. BBT is used mainly to safeguard a solution reasonably close to the optimum. Despite the success reported by Lauer, the BBT algorithm is still considered as ineffective for large scale power systems.

The inflexibility of BBT to consider multiple constraints is the major criticism. Multiple constraints require a more sophisticated solution method for the sub-problems, more relaxation and probably more branching which all lead to further computational effort. The BBT approach, however, has two inherently desirable characteristics. It gives an indication of the proximity of the optimal solution by virtue of lower/upper bound values of their subproblems. It also has the unique feature that during high computing loading periods, the solution process may be stopped temporarily before completion. The best upper bound solution obtained so far can be saved and made use of when the unit commitment scheduling process is resumed as the computer loading is reduced.

### 3.4 Lagrangian Relaxation

In the search for a rigourous approach to the unit commitment problem, Lagrangian relaxation[41,118,160] methods have the significant contribution of providing a theoretical foundation to decompose the problem into separable smaller optimization problems. Using the Lagrangian multipliers, Merlin and Sandrin[145] include the system coupling constraints, i.e. power balance and spinning reserve requirements in the objective function and proposed a max-min formulation of the form:

$$\operatorname*{Max}_{q^t, m^t} \{ \operatorname*{Min}_{P_g^t, \mu_g^t} L(P_g^t, \mu_g^t, q^t, m^t) \} \tag{3.5}$$

where the Lagrangian function

$$L(P_g^t, \mu_g^t, q^t, m^t) = \sum_{t=1}^{T} [F_g(P_g^t, \mu_g^t) -$$

$$q^t ( \sum_{g=1}^{N_g} P_g^t - D^t) + m^t ( \sum_{g=1}^{N_g} \mu_g^t P_g^{max} - D^t - R^t)]$$

$F_g(P_g^t, \mu_g^t)$ = unit generation cost function including start up, shut down or banking costs.

$P_g^t$ = generator output levels

$\mu_g^t$ = an integer indicator which equals 1 for unit $g$ in operation at time $t$.

$D^t$ = system load demand at time $t$

$R^t$ = system spinning reserve requirement at time $t$

This is subject to the normal unit minimum and maximum output constraints. Note that the system "coupling" constraints:

$$0 = \sum_{g=1}^{N_g} P_g^t - D^t \qquad t = 1, 2, 3, \ldots, T$$

and

$$0 \leq \sum_{g=1}^{N_g} \mu_g^t P_g^{max} - D^t - R^t$$

are included in the Lagrangian function using Lagrangian multipliers $q^t$ and $m^t$. The essential characteristic of the approach is that for a given set of $q^t$ and $m^t$, the minimisation of the Lagrangian functions may be decomposed into $N_g$ additively separable local minimisations, each with respect to a generator, of the form:

$$\underset{P_g{}^t,\mu_g{}^t}{\text{Minimize}} \quad \sum_{t=1}^{T} \{F_g(P_g{}^t,\mu_g{}^t)-q^tP_g{}^t+m^t\mu_g{}^tP_g{}^{max}\} \qquad (3.6)$$

subject to unit constraints such as output range limits. Other unit constraints such as minimum up/down times can also be included in the local minimisation problems. Because there are no coupling constraints with other units in the system, the local minimisation problems may be solved relatively easier than the original problem. Nieva[160] and Merlin et al [145] suggested that dynamic programming is an efficient computational algorithm for the local minimisation problems. The Lagrangian multipliers are updated using a subgradient technique to ensure their convergence and hence to provide a feasible solution in a finite number of iterations. The method generally does not give an optimal solution because of non-convexity of the original problem and also because of the difficulty of locating the optimal values of the Lagrangian multipliers $q^t$ and $m^t$. It however can produce lower and upper bounds of the optimal solution within 0.5% of the optimum. Lauer[118] showed that Lagrangian relaxation approaches are capable of scheduling system sizes to 250 units within acceptable computation time.

### 3.5 Dynamic Programming

Dynamic programming (DP) is the brain-child of Professor Bellman. The essence of DP approach is centred on the concept of the PRINCIPLE of OPTIMALITY. Unlike other mathematical programming methods, such as the linear programming approach, there is not a definite form of a problem which can be readily recognized as solvable by the DP approach, nor are there any necessary and sufficient conditions which can be examined to conclude that a problem is not solvable by DP methods. Appendix C gives a brief introduction to the principle of optimality by a simple example. For detailed exploration of the DP methods, the interested reader will be able to find much material in text books[11] and various publications.

Briefly, DP solves a problem by subdividing the problem into a number of subproblems called stages. In each stage, the solution variables may assume many possible states. Each state is associated with a status cost (also commonly referred to as return). DP approach is characterized by a recursive formula which is used to compute the optimal status cost of a state. An optimal solution to the original solution is obtained when the minimum accumulated cost is incurred by following an optimal trajectory span from the first to the last stage of the problem.

The difficulties of utilising a DP formulation to solve an optimization problem include the following:

(1) The way to subdivide the original problem into a number of subproblems is not obvious.

(2) The efficacy of a DP approach depends heavily on how the subdivision is done. Indeed the way to subdivide the problem has a direct effect on whether the problem is solvable by a DP method. The same problem subdivided in

one way may lead to infeasibility while formulated in another way the optimal solution may be obtainable.

(3) The recursive formula for optimal status cost calculation depends on how the original problem is broken down into stages. It has the most profound effect on the efficiency of the method. However, there is not a unique nor structured way to derive such an important formula.

(4) The number of possible states in each stage can be very large. DP methods generally require enormous memory to store the many possible states and long computer time to determine their respective status cost. This is the well known "curse of dimensionality" problem of DP implementations.

Despite the difficulties mentioned above, DP has found many applications in electricity supply industries both in the planning phase and operational control. The earliest applications of DP technique on unit commitment problems were perhaps by Udo[212] and Lowery[130] who by coincidence both published their algorithms in 1966. These two pioneers applied the DP optimality principle in two completely different manners. In this thesis, Udo's approach is arbitrarily classified as a time variant implementation and Lowery's technique as a time static implementation. In the time variant implementation, each subinterval of the forecast load period is regarded as one DP stage in the sense discussed above. The accumulated operational cost over the whole study period for all unit combinations in all stages is formulated as one objective function. While theoretically such a formulation will give a true optimum to the problem, there are computational difficulties associated with it. In the time static implementation, each subinterval is treated separately. Within each subinterval, DP optimization is carried out to find the optimal unit combinations to give the minimum operational cost for that interval. The linkage between the sub-intervals is considered based on a heuristic approach similar to the merit order scheme. While the time static approach is not an ideal solution, it has certain advantages over the former formulation. In the following sections, both techniques will be reviewed in greater detail and the advantages/disadvantages of each formulation will be discussed.

### 3.5.1 Time Variant DP Implementation

As stated earlier, the unit commitment problem is to determine the on/off schedule of the generators for a period of 24 hours or longer in the future so that the forecast load will be met at minimum cost. Since the daily load curve can be approximated by subdividing it into hourly intervals with constant load within the hour, the obvious DP formulation is to treat each interval as a stage and a recursive DP optimization process is applied from the first interval to the last[21,86,187,135,165,213,217,218,221]. The method is summarized as follows.

**Step 1 : Forward Optimal Status Cost Calculation**

Starting from the first interval, determine the feasible on/off combinations of the generators for each interval (or stage). Since the minimum on/off time constraints restrict the allowable start up/shut down of a unit in the subsequent intervals, the historical data of how long a unit has been on/off is part of the attributes of the generator on/off combinations. The same on/off unit combination might occupy many states in an interval because the units have been on/off for different durations. Each of these states has to be treated separately because each of them may lead to different feasible unit combinations, operational histories and hence different costs at the later intervals. For each feasible state of any interval k, an optimal status cost is calculated. This optimal cost is the minimum fuel and start up/shut down costs accumulated from the first interval to this state of interval k. The optimal status cost of state j in interval k is determined by:

Optimal status cost of state j in stage k =

min { optimal status cost of state i in stage (k-1) +
transition cost from state i of stage (k-1) to
state j of stage k }

for all states i belonging to stage (k-1) (3.7)

A graphical interpretation of the formula is shown in Fig.3.7. The transition cost is the operational cost to satisfy the forecast load demand of interval k plus any start up/shut down cost that may be incurred.

**Step 2 : Back Tracking**

When all possible states and their respective status costs are computed, the minimum status cost at the last interval is the optimal operational cost for the whole study

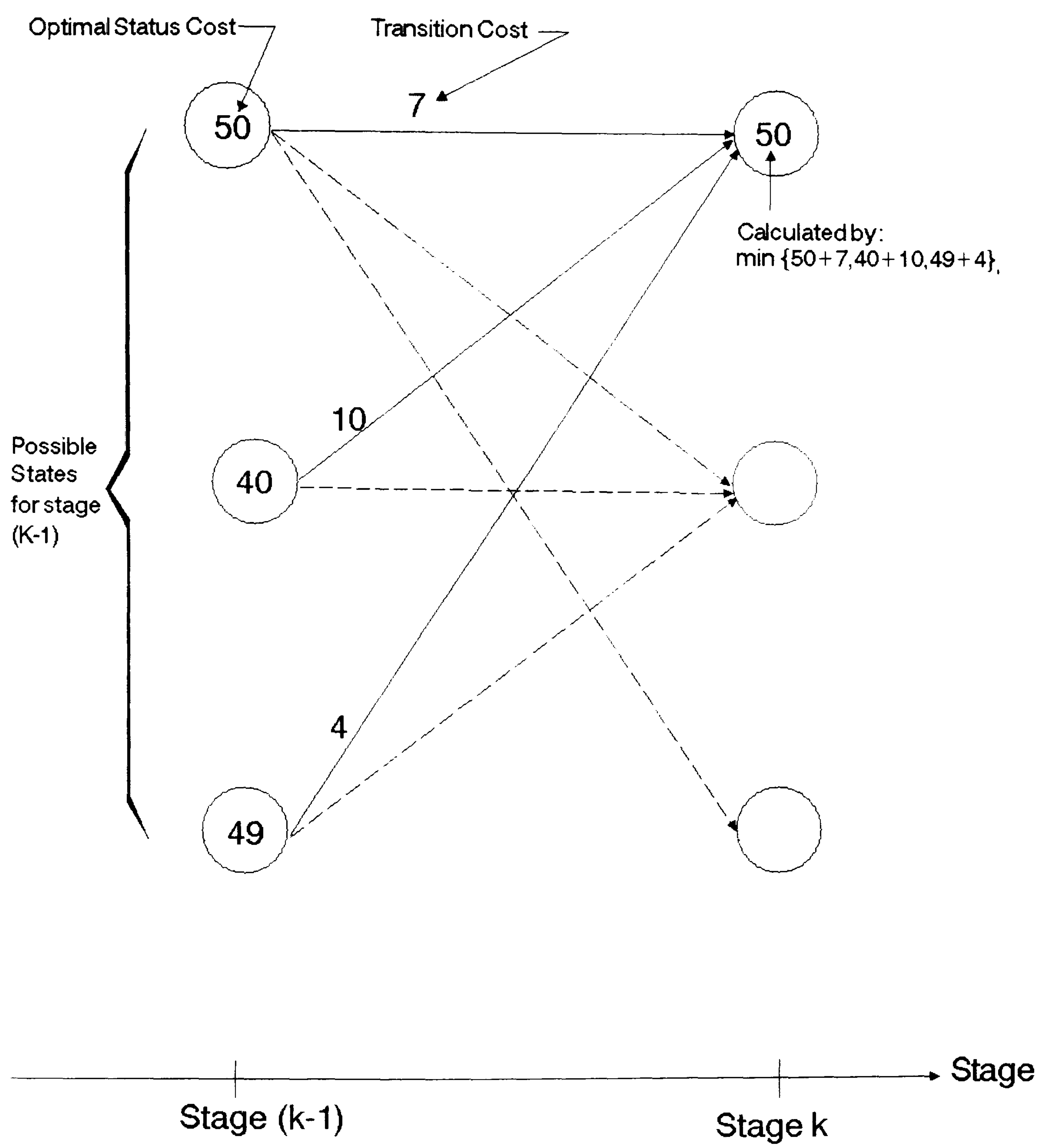



Fig.3.7 Optimal Status Costs in a Time Variant DP Implementation

period. The trajectory from the first interval leading to this minimum total cost at the last interval is the optimum path. The optimal unit commitment schedule for all intervals is obtained by back tracking this optimal trajectory from the last stage.

The pioneering work by Udo[212] is similar to the solution scheme described above. This is known as a full DP solution because the approach attempts to include every feasible combination and state of the generating units for all time intervals in the solution process. Theoretically a commitment problem of any complexity and constraints may be modelled using this approach and a global solution also may be obtained. The difficulty associated with this method is the inherent so called "curse of dimensionality". When all possible unit on/off combinations together with all linear and non-linear constraints are considered, the number of possible states in each interval will be astronomically large even for a medium size power system. As a result, even assuming that an algorithm has been worked out to overcome the mammoth memory and storage problems, the time taken to solve the commitment problem may be so long that by the time the schedule is available, the schedule may be possibly already out of date.

For the time variant DP approach to be practical, the number of states in each interval must be reasonably small. Various approximations have been proposed in an attempt to make the algorithm computationally manageable. The corner stone perhaps may be due to Pang, Chen, Sheble and

Albuyeh[164,165]. They proposed a "windowing" technique in which a restrictive set of generators will be searched for the optimal combinations. Utilizing the priority list, they proposed that this restrictive set shall consist of a fixed number of units above and below the minimum capacity requirement to satisfy load and spinning reserve as shown in Fig.3.8. With this restrictive generator set, the possible on/off combinations can still be very large. Further reduction on the number of states is achieved by considering a limited number of possible combinations of these few selected generators. Sequential combination search and truncated combination search[165] techniques are proposed. Sequential combinations are generated by turning on each unit in the window in descending merit order and truncated combinations are generated by considering a fixed smaller number of units from those units in the search window. The methodology is demonstrated to be effective using an example system of 96 units giving a solution better than those of a merit order scheme by up to 0.79%. Villaseca and Fardanesh[218] improve Pang's approach by allowing a variable search window pending on the rate of change of load. When the rate of change of load is greater than the average rate of change, a larger search window is imposed and vice versa. This significantly reduces the computational time requirements.

There are other improvements proposed by various workers. Van Meeteren[216], Waight et al[221] incorporated network constraints in the solution process by executing LP dispatches on all feasible combinations in each DP stage. Hobbs, Hermon and Sheble[86] re-addressed the importance of ramping rate

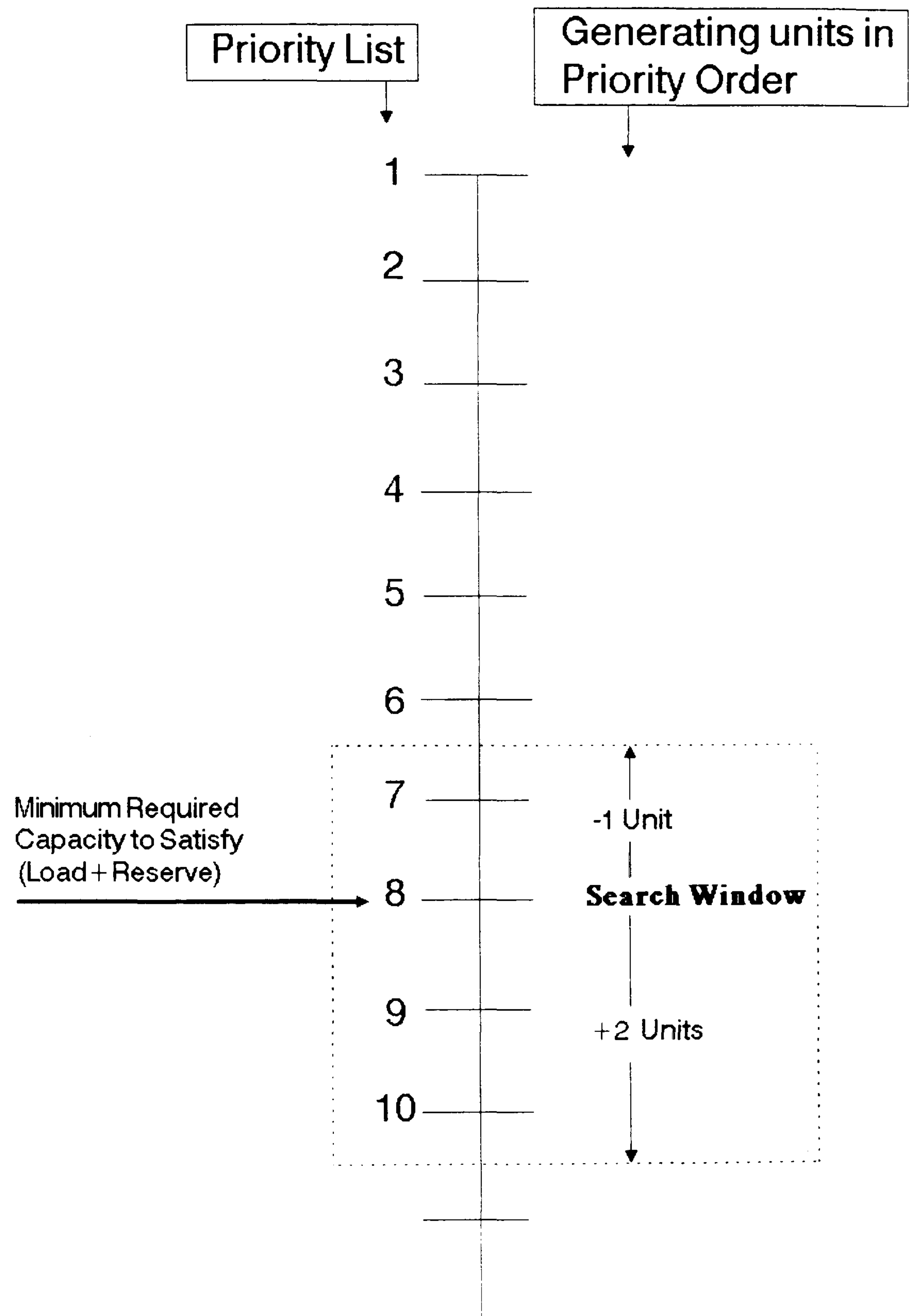



Fig.3.8 Search Window Example for a Restricted Search DP Approach

restrictions and kept track of more than one state of the same unit combination but with different operating history in each time interval. This consideration inevitably led to higher computational time but was shown to be able to produce a feasible commitment schedule which may not have been possible otherwise.

To date, the time variant DP approach is a well matured technique for unit commitment scheduling. It is a strong contender to the conventional merit order scheme and is adopted in some commercially available Energy Management System software packages.

### 3.5.2 Time Static DP Implementation

As mentioned in section 3.1, the merit order table is obtained by ranking the average full load costs of the units in the system. This merit order table has at least two major deficiencies. On the unit level, the non-linear output dependent incremental cost information is lost during the ranking process. On the system level, the ranking gives the false impression that minimum generation cost can be achieved by loading and unloading the units by following the merit order. The inaccuracy in merit order representation of the generation system is probably one of the causes which leads to sub-optimality in a merit order unit commitment solution. Lowery proposed in 1966 that DP is a feasible technique to establish an optimal unit combination table[7,130] for the generating system without using the average full load cost approximations. His method centres on the application of the following DP recursive formula.

$$G_N(x) = \text{Min} \{ F_N(y) + G_{N-1}(x-y) \} \quad \text{for } N=2,3,\ldots N_g \tag{3.8}$$

where

$G_N(x)$ = the minimum running cost of carrying x Mw load on N generating units.

$F_N(y)$ = the cost of carrying y MW load on unit N.

$G_{N-1}(x-y)$ = the minimum cost of carrying the remaining $(x-y)$ MW load on the (N-1) generating units already considered..

$N_g$ = Number of generating units available in the system.

y = subject to the minimum and maximum stable output constraint of the unit.

The expression describes a simple computational algorithm which can be used to construct an input-output curve of all the available units in the system by building up from a single unit. Knowing the input-output curve of one unit, one can determine the minimum cost-curve for two units by the above expression. When the minimum cost curve of two unit is found, a third unit can be considered again with the above formula. The expression is applied recursively until all units in the system have been considered.

Using the optimal fuel cost table so obtained, Ayoub and Patton[7] proposed a "multi-pass" unit commitment method. Their method firstly establishes an initial schedule based on the optimal unit combinations for each time interval suggested by the optimal fuel cost table. The schedule is then scanned considering the probabilistic security constraints in every time interval of the study period. The commitment schedule is modified if security constraints are not satisfied. A third pass is then activated which takes into account the effect of start up cost and other operational constraints, and the schedule is modified again if needed.

The advantage of this class of DP approach is its simplicity amenable to the merit order scheme. Furthermore, the method is applicable to nonlinear cost functions of the generators. Similar to the time variant DP methods, the disadvantage of the method is the computer time and memory requirements. For very large systems, the optimal fuel cost table could take a very long time to construct. The largest system that has been tested using this approach is only 20 units.

### 3.6 Summary

This chapter has outlined some of the prominent algorithms employed by the electricity industry to schedule the on/off of the thermal generating units so that the accumulated operating cost is minimized over the study period. The methods reviewed utilize a wide range of techniques including heuristic merit order schemes, mixed-integer programming, branch-and-bound, Lagrangian relaxation and dynamic programming. Among these, merit order schemes represent the traditional approaches and are still widely used by many utilities because of their simplicity, practicality and computational efficiency. To date, mixed-integer and branch-and-bound methods are applicable to smaller systems only because of their inefficiency. Lagrangian relaxation is perhaps the most rigourous of all the methodologies. It is recently reported to apply successfully for a system of 250 units. The disadvantage of the method is the difficulty to include many operational constraints in the solution process. Dynamic programming is gradually becoming the industry accepted methodology to solve thermal unit commitment scheduling problems. In particular, restricted search time variant DP implementations are adopted in some commercially available Energy Management System packages. The time static DP approach has the advantage of using an optimal unit combination which can deal with highly non-linear generation cost models. The method is relatively simple but computationally inefficient. The technique proposed by the author in the next chapter may be included in the same time static DP category. The optimal fuel cost table however is determined by a different formulation which greatly improves the execution time.

# CHAPTER 4

## UNIT COMMITMENT USING COMPOSITE COST DYNAMIC PROGRAMMING APPROACH

Operating cost saving can be achieved by proper scheduling of the start-up and shut-down of available generating units. In the last chapter, the existing methods which aim to produce the optimal on/off schedule of the generators were reviewed. This chapter describes an original computational algorithm[36] based on the dynamic programming (DP) principle for selecting <u>and</u> assigning loading levels of generators to obtain the optimal commitment schedule. It makes use of a composite generator operating cost model to combine fuel cost and other operating costs to represent the generation cost. The method brings the dimensionality problem normally associated with the DP technique under control by storing only an appropriate range of stages and states necessary to allow the computation to proceed. Experience of the algorithm shows that the computer time required to obtain the optimal unit combination is <u>independent</u> of the number of generators in the system but depends on the total generating capacity and the required accuracy. An approximation formula is presented for estimating the computer time requirement. A test system which has 224 units and 51,750 MW installed capacity is used to demonstrate the potential practicability of the technique to a large system.

## 4.1 Limitations of Existing DP Approaches

The unit commitment problem is a highly complex optimization problem. Due to the non-linearity and time variant characteristics of various constraints governing the operation of the generating units and of the system, rigourous mathematical optimization methods are in general impractical for the solution of large scale systems. With the advance of computer aided control in power systems, many different solution methods with varying degrees of simplification have been proposed. Among these existing techniques, the dynamic programming technique has attracted considerable interest because of its inherent flexibility to consider the non-linearity and time dependent nature of the constraints. In particular, the time variant DP implementation as discussed in the last chapter has become increasingly popular. To overcome the excess computer storage and processing time requirements, various approximations have been devised in the time variant DP approaches. The most widely adopted simplification is to use a 'windowing' technique such that only a small subset of the possible generator on/off combinations is considered in each time interval. An optimal schedule for the entire study period is then computed assuming those selected combinations are the only possible generator combinations. Such truncation of many possible generator combinations will probably introduce suboptimality to the solution. Another disadvantage of the time variant DP approach is the problem of assigning the appropriate generation levels among the 'on' units. This is overcome by assuming a linear or piece-wise incremental cost model for the generators. Merit order or linear programming economic dispatch techniques are then applied to each possible unit combination to obtain the optimal sharing of generation output among the 'on' units satisfying the system load demand. This complication adds substantial computational effort to the solution process and introduce further approximation to the modelling.

The time static DP implementation, on the other hand, treats each time interval independently in the DP process. As illustrated by Ayoub and Patten[7], the scheduling procedures for this class of DP approach are similar to the heuristic merit order schemes. The significant difference of the static DP approach is that an optimal generation table is used instead of a priority list. The importance of the heuristic approach for unit commitment solution cannot be over emphasised. With the increasing volume of sophisticated methods available, the merit order scheme is still widely regarded as the only practical solution for some large systems. For example CEGB in the U.K. uses a program called GOAL which is based on priority order concept. The essential advantage of time static DP unit commitment method is that it maintains the inherent simple approach of a merit order scheme and tends to eliminate the average full load cost approximation imbedded in a priority list. The generator combination table relates the optimal unit combination together with optimal load sharing to the total generation requirement. The rigid loading/unloading order of generators in a priority list approach is abandoned and non-linear generation costs can be modelled accurately. Time static DP, however, has not gained a parallel recognition as merit order approaches. The most important factor which contributes to the situation perhaps is that the execution time of a time static DP approach is significantly more than a merit order scheme. The CPU time required to construct an optimal generation table is much longer than to build a priority list. The optimal generation cost table is therefore generally compiled off-line or once at the beginning of the computer program execution. As a result, the optimal generation table tends to become inflexible to deal with practical issues such as derated generator output at certain time of a day, start-up/shut-down costs and must on/off instructions etc. The time static DP method therefore does not inspire the solution speed which is of paramount important for large system applications. The subject of the thesis is to overcome the computational problem of building this optimal generation table. An original DP recursive formula is devised. A novel composite cost dynamic programming (CCDP) method utilizing this new DP recursive formula to schedule the unit on/off is proposed. In the following sections, the unit commitment problem is formulated and the solution steps of the CCDP method are described.

## 4.2 Objective Function

As in most unit commitment algorithms, the CCDP approach subdivides the scheduling horizon into a number of intervals, say T. The objective function of unit commitment is then to minimise the total operating cost over the entire period satisfying the load demand at all times.

$$\text{Minimize} \sum_{t=1}^{T} \{ \sum_{g=1}^{N_g} F_g(P_g^t) + S_g(P_g^t) \} \tag{4.1}$$

where

$P_g^t$ = generator output (MW) at time interval t.

$F_g(P_g^t)$ = fuel cost of generator g supplying $P_g^t$ MW at interval t to the system

$S_g(P_g^t)$ = operating costs other than fuel cost to allow unit g to generator $P_g^t$ MW. These include start up, shut down, maintenance costs etc.

$N_g$ = Number of generating units in the system.

## 4.3 Operational Constraints

The objection function of Eq.(4.1) is subject to many equipment and system operating limitations. The CCDP method which has been implemented schedules the units with the following constraints taken into consideration.

### 4.3.1 Unit Minimum and Maximum Output Limits

These output limits define the allowable output power of the generating units for the studying period. These limits are normally static, specified by the manufacturer. But as the generating unit ages, these limits may vary and must be verified by the power station manager from time to time. Outage of auxiliary equipment also temporarily affects the output power range of the plant. GT's outputs are sensitive to ambient temperature. The maximum output of GTs may need to be estimated in advance in associated with the forecast weather conditions. These practical considerations may be specified as part of the input data to the unit commitment computer program.

$$P_g^{min} \le P_g^{t} \le P_g^{max} \qquad \text{for } g=1,2,3...N_g \qquad (4.2)$$

### 4.3.2 Fuel Cost Non-linearity

The fuel cost to output power relation is often non-linear. Non-linearity can be due to various reasons such as the multi-valve steam throttle design commonly used in the U.S.A. and multi-fuel intake at various output levels. In the present implementation, a generally accepted approximated fuel cost function in quadratic form is assumed.

$$F_g(P_g^{t}) = A_g + B_g\, P_g^{t} + C_g(P_g^{t})^2 \qquad g=1,2,3....N_g \quad (4.3)$$

where

$A_g$, $B_g$, $C_g$ = fuel cost coefficients representing the constant, proportional and quadratic cost multiplying factor.

It should be noted that the proposed unit commitment method is not restricted to quadratic fuel cost functions but is applicable to practically any form of fuel cost function. It may be non-differentiable, non-convex or empirical. One of the essential feature of the proposed algorithm is this generation cost handling capability.

### 4.3.3 Start-up Cost

The start-up cost of a unit depends on the length of time the unit has been shut-down prior to starting up. Without loss of generality, the following start-up cost function is adopted :

$$S_g^U(t) = \mu_g \Gamma_g t/(1+\Gamma_g t) \qquad g=1,2,3,\ldots.N_g \qquad (4.4)$$

where

$S_g^U(t)$ = Start-up cost of unit g

$\mu_g$ = Cold start up cost

$\Gamma_g$ = Cooling rate

t = Time passed since the unit shut-down

### 4.3.4 Banking Cost

The method also considers, instead of complete shutting down of a boiler-turbine generating set, the option of maintaining the temperature and pressure of the boiler so as to provide a hot standby to the system. The main consideration is whether it is more economical to shut down the unit first and then restart it later or to bank the unit for some period and then bring it back on line later. A fixed fuel cost rate per unit time, $\beta_g$, is used to model the banking cost characteristic.

$$S_g^B(t) = \Phi_g + \beta_g t \qquad (4.5)$$

where $\Phi_g$ represents a fixed lump sum cost involved in changing the status of the unit from synchronization to banking and back to synchronization at a later time.

### 4.3.5 Shut-down Cost

The shut-down cost of a thermal unit is normally small compared with its start up cost. A fixed shut-down cost, $S_g^D$, may be used to reflect the labour cost and residual heat lost involved in shutting down a unit.

### 4.3.6 Minimum up/down Time

In daily operation there is generally a requirement that a unit runs or stays shut-down for a certain minimum period of time before it changes status again. There may not be any technical reason why such restrictions should be imposed. However, frequent start-up and shut-down will cause the following problems to the station operation. They increase the thermal stress of the boiler and generator housing and hence reduce the expected operating life of a generating plant. They reduce the time period between scheduled maintenance outage and drain the limited resources on crew availability. Minimum on/off period is therefore generally specified by station managers.

### 4.3.7 Fixed Generation Units

These are the pre-scheduled units. The system operator may pre-schedule certain units to must be "on", must be "off" or fixed generation for certain intervals of the study period. Specification of such requirements are frequently issued by the system operators in the light of new data on the generation system. Scheduled out or forced out units can therefore be treated as must be "off" units. Units which are pre-specified on/off will reduce the commitment problem to certain extend. However, the output level of the must be "on" units affects the generation levels of the other synchronized units, the must be "on" units are necessarily included in the unit commitment decision process.

### 4.3.8 Derated Capacity

Outages of the auxiliary equipment frequently lead to reduced output capacity of a generating unit. Derated capacity, or changes from derated state to full capacity state or to another derated state in certain intervals of the study period also can be specified by the system operators. The inclusion of derated capacity does not affect the overall optimization scheme but changes the maximum output available from the unit. For any unit at any interval, its maximum available generating power becomes:

$$P_g^{min} \le P_g \le \text{Min}\ \{P_g^{max}, P_g^{Derated\ Cap}\} \qquad (4.6)$$

$$\text{for } g=1,2,3...N_g$$

### 4.3.9 Power Balance

Since electrical energy cannot be stored in an appreciable amount economically, the total generation available from the committed units in any interval must equal to or exceed the total system load demand. More specifically, the summation of the assigned generation output of the synchronized units must balance the system load. In order to achieve optimal generation cost, accurate determination of the expected MW output of the units must also be computed in the scheduling process. The proposed CCDP algorithm has the advantage of combining unit selection and optimal generation sharing between units into one integral step.

$$\sum_{g=1}^{N_g} P_g^{\,t} = D^t \qquad \text{for } t=1,2,3,\ldots T \tag{4.7}$$

### 4.3.10 Security Requirement

One of the primary objectives of unit commitment is to ensure the security of system to withstand predicted load uncertainty and generation forced outage. Spinning and non-spinning generation reserve must be allocated in the unit commitment schedule to meet the security criteria set down by the management. Without lost of generality, spinning reserve can be defined as the extra generation available on demand from the synchronized generators within a short time period typically 1 to 5 minutes. Non-spinning reserves are the extra capacity available from the synchronized <u>and</u> off-line units within a time span in the region of 20 to 120 minutes. The reserve level is quite often a compromise between engineering and economic constraints and is itself a complex optimization problem. Once the criterion is decided, it can be realised in the daily unit commitment program. In most existing unit commitment methods, simple reserve models are used in which the spinning reserve requirement is satisfied by specifying the total synchronised generation capacity to exceed the total load demand by a certain margin. Similarly the non-spinning reserve is satisfied by specifying the amount the total generation available from both the on-line and off-line units within the limiting time criterion to exceed the predicted load. Another popular spinning reserve model is to have the on-line capacity to exceed the system load by the largest capacity of any synchronized unit for each respective interval. This criterion makes use of the fact that the largest unit is usually the most efficient unit in the system and is most likely loaded to its maximum capacity. The outage of such a unit will probably produce the worst disturbance to the system. In the proposed unit commitment algorithm, two spinning reserve conditions may be specified by the system operators. Firstly the total capacity of the committed units at any time interval must exceed the forecast load of that interval by a certain percentage. This is designed to provide the option of specifying a reserve requirement similar to the conventional approach. The second condition is that the loss of generation of any loaded unit must be able to be picked up by the remaining on-line units within a specified short time period. The computational mechanism for this second consideration will be described in more detail later. Non-spinning reserve is not included in the present implementation but a similar approach can be applied to this security requirement.

Figure 4.1 depicts the various input data required by the CCDP unit commitment program. The program produces two results, namely, the commitment schedule and the estimated production cost for the forecast load. The commitment schedule feeds the economic dispatch program for finer tuning of the load sharing between the committed units.

### 4.4 Scheduling Algorithm

The practicality of the merit order approach in scheduling unit commitment for large scale systems is cited in many publications. Many new developments of sophisticated

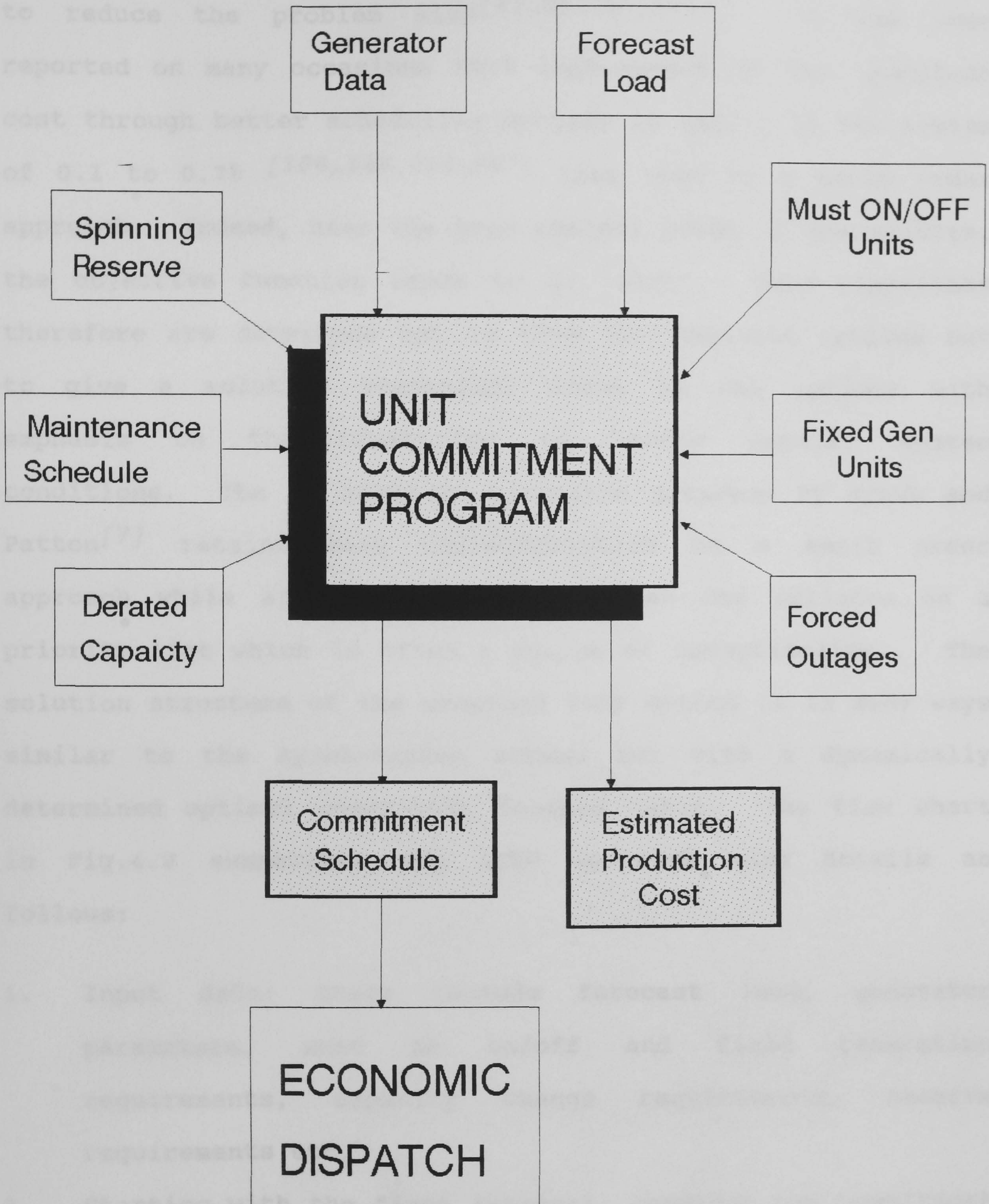



Fig. 4.1 Input and Output data of Unit Commitment Program

algorithms are still making extensive use of the priority list to reduce the problem size[41,68,118,217]. It has been reported on many occasions that improvement of the operation cost through better scheduling methods is small; in the region of 0.1 to 0.7% [106,118,165,227] less than by a merit order approach. Indeed, near the true optimal point of the problem, the objective function tends to be 'flat'. Many algorithms therefore are developed not to give the absolute optimum but to give a solution reasonably close to the optimum with emphasis on the capability to handle special system conditions. The DP solution algorithm proposed by Ayoub and Patton[7] retains many characteristics of a merit order approach while at the same time reduces the reliance on a priority list which is often a source of suboptimality. The solution structure of the proposed CCDP method is in many ways similar to the Ayoub-Patton scheme but with a dynamically determined optimal generation loading table. The flow chart in Fig.4.2 summarizes the CCDP approach with details as follows:

1. Input data: these include forecast load, generator parameters, must be on/off and fixed generation requirements, capacity change requirements, reserve requirements etc.
2. Starting with the first interval, consider the commitment problem interval by interval. Check unit availability and form composite cost functions of each unit for the interval.

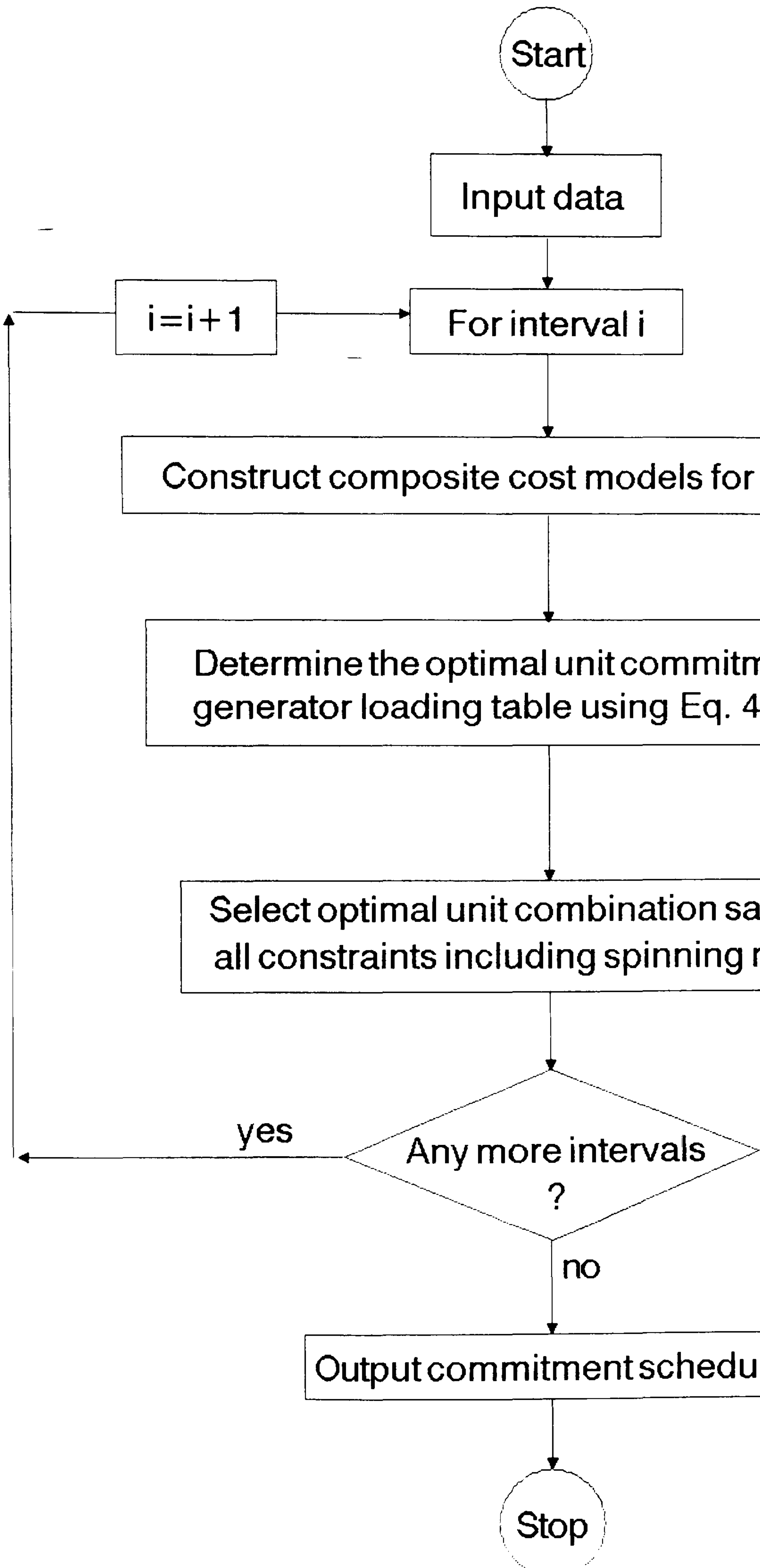



Fig.4.2 Flow Chart of CCDP Unit Commitment Algorithm

3. With the composite cost functions obtained in Step 2, determine the optimal unit combination using the dynamic programming recursive formula described in the next section so that forecast load for that interval is satisfied at minimum cost.
4. Check that the optimal unit combination obtained in Step 3 satisfies reserve requirements. If these are not satisfied, go back to Step 3 to find another combination. If reserves are satisfied, go to Step 2 for the next interval.
5. When the unit commitment for all intervals of the study period is scheduled, the overall production cost is calculated and the commitment schedule is completed.

### 4.4.1 New DP Recursive Formula

The dynamic programming approach is characterized by stages, states and decision variables at every stage, and a recursive relationship. The recursive relationship identifies the optimal sub-policy for each state of stage n based on the optimal sub-policy obtained for each state of the $(n-1)^{th}$ stage. Lowery[130], Ayoub and Patton[7] used a dynamic programming based technique to determine the optimal combination and loading of the generating units which is then used to schedule the unit commitment sequence. In the CCDP, a new computational DP recursive formula is used which builds the optimal unit combination much more efficiently. This new DP recursive formula is described in the following paragraphs.

Let there be $N_g$ units in a system with each unit having a known operating cost characteristic such as that described by

Eq.(4.3). If the generation output of a unit can be discretized to a multiple of $x$ MW steps, then its fuel cost function may now be described as costs at different output levels as shown in Fig.4.3. To find the optimal combination of the $N_g$ generating units to give a minimum fuel cost for a total system load demand $D$, the following DP recursive formula is proposed:

$$G(D) = \text{MIN} \{ G(D - jx) + \triangle F_g(jx) \} \qquad (4.8)$$

$$g=1,2,3...N_g \; ; \quad j=1,2,3...M$$

where

$G(D)$ = Optimal total fuel cost for load $D$

$D$ = 0, $1x$, $2x$, $3x$, ...$Nx$

$Nx$ = total generating, $G_{total}$, from all units

$\triangle F_g(j.x)$ = Additional operating cost for unit g to generate further $jx$ MW from its optimal loading at $G(D-jx)$

$M$ = Highest generation level of the largest unit

$G(D) = 0.0$ for $D$ 0.0

In the above equation, a stage is defined as a multiple of the generation discretization step $x$. For a system with a total capacity of $G_{total}$, the number of stages is $G_{total}/x$=N. There are two unusual characteristics of the above DP formulation. The first is that in each stage there is only <u>one</u> state which associates with a load level D. The second special feature is that the optimal sub-policies of <u>M previous stages</u> are utilized to evaluate the optimal cost of the current stage. Most DP recursive formulae have multiple states for each stage and the optimal policies for all states of <u>one previous stage</u>

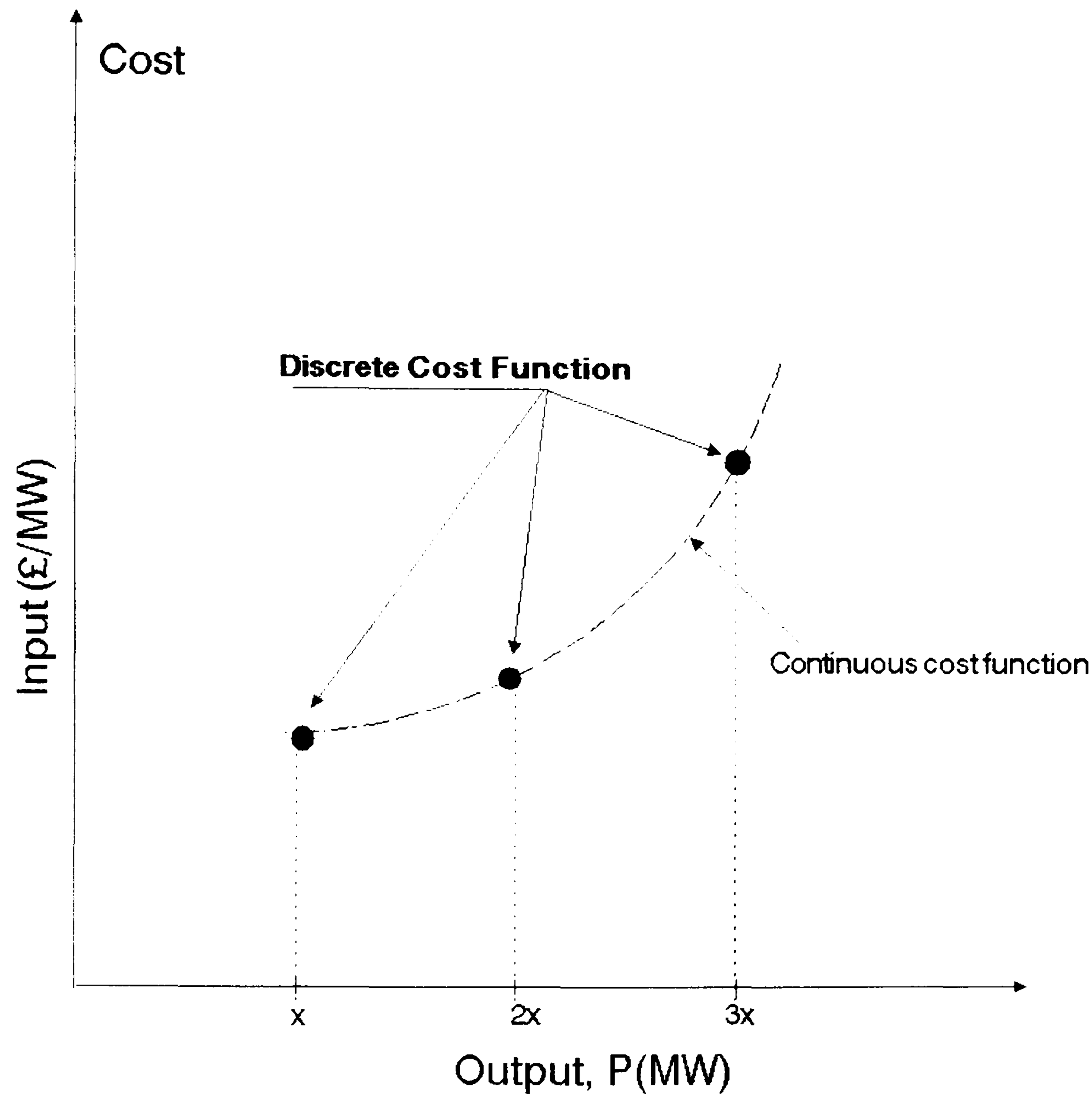



Fig.4.3 Discrete Production Cost Model for Static Time DP Unit Commitment Approaches

are used to compute the optimal cost of each state of the current stage. The DP recursive formula implemented by Lowery, Ayoub and Patton has such "normal" characteristics. The proposed equation, however, also satisfies the DP principle in that the cumulative cost of the destined state depends only on the transition cost and the optimal cost of the incident state. The optimal path leading to the incident state is not of consequence. By inspecting Eq.(4.8), it is apparent that the maximum number of possible paths from each incident stage to the destined stage is the number of units in the system. Many of such paths are infeasible because the unit may be at its maximum output at the incident stage. The transition cost, $\Delta F_g(j.x)$, associated with a feasible path is the additional operating cost incurred for the unit at the incident stage to increase its output by an additional $(jx)$MW. Since G(D) is known for $D$=0.0, optimal operating cost and the corresponding optimal unit combination at load levels $1x$, $2x$, $3x$ ... $Nx$ can be calculated using the given cost function of the units, $F_g(P_g)$, and applying Eq.(4.8) iteratively. Fig.4.4 gives a pictorial impression of Eq.4.8.

As mentioned earlier, in determining G(D), M optimal unit combinations and costs at stages having loads $D-x$, $D-2x$, $D-3x$, ... $D-Mx$ are needed. In other words, besides storing the cost functions of each unit and other necessary data, the computer memory requirement for the algorithm is only $(N_g+1)M$ words. In the U.K., CEGB has approximately 90 plants with the largest station of 4000 MW. For an accuracy of 10MW step size, the computer storage requirement will be 36.4k words. It is obvious that further memory reduction can be achieved by

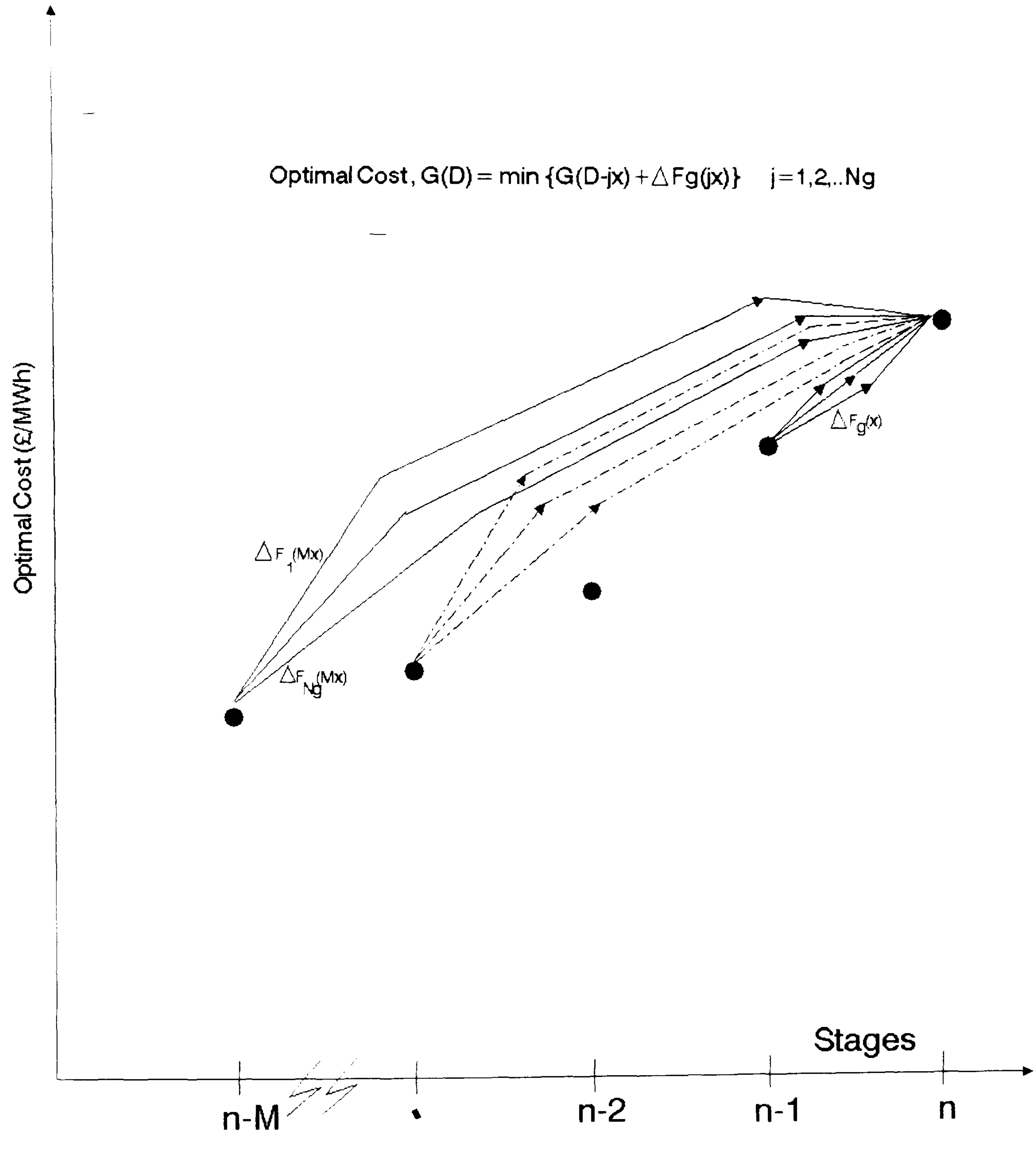



Fig. 4.4 Graphical Interpretation of the Proposed DP Recursive Formula

breaking up the largest plant into several smaller plants so that the maximum generation level, M, needed to represent the largest plant is smaller. For example, if the 4000 MW station is represented by two equal size but smaller plants, then the memory requirement will then be 18.4k words. Trials with the algorithm indicate that the number of generators in the system does <u>not</u> contribute significantly to the computer time required to determine the optimal generator combination. As depicted in Fig.4.5, the computer time requirement is a function of system capacity and desired accuracy. In the figure, the CPU time is the computer processing time required using a *Perkin Elmer 3230* to obtain the optimal generator combinations and costs for generation levels from 0 MW to total system capacity in a step size chosen. The number of stages is the total system capacity divided by the step size. The curve in the figure may be approximated by:

$$\mathrm{Log}(t)=1.843\ \mathrm{Log}(\text{total capacity/step size})-4.465 \qquad (4.9)$$

where t is the CPU time in seconds and can be used to estimate the computer time required to execute the unit commitment program. The validity of this estimation is illustrated in the results section.

### 4.4.2 Composite Cost Function

Due to the computational efficiency of the proposed DP recursive formula, it becomes feasible to calculate the optimal generation loading table for each interval of the study period. Building a new optimal generation table for each interval has the significant advantage of providing an

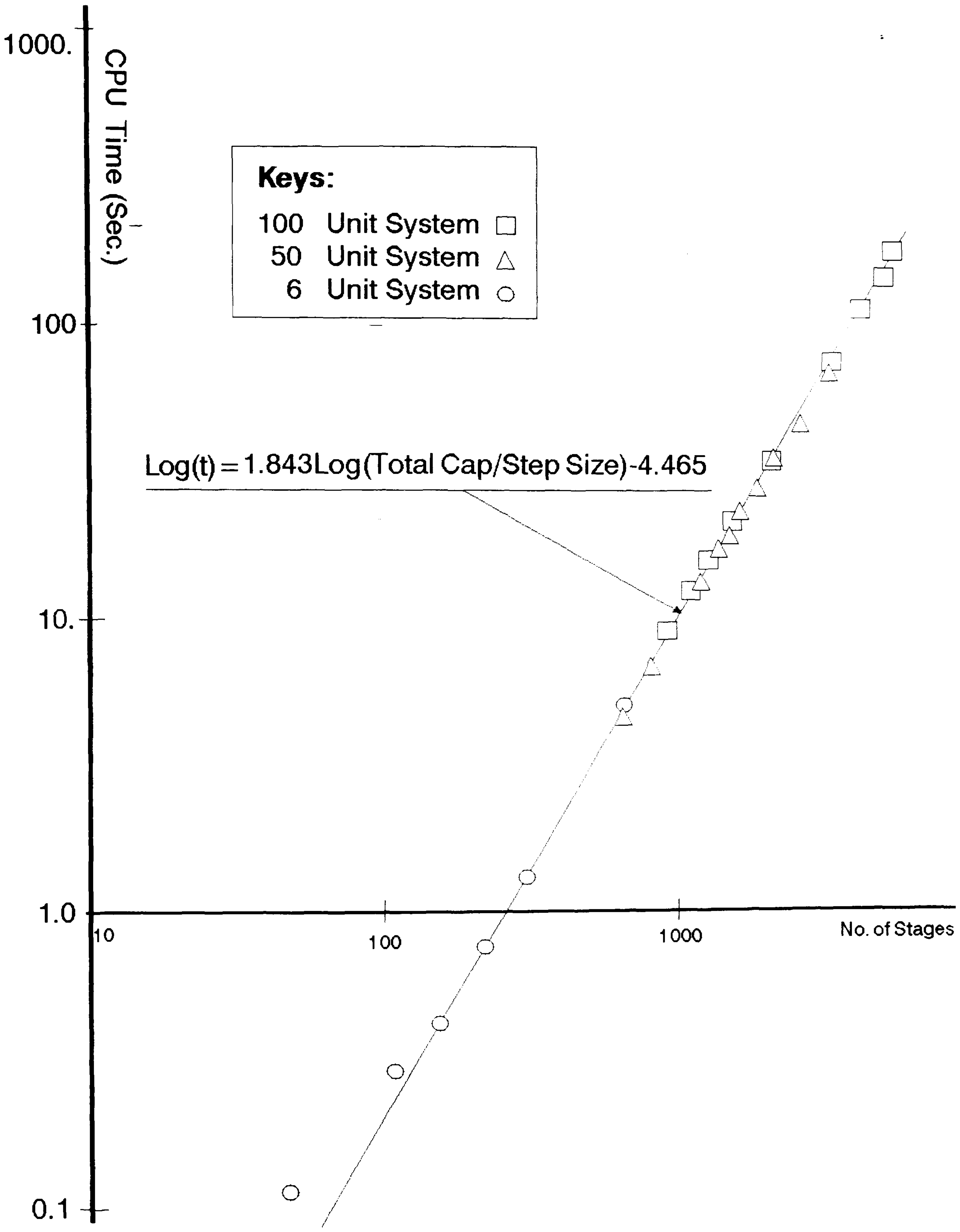



Fig. 4.5 Graph - Computer Processing Time Versus Number of Stages

opportunity to take into account any changes in unit availability, output limitations and generation model at different time intervals. Unlike the conventional time static DP approach, in the proposed CCDP approach, a new optimal generation table is constructed in every successive interval to maximize the flexibility to include changing system conditions. Furthermore, for each time interval j, a composite operating cost function of each unit i is utilised. A composite generation cost model is a combination of fuel cost function and other time dependent operating cost and is defined as followed.

1. If the unit was "off" in interval (j-1) :

$$W_i(P_i) = F_i(P_i) + S_i^U(t)/H_i \qquad (4.10a)$$

where

$W_i(P_i)$ = composite cost of unit i at output $P_i$ MW

$F_i(P_i)$ = fuel cost at output $P_i$ MW described by Eq.(4.3)

$S_i^U(t)$ = start up cost calculated using Eq.(4.5)

$H_i$ = estimated number of intervals the unit will be up if it were started up in interval j.

2. If the unit was "on" in interval (j-1):

$$W_i(y) = F_i(y) - S_i^D \qquad (4.10b)$$

where $S_i^D$ is the shut-down cost of unit i.

The composite cost model of Eq.(4.10) is an artificial operating cost function derived by using intuitive reasoning outlined as follows.

Assume that there are K units which were in "off" status during interval (j-1). The unit commitment problem is to determine which of these units, if any should be started up in interval j so that the overall cost will be minimum. Assuming that any unit started up at interval j will be shut down at a later interval (j+H) when load returns to the same level as in interval (j-1) then $H_i$ for all units will be equal to H. The total operating cost of any of these units i supplying energy in the H intervals will be:

$$W_i^{total} = F_i^j + F_i^{j+1} + \ldots + F_i^{j+H-1} + S_i(t_i)^{j+H}$$

where

$i$ = index of "off" unit. i = 1,2,3,.....K

$F_i^j$ = fuel cost of unit i at interval j

$j+H$ = interval at which load resumes to level as in interval (j-1)

$S_i(t_i)$ = start-up cost after unit i which has been shut-down for $t_i$ hours at the beginning of interval (j+H)

The effective operating cost of unit i at each interval between j and (j+H) is therefore the fuel cost plus an average start up cost as described by Eq.(4.10a). In the above description, the estimated up time of all units started up in interval j is H. For those units which are pre-scheduled to shut down before (j+H) will have a smaller expected up time and the contribution from their start up costs to the resultant composite costs will be larger than it would have been should these pre-scheduled shut-down constraints not have been there.

For a unit which was already "on" in interval (j-1), there is no start up cost involved for it to continue to operate in interval j. The unit, however, could be shut down in interval j and incur a shut-down cost to the system. Therefore, if the unit is to continue to operate in interval j, the cost to the system is effectively the fuel cost minus the shut down cost i.e. Eq.(4.10b).

The composite cost of an unit available in interval j is depicted in Fig.4.6 and is essentially the fuel cost function plus a constant component. The shape of the fuel cost curve remains the same. The constant component added to the fuel cost will therefore affect the selection of units but not the optimal loading level of the selected units. In effect, the constant components encourage the "ON" units to stay on and discourage the "OFF" units to be brought on-line unless it is financially very attractive or because of other factors.

### 4.4.3 Spinning Reserve

Spinning reserve is the excess capacity of synchronized generators above the load. Spinning reserve is costly as it implies that some units will be partially loaded at which fuel efficiency is usually less than at higher loading points. Hence it is desirable, from an economic point of view, to have a minimum amount of spinning reserve subject to acceptable risk. The proposed unit commitment algorithm commits thermal units to satisfy two spinning reserve criteria. These are described as follows:

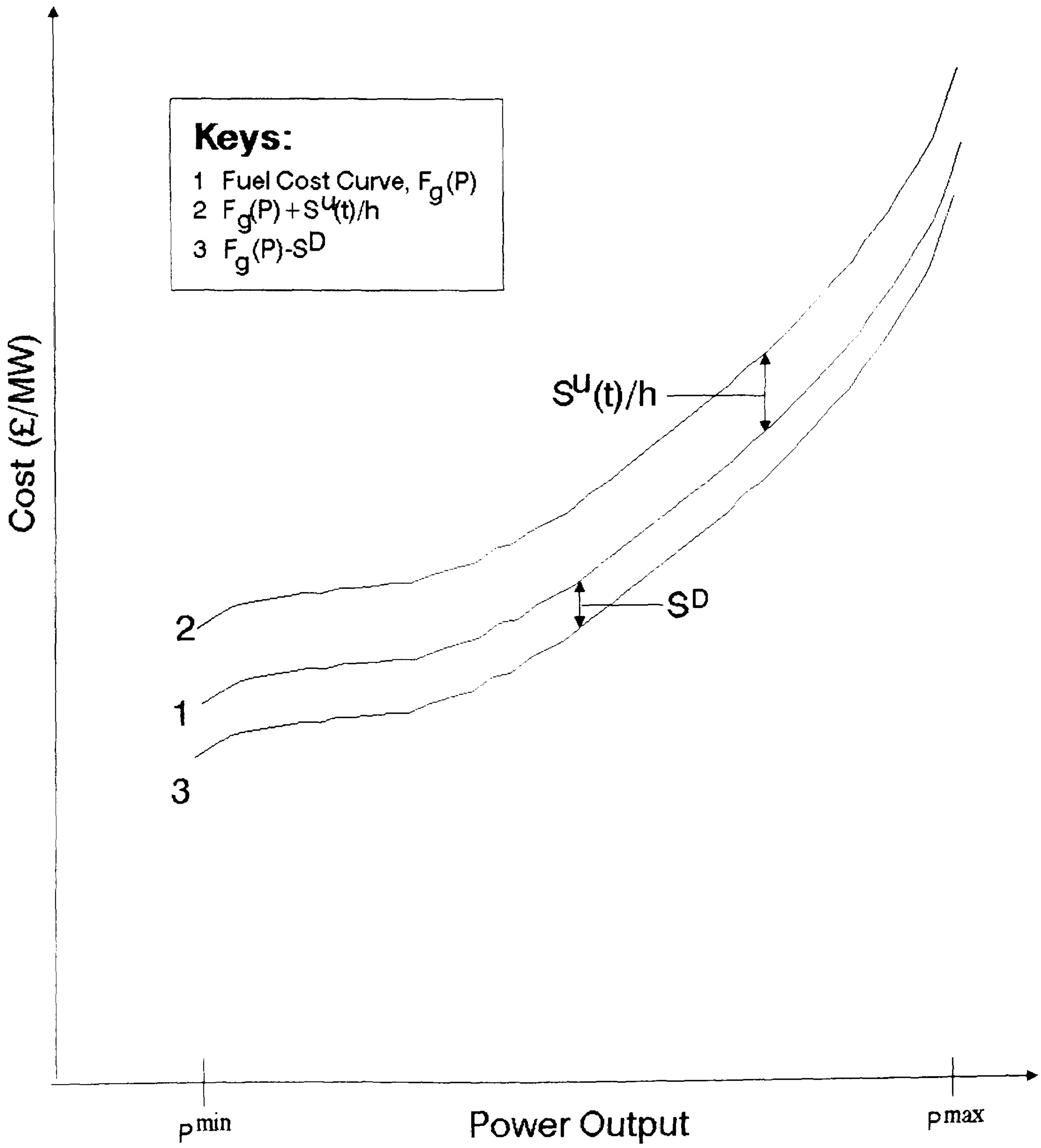



Fig. 4.6 Composite Cost Function for units was "ON" and was "OFF" conditions

1. Fixed percentage over forecast load: The total on-line capacity commitment at any interval is equal to or exceeds the expected load of that interval plus a certain percentage. This can be easily assured by checking the total on-line generation capacity against the total capacity requirement. The on-line units are selected by the DP recursive formula described above which minimizes the overall operating cost to the system.
2. Loss of generation: The second requirement is that the total pick up capacity should equal to or exceed the loading of any on-line unit within a pre-specified short time period. The spinning reserve available from a unit is the spare capacity available from the unit or the ramping capacity of this unit within the specified time whichever is smaller, i.e.

   Unit spinning reserve = *MIN* { (capacity-loading),(ramping rate x time) }

To satisfy the second criterion the total spinning reserve available from the remaining on-line units must be greater than or equal to the pre-outage loading of the unit under consideration. A computational technique is developed to ensure that the spinning reserve of a unit combination is adequate to cover the loss of any units. The method used is best illustrated by an example as given below.

Example: Consider that three generators depicted in Table 4.1 are selected to supply a forecast load. Assuming that a response requirement to pick up the loss of any generator within 10 minutes is specified. Then the load carrying capability of the generation configuration can be determined as shown in Table 4.2.

Table 4.1 Example Generator Data

| | | Unit 1 | Unit 2 | Unit 3 |
|---|---|---|---|---|
| Capacity (MW) | *min.* | 20.00 | 30.00 | 40.00 |
| | *max.* | 100.00 | 150.00 | 150.00 |
| Ramping rate (MW/min) | | 2.50 | 4.00 | 4.50 |

Table 4.2 Load Carrying Capability Example

| | Unit 1 | Unit 2 | Unit 3 |
|---|---|---|---|
| Capacity1 | 100.0 | 150.0 | 150.0 |
| Ramp cap2 | 25.0 | 40.0 | 45.0 |
| Avail Spin3 | 85.0 | 70.0 | 65.0 |
| Output4 | 75.0 | 70.0 | 65.0 |
| Difference5 | 10.0 | 0.0 | 0.0 |

Notes:
1 - Maximum output of the unit.
2 - Ramping capacity of the unit within specified time. (=Ramping rate x 10 min.)
3 - Total spinning reserve available to cover the loss generation of the unit.(E.g. unit 1 is covered by the total spinning reserve of units 2 & 3)
4 - Maximum output of the unit without reducing its contribution to overall spinning reserve available to the other units.
5 - Difference between (3) and (4).

The maximum load the units can supply without violating the 10 minute pick up time requirement is (a)+(b) where (a) is the summation of row 4 and (b) is the minimum of row 5. In this example (a)=75+70+65=210MW, (b)=0.0. The maximum load these three units can supply is therefore 210MW. Should this maximum load be less than the forecast load of the interval, the CCDP algorithm can be used to find a new unit combination and the spinning reserve response time requirement then checked.

### 4.5 Computational Results

The method proposed has been programmed in FORTRAN 77 on a *Perkin Elmer 3230* computer. To demonstrate the effectiveness of the proposed technique, a system with 12 thermal units shown in Appendix G is used as an example. The computer processing time required to schedule the commitment of these units for a 24 hour period and a step size of 5MW is less than 10 seconds. A comparison of the proposed method with a priority order scheme is shown in Table 4.3.

**Table 4.3** Comparison of operating cost using commitment schedules obtained by CCDP and Priority Order techniques

| Method | Fuel Cost | Start up Cost | Shut-down Cost | Total Cost | Time (Sec.) |
|---|---|---|---|---|---|
| Priority | 28452.30 | 0.00 | 11.00 | 28463.30 | 5.9 |
| CCDP | 28019.07 | 193.73 | 31.00 | 28243.80 | 9.6 |
| Diff % | -1.55 | - | - | -0.78 | - |

All on-line units, in the two methods, are optimality loaded satisfying both spinning reserve requirements. Various studies have shown that the proposed method has an overall cost improvement over the priority order scheme ranging from 0 to 2% depending on system generating unit characteristics etc.

The CCDP technique is robust with respect to the step size chosen. Table 4.4 shows that a 5 fold change in step size has only a marginal effect on the overall operating cost. Closer examination of the two commitment schedules for the two step sizes reveals that the two schedules are in fact identical as far as generator start-up and shut-down time are concerned. It is likely that the best step size is system dependent. Too big a step size will introduce suboptimality to the commitment schedule and too small a step size will increase the computational effort. For real system application, extensive tests should be carried out to determine the optimal step size.

**Table 4.4** Effect of step size on operating cost

| Step Size | Fuel Cost | Start up Cost | Shut-down Cost | Total Cost |
|---|---|---|---|---|
| 2.0 MW | 28,019.88 | 193.73 | 31.00 | 28,244.61 |
| 10.0 MW | 28,042.80 | 193.73 | 31.00 | 28,267.53 |
| Diff % | 0.082 | - | - | 0.081 |

To investigate the practicality of the method for a large scale system the commitment program has been applied to the EPRI Scenario System A[230]. In this test system, there is 224 thermal generating units with total capacity of 51,750 MW. Production cost results for one of the tests carried out are given in Table 4.5. The computer time required for this study is 20.9 minutes. Using Eq.(4.9), the estimated CPU time is:

System capacity = 51,750 MW
Generation step size chosen = 25 MW
Number of stages = 51,750/25 = 2,070
By Eq.(4.9), CPU time/interval = 44 sec
24 hourly intervals, total CPU time = 44 x 24 = 17.6 min.

The actual computer time used is greater than that estimation because of the additional processing time for data input/output, spinning reserve calculation etc.

**Table 4.5** Sample operating cost result for a 224 unit system

| Method | Fuel Cost | Start up Cost | Shut-down Cost | Total Cost |
|---|---|---|---|---|
| Priority | 1,181,971.0 | 7,451.0 | 3,069.0 | 1,192,491.0 |
| CCDP | 1,179,229.0 | 7,875.0 | 3,381.0 | 1,190,491.0 |
| Diff % | 0.23 | - | - | 0.17 |

## 4.6 Summary

A new unit commitment approach based on the dynamic programming principle has been presented. The method is centred on a novel DP recursive formula which is used to compute the optimal generation table of the available generators at any sub-interval of a study period. Because of the efficiency of this new recursive formula, the optimal generation table is renewed for each sub-interval as against the traditional way of computing a fixed optimal combination table once at the start of the unit commitment scheduling process. The enhancement gives the approach the flexibility to deal with the changing condition of the system with time. One of the most interesting aspect of the proposed DP formula is that the computer time requirement has been found to be largely independent of the number of units but rather a function of total system generating capacity and required accuracy. The computer execution time can therefore be controlled by adjusting the step size to the appropriate accuracy. A technique for inclusion of the ramping rate of on-line units and response time required to pick up load shed by any loaded generator has also been described. A spinning reserve constraint considered in the approach includes such response time requirements in the scheduling process.

# CHAPTER 5

# SURVEY OF ACTIVE POWER ECONOMIC DISPATCH

Economic dispatch is the heart of the application programs in a power system EMS control centre. It plays the most important role of sharing the load among the synchronized units to minimize energy cost while taking into consideration the security of the system. An economic dispatch algorithm normally makes two assumptions. The first is that the system topology is assumed unchanged from it present configuration or to change to a known state. The second is that the load at target time is known a priori which is generally based on results of a load forecast algorithm. A dispatch solution is therefore optimal with respect to a destined network topology and load demand distribution only. Since the system conditions vary continuously, the optimization process is carried out repeatedly in an EMS in order to track the changing operating environment. The relevant time horizon for this optimization process in real time control is about 5 to 30 minutes. The appropriate execution frequency of the dispatch function is mainly dictated by the availability of computer resources. Any frequency would be too fast when there is not much activity in the system and would be too slow at periods of fast load changes or component failure. One way to overcome this problem is to adapt the execution frequency to the system conditions. A possible adaptive scheme is to execute the application program when one of the following condition is satisfied:

- a change in system topology or unit availability
- a pre-specified amount of load change since last execution
- a pre-specified time has escaped since last execution
- other conditions warrant the need to execute the dispatch program

The important advantage of the adaptive approach is to eliminate unnecessary execution of the dispatch program and thereby leaving more computing power to carry out other computationally intensive tasks such as state estimation, security assessments etc.

The coupling between a dispatch function and other power application functions in a typical integrated EMS scheme is shown in Fig.5.1. The unit commitment function described in the previous chapters pre-dispatches the units availability at different times of a day and is part of the overall hierarchical control scheme to ensure the economic and secure operation of the system. The utilization of a unit commitment module has the benefit of relieving the complicated discrete unit selection problem from the generation dispatch problem. With more "up-to-minute" system states and accurate load forecast than those available to the unit commitment, advance dispatch algorithms deal explicitly with transmission limitations, losses[189] and regulation margin[192] requirements of the system. As shown in the figure, the economic dispatch solution comprises two sets of results. The first is the target generation operating points for all those units not under automatic generation control, referred to as

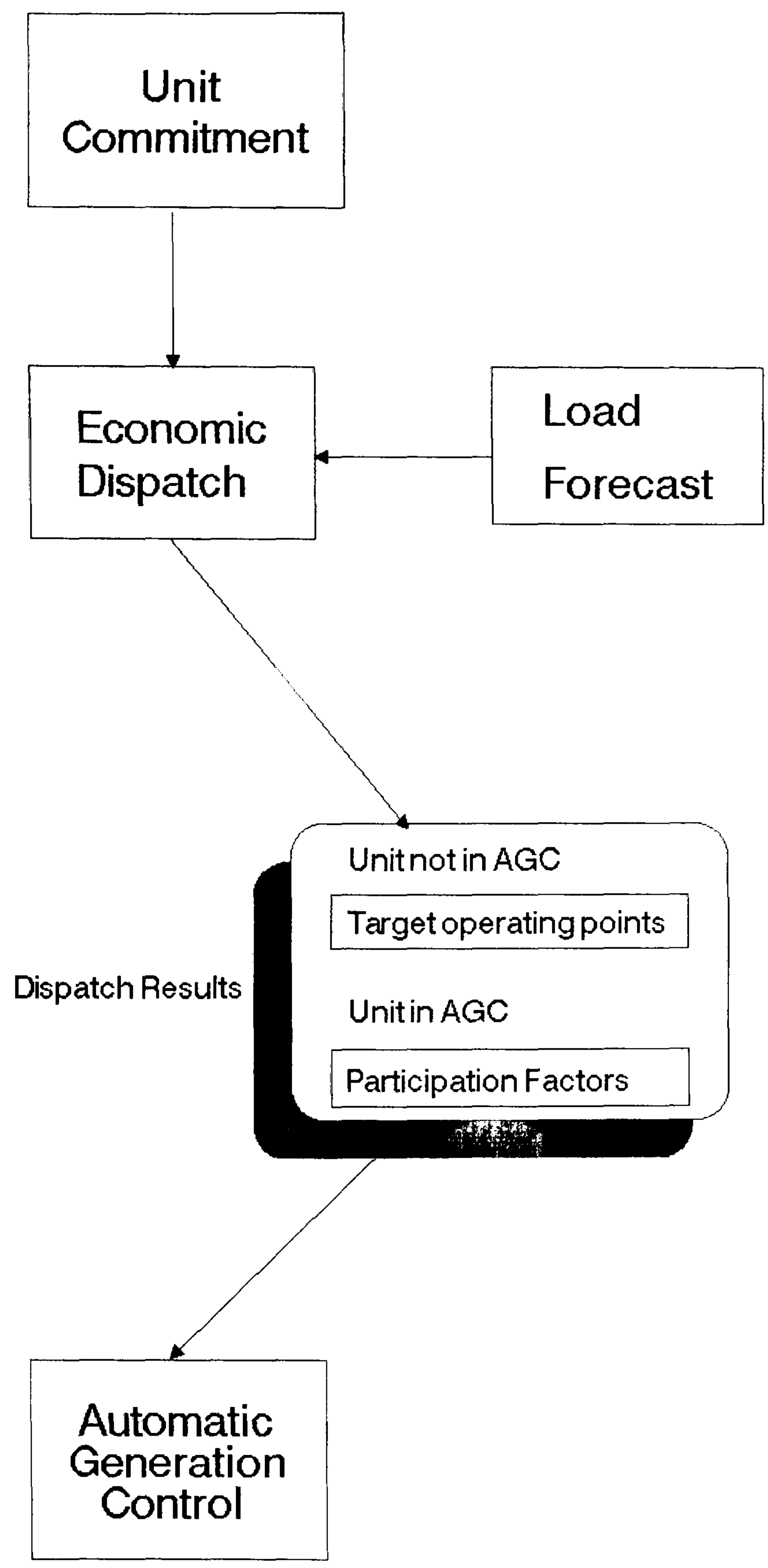



Fig.5.1 Data flow between Economic Dispatch and Automatic Generation Control

fixed ramping units, at target time, $t^{target}$. The second is the constrained participation factors for those units participating in the automatic generation control (AGC) (also known as Load Frequency Control, LFC) scheme. Load prediction error and imperfect control of physical response rates of any units for various reasons are taken care of by the automatic generation controlled units. Wollenberg and Stadlin[227] suggested a simple and practical formula for calculating the participation factor of unit $i$, $\alpha_i$ :

$$\alpha_i = P_i^{target} / P_{AGC}^{target} \tag{5.1}$$

where

$$P_{AGC}^{target} = \sum_{i \in AGC} (P_i^{target} - P_i^{present})$$

$P_i^{present}$ = generator output at time of dispatch execution

$P_i^{target}$ = economic dispatch solution for target time condition

At any instant $t$ between dispatch program execution and target time, a fixed ramping unit $j$ shall generate at a output level linearly proportional to the difference between the target operating point and its initial output, i.e.

$$P_j^t = P_j^{present} + (P_j^{target} - P_j^{present}) \frac{t - t^{present}}{(t^{target} - t^{present})} \tag{5.2}$$

The AGC units will shift their outputs according to:

$$P_i^{output} = P_i^{present} + \alpha_i P_{AGC}^t \tag{5.3}$$

where $P_{AGC}{}^{t}$ is the total generation requirement from the AGC units to balance the load. Since the participation factors are based on an economic dispatch solution, the generation shift in Eq.(5.3) has implicitly considered all the physical and operational constraints that are included in the dispatch methodology used.

Over the years, the problem of optimal dispatch has been considered by a large number of authors. It has been estimated that 1 to 2%[196] savings of the total generation cost can be realistically achieved by utilizing an appropriate economic dispatch algorithm. In the simplest equal incremental cost approach, the transmission capacity of the system is ignored. The application of more advanced mathematical optimization algorithms together with suitable approximations would enable the inclusion of many complex operational constraints in the solution process. This chapter reviews the significant dispatch techniques reported in the literature. In general, the dominant techniques may be grouped into five categories:

1. Equal incremental cost
2. Gradient Methods
3. Linear programming (LP)
4. Quadratic Programming (QP)
5. Dynamic Programming (DP)

The merit of considering transmission losses in an economic dispatch will be examined first. The different dispatch algorithms are then reviewed in the order as shown above.

## 5.1 Transmission Losses

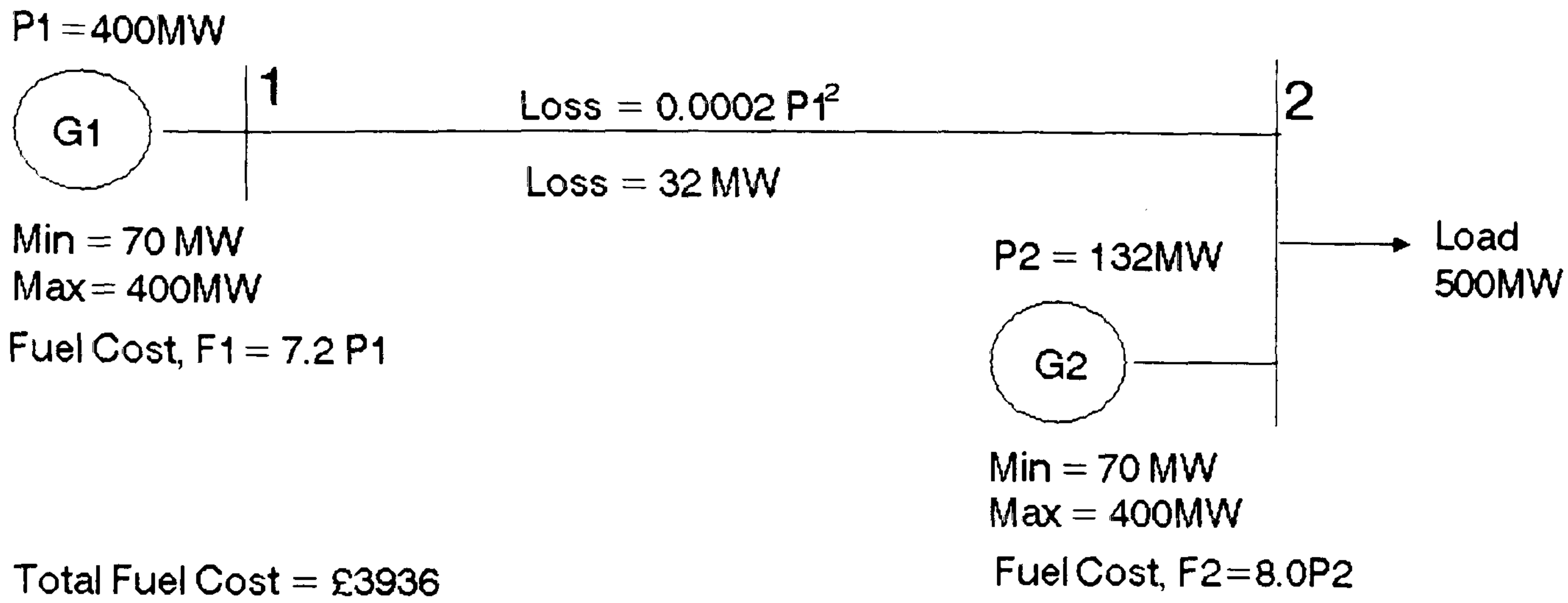
Transmission losses on the lines are proportional to the square of power flow. The intrinsic relationship between transmission losses and economic dispatch solution may be illustrated by a simple example. Let an electric supply system has two generators and a major transmission line. The single line diagram together with generator fuel cost functions, load and transmission losses function of the system are depicted in Fig.5.2(a). Intuitively, one would load unit 1 to its maximum capacity because it has much lower incremental cost than unit 2. For this particular dispatch, transmission loss incurred would be 32.0MW and can be supplied as extra generation by unit 2. The total production cost of this condition is £3936, also shown in Fig.5.2(a). Another obvious option is to minimize the transmission loss by loading unit 2 to its maximum. In this case, the loss is 2.1MW and the total production cost is slightly reduced to £3935 as shown in Fig.5.2(b). The optimum load sharing for this simple system, however, can be obtained analytically using calculus. The total production cost of the system is

$$C = 7.2P_1 + 8.0P_2 \tag{5.4}$$

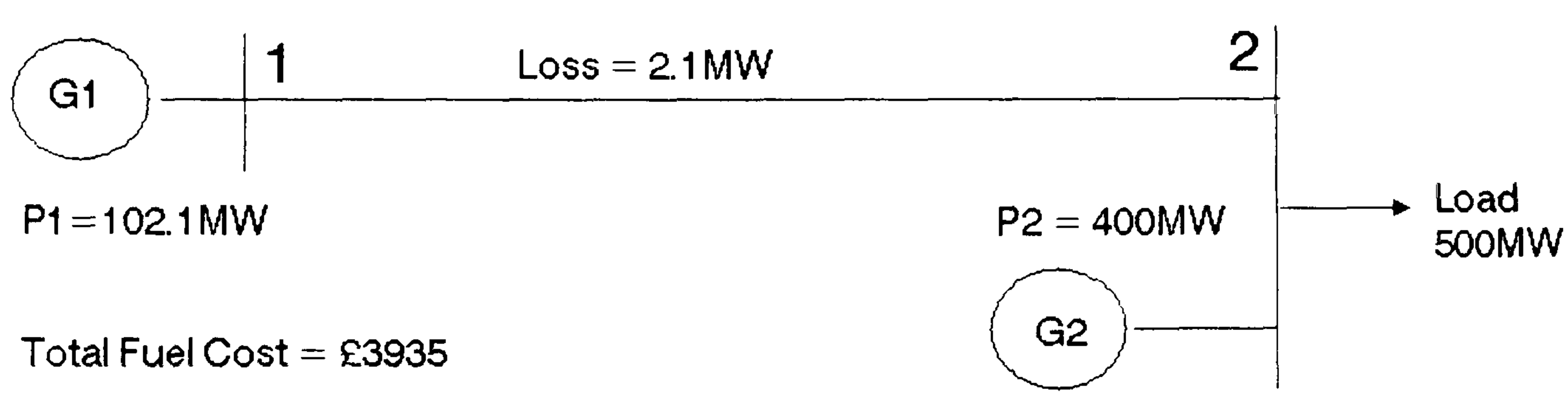
Total generation requirement is

$$P_1+P_2 = 500 + 0.0002{P_1}^2$$
$$\Rightarrow P_2 = 500 - P_1 + 0.0002{P_1}^2 \tag{5.5}$$

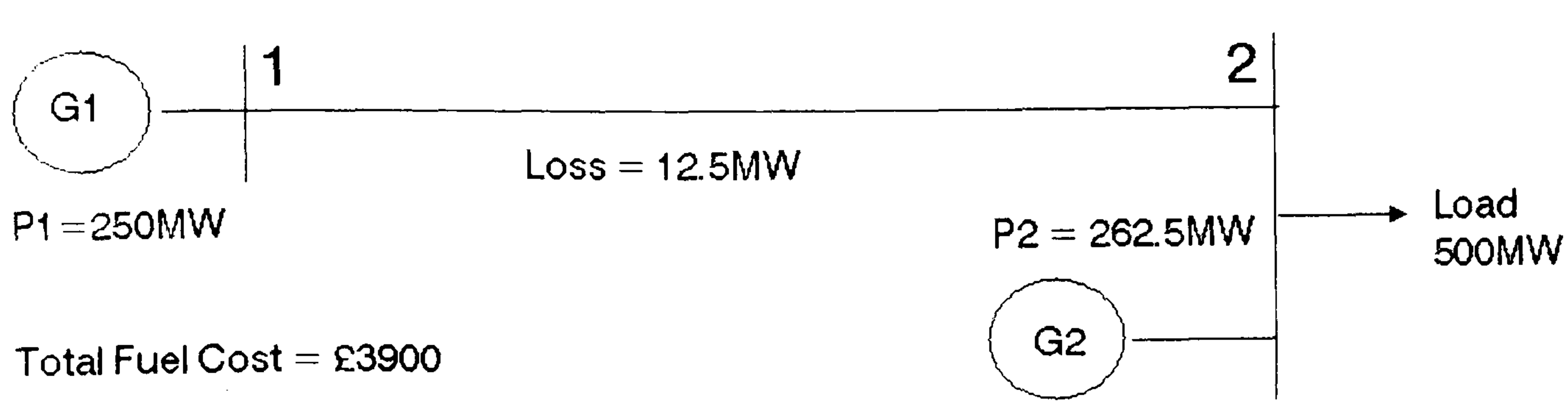
Substitute this in Eq.(5.2) and take the differential with respect to $P_1$. We have




(a) Case 1: Cheapest Unit Loaded to Maximum First




(b) Case 2: Minimum Transmissiion Loss




(c) Case 3: Optimum Dispatch Considering Losses

**Fig.5.2 Example system to illustrate effect of considering transmission losses on economic dispatch solution**

$dC/dP_1 = 0.0032P_1 - 0.8$

Setting $dC/dP_1=0$ for minimum cost, we have

$P_1 = 250$ MW

This is substituted back to Eq.(5.5) to obtain $P_2$=262.5MW. The total cost for this dispatch solution is £3900. This condition is shown in Fig.5.2(c). A summary of the three dispatches is shown in Table 5.1.

**Table 5.1** Summary for three dispatch solutions for a 2-unit example system

| Case | Losses | Cost | Diff* |
|---|---|---|---|
| Cheapest unit loaded to max | 32 MW | £3936 | +0.92 % |
| Minimum losses | 2.1 MW | £3935 | +0.90 % |
| Optimal dispatch | 12.5MW | £3900 | - |

* Diff = comparison with the optimal dispatch

From the above simple example, we can derive certain characteristics of transmission losses:

1. Minimum transmission losses do not imply minimum overall production cost. For the example system and loading condition examined, the error in comparison to the true optimum is as much as nearly 1% of the total production cost.
2. Dispatching without properly addressing the transmission losses problem but by considering only the relative

generation cost of the available units can also lead to a very expensive solution.

In the example system, loss is a simple function of unit 1. In a practical system, a transmission network is mesh and losses are a function of system topology, load distribution and generator loading. The inclusion of transmission losses in an automatic economic dispatch will normally reduce the overall operating cost but also implies the necessity of detailed analysis of the transmission network and availability of reliable real time topological and analogue system data. The benefit of optimizing transmission losses therefore must be set against the capital cost to provide the computer resources and the associated SCADA equipment to enable data collection and generator dispatch decision to be carried out. For a system with high losses either due to long transmission lines or heavy loading of its network, the financial savings by considering losses in a generation dispatch scheme will greatly off set the total cost of installing a real time energy management system.

### 5.1.1 Transmission Loss Formulae

Because of its simplicity, transmission loss formulae in one form or other are utilized in many economic dispatch solution schemes. The [B] matrix loss formula represents the classical approach to estimate the active power losses in terms of active power generation sources. The [B] matrix loss formula generally has the form:

$$P_L = [P]^T[B][P]+[P]^T[B_O]+B_{OO} \tag{5.6}$$

where

$[P]$ = vector of nodal active power generation

$[B],[B_0]$ = loss coefficient matrices for quadratic and linear functions of $[P]$.

$B_{00}$ = constant

Eq.(5.6) can also be written as:

$$P_L = \sum_{i=1}^{N_n} \sum_{j=1}^{N_n} P_i B_{ij} P_j + \sum_{i=1}^{N_n} B_{i0} P_i + B_{00} \tag{5.7}$$

There are different techniques[84,85,108,211] of calculating the loss coefficients. As mentioned earlier, transmission losses are a function of system configuration, load distribution and generator outputs. In deriving a simple expression of losses with generator active power as independent variable only, certain fundamental assumptions are generally required:

1. The loss coefficient matrices are established for a base case and the resultant loss formula therefore will be of acceptable approximation only if the system conditions have not deviated significantly from this reference frame. This implies that:
2. System topology remains mainly the same as the base case in particular for bulk supply transmission lines.
3. Load demand at each node conforms to a constant complex distribution factor, $\mu_i$, of the total system load. i.e.

$$D_i = D_i^0 + \mu_i D^{total} \tag{5.8}$$

4. Generator real and reactive powers are related to each other by a constant ratio, $\beta_i$, i.e.

$$Q_i = Q_i^o + \beta_i P_i \tag{5.9}$$

5. Bus voltages remain constant in magnitude and phase angle.

The $\mu_i$, $D_i^o$, $\beta_i$ and $Q_i^o$ quantities can be derived from two load flow calculations, one called the "base case" and other called "off-base". The base case may be the peak load condition and the off-base may be the low load condition. Several sets of loss coefficients are often employed to cover a range of system conditions for improved accuracy.

Brownlee, Cahn, Early et al, Shirley, Lubisich and others [26,30,57,69,81,188,224] used different combinations of the above assumptions and other system conditions to arrive at similar loss formulae. However, not all system conditions can be predicted a priori and the assumptions used for the derivation of the [B] matrices may be invalid. Loss formulae therefore have to be re-evaluated from time to time to capture new system conditions into the simple expression.

Nicholson and Sterling[159] derived a more general real power loss formula which requires no assumptions about the network but instead uses the bus impedance matrix together with A.C. load flow solutions. The loss formula derived is:

$$\begin{aligned} P_L = {} & [P]^T[\alpha][P]+2\{[D]^T[\alpha]-[Q]^T[\beta]\}[P]+ \\ & \{[D]^T[\alpha]-2[Q]^T[\beta]\}[D]+[Q]^T[\alpha][Q] \end{aligned} \tag{5.10}$$

where

[P] = nodal active power generation

[D] = nodal active power load

[Q] = nodal reactive power load

[$\alpha$],[$\beta$] = symmetric matrices of coefficients derived from the current network voltage distribution and configuration.

The algebraic form of Eq.(5.10) is similar to the general loss formula in Eq.(5.6). The significant advantage of Nicholson and Sterling's derivation is that Eq.(5.10) is responsive to the current network topology, load distribution and voltage distribution via an A.C. load flow. As a result, a more accurate dispatch solution may be obtainable.

## 5.2 Equal Incremental Cost Techniques

In the late 1950's, Kirchmayer[107] proposed the fundamental equal incremental cost method for active power dispatch; which subsequently becomes the basis of many sophisticated techniques. The approach can be summarized by the statement: *Optimal operation cost is achieved if the generating units are generating at such level that their respective incremental costs are the same.* Mathematically, the incremental cost of each unit, according to this dispatch criterion, is set to :

$$\mu = dF_g/dP_g \qquad g = 1,2,3, \ldots N_g \qquad (5.11)$$

where

$F_g$ = operation cost of generating unit g

$P_g$ = generator output subject to the minimum and maximum limitations

The equal incremental cost criterion provides a simple operating strategy for each generator in the system. The desired generator outputs can be obtained simply by choosing a trial value for the Lagrangian multiplier $\mu$ and solving for $P_g$. The multiplier $\mu$ is then adjusted up and down so that the total generator output is equal to the forecast load demand. It is obvious that the method can easily cope with individual generator output limits. When a unit reaches its lower or upper operating limit, the unit is scheduled to generate at this limit.

A variant of the equal incremental cost concept is a merit order approach. Assuming that the operating cost of a unit can be approximated by a linear cost function, the incremental cost of a unit is the slope of the linear cost function. When the units are ranked in increasing order of their incremental costs and all units are initialized to their lower output limits, the generating units can be considered for loading to their maximum limits in the order of merit until the demand is satisfied. One generator will usually partly loaded and this is called the 'marginal' unit. The incremental cost of the marginal unit is the 'marginal' cost which is equal to the Lagrangian multiplier $\mu$ in Eq.(5.11).

It has been shown[107] that transmission losses can be included in the equal incremental cost or merit order approaches by charging the incremental losses at a rate equal to the incremental cost of <u>received</u> power. The generator co-ordination equation in Eq.(5.11) then becomes:

$$\mu = dF_g/dP_g + \mu(dL/dP_g) \qquad i = 1,2,3, \ldots N_g \qquad (5.12)$$

$$\Rightarrow \mu = (dF_g/dP_g)/(1-dL/dP_g)$$
$$\Rightarrow \mu = (dF_g/dP_g).pf_g$$

where $pf_g$ is known as the penalty factor of unit g and is equal to $1/(1-dL/dP_g)$. Given the loss formula of the form in Eq.(5.6) and an initial dispatch solution, the penalty factors may be estimated. Eq.(5.12) can then be applied to obtain a new dispatch solution. An iterative procedures therefore can be set up to update the penalty factors and the generator loading until convergence.

The advantage of equal incremental cost and merit order approaches are their extreme simplicity resulting in trivial computational algorithms. The methods therefore have no difficulty in dealing with large scale problems. The disadvantages are that functional constraints such as transmission limits are precluded and that non-differentiable cost functions may not be easily considered. Nevertheless, because of their simplicity and computational effectiveness, the techniques are employed frequently to initialize a trial solution in more sophisticated techniques.

## 5.3 Gradient Methods

Gradient methods are formalized direct enumeration search procedures. Most gradient search techniques starts off from a feasible solution and search for the optimum solution along a monotonously decreasing (for minimisation problem or vice versa) trajectory while maintaining feasibility all the time. Consider the economic dispatch problem:

Objective function:

$$\text{Minimize } C = \sum_{g=1}^{N_g} F_g(P_g) \tag{5.13}$$

Subject to constraints:

(1) Load balance: $\sum_{g=1}^{N_g} P_g = D$ (5.14)

(2) Generator power limits: $P_g^{min} \leq P_g \leq P_g^{max}$ (5.15)

The Taylor-series expansion of Eq.(5.13) about an initial operating point is:

$$\begin{aligned} C+\triangle C = {} & F_1(P_1)+F_2(P_2)+F_3(P_3)+\ldots+F_{Ng}(P_{Ng})+ \\ & dF_1/dP_1(\triangle P_1)+dF_2/dP_2(\triangle P_2)+\ldots+dF_n/dP_n(\triangle P_{Ng})+ \\ & \tfrac{1}{2}\{d^2F1/dP1^2(\triangle P_1)^2+d^2F_2/dP_2^2(\triangle P_2)^2+\ldots\}+\ldots \end{aligned} \tag{5.16}$$

Neglecting the second order and higher order terms, the change in the total operating cost is:

$$\triangle C = dF_1/dP_1(\triangle P_1)+dF_2/dP_2(\triangle P_2)+\ldots+dF_n/dP_n(\triangle P_{Ng}) \tag{5.17}$$

Given an initial feasible solution, the optimal solution can be approached by allowing the power output of the generators to perturb about the initial operating point such that:

$$\sum_{i=1}^{N_g} \triangle P_i = 0 \tag{5.18}$$

and that the change in the total operating cost in Eq.(5.17) is negative representing an improvement in the dispatch solution. The search process can be started with selecting an dependent unit, x, such that

$$\Delta P_x = - \sum_{i \neq x}^{Ng} \Delta P_i \tag{5.19}$$

Substituting this in Eq.(5.17), we have

$$\Delta C = \sum_{i \neq x}^{Ng} [dF_i/dP_i - dF_x/dP_x] \Delta P_i = [A_i][P_i] \tag{5.20}$$

The coefficient $A_i$ for each unit can be evaluated after unit x is selected and the magnitude of the coefficient indicates the reduction in the total energy cost by varying the output of another unit with an opposite and equal change in the $x^{th}$ unit. Figure 5.3 shows a flow chart of a computer programof the concept. As shown the unit with the least incremental cost and hence largest cost reduction, say y, will be chosen to increase its output level first. The amount of movement must be checked against the operating limits of both unit y and the dependent unit x. The procedures depicted is simple and straightforward; but requires a large number of iterations to converge to a satisfactory optimal solution.

Second order gradient methods can also be implemented. Substitute equation Eq.(5.19) in Eq.(5.16) and retaining the second order terms, the change in operating cost in this case is:

$$\Delta C = \sum_{i \neq x}^{Ng} [dF_i/dP_i - dF_x/dP_x] \Delta P_i + \tag{5.21}$$

$$\tfrac{1}{2}\{ \sum_{i \neq x}^{Ng} [d^2F_1/dP_1{}^2 (\Delta P_1)^2 + d^2F_2/dP_2{}^2 (\Delta P_2)^2 + \ldots] +$$

$$d^2F_x/dP_x{}^2 [\Delta P_1{}^2 + \Delta P_2{}^2 + \ldots + 2\Delta P_1 \Delta P_2 + 2\Delta P_1 \Delta P_3 + \ldots] \}$$

At the optimum the incremental cost of all units are identical ignoring the unit output limits. This means that the partial derivative of the change in the total operating cost, $\partial \Delta C/\partial \Delta P_i$, with respect to each independent variable for all $i \neq x$, is zero bearing in mind that any change of output in one unit is balanced by an opposite and equal magnitude change in the other units. These partial derivatives result in a set of simultaneous equations.

$$\partial \Delta C/\partial \Delta P_1 = 0 = (dF_1/dP_1 - dF_x/dP_x) + d^2F_1/dP_1^2 \Delta P_1 + d^2F_x/dP_x^2 \sum_{i \neq x} \Delta P_i$$

$$\partial \Delta C/\partial \Delta P_2 = 0 = (dF_2/dP_2 - dF_x/dP_x) + d^2F_2/dP_2^2 \Delta P_2 + d^2F_x/dP_x^2 \sum_{i \neq x} \Delta P_i$$

$$\vdots \qquad (5.22)$$

Let $F_i' = dF_i/dP_i$ and $F_i'' = d^2F_i/dP_i^2$, then Eq.(5.22) written in matrix form becomes

$$\begin{bmatrix} F_1''+F_x'' & F_x'' & F_x'' & \cdots \\ F_x'' & F_2''+F_x'' & F_x'' & \cdots \\ F_x'' & F_x'' & F_3''+F_x'' & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \vdots \end{bmatrix} = - \begin{bmatrix} F_1'-F_x' \\ F_2'-F_x' \\ F_3'-F_x' \\ \vdots \end{bmatrix} \qquad (5.23)$$

Solving Eq.(5.23) will give the movement of the generators from their initial output to $(P_i+\Delta P_i)$ for all $i \neq x$. The movement of unit $x$ is given by Eq.(5.19). Iterative computational procedure similar to the flow chart shown in Fig.5.3 for first order gradient approach may also be developed for this case.

One of the advantages frequently mentioned of gradient search procedures is that the search process may be interrupted at any time and the most recent solution will

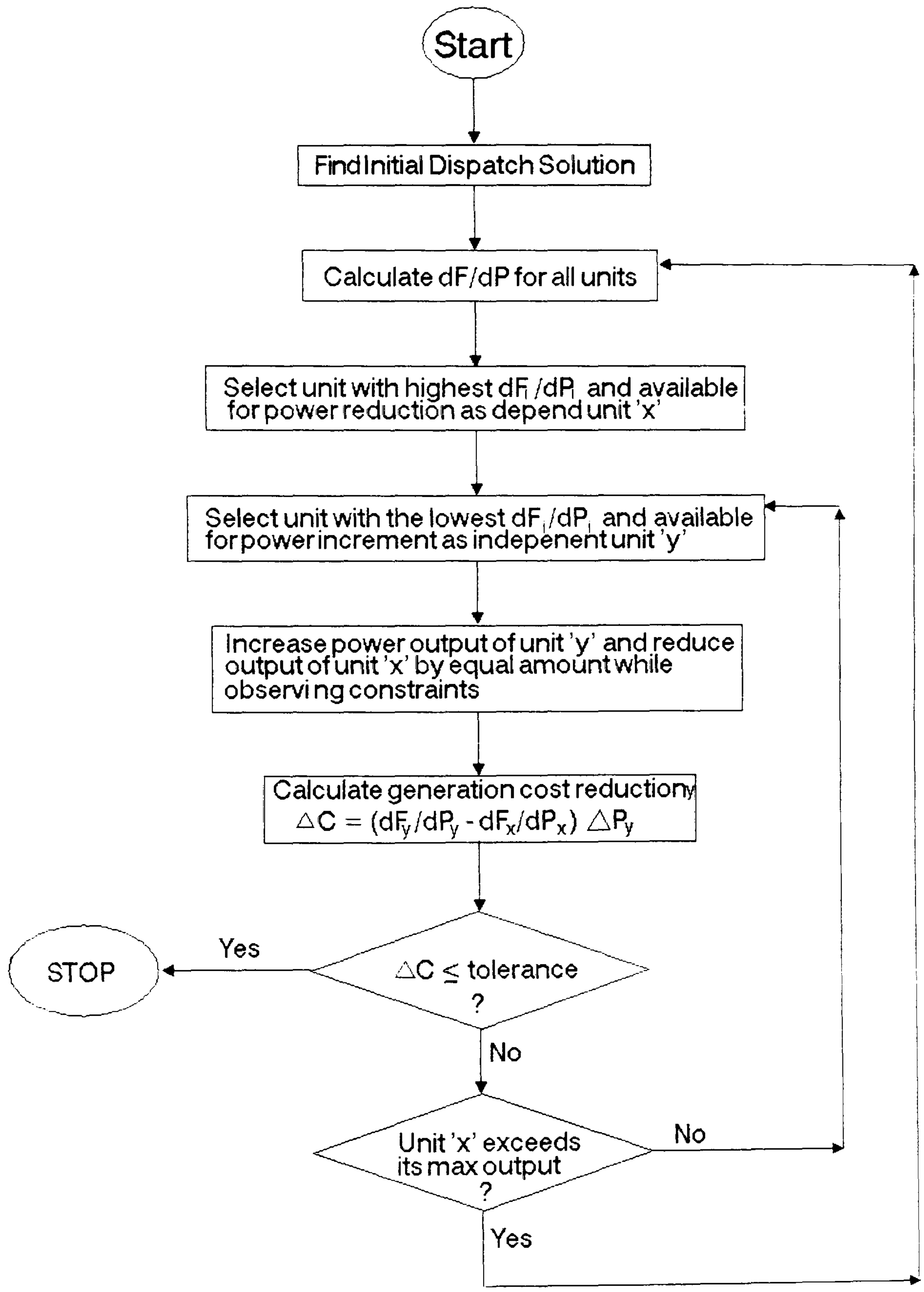



Fig. 5.3 Flow chart of a economic dispatch algorithm using a first order gradient search

still be feasible and a reasonable operating point of which to make use of. This argument is not particularly valid since, using a modern powerful digital computer, the execution of economic dispatch normally completes in the order of seconds. The disadvantages of gradient search methods, however, include most problems associated with mathematical optimization approach. There is no clear stopping criterion: usually the optimization process is allowed to continue until there is no significant generation cost reduction in a number of successive iterations or that a fixed reasonably high number of iterations has exceeded. The approaches are also restricted to linear or smooth analytical differentiable generation fuel cost functions for obvious reasons. Furthermore, similar to incremental cost techniques, the incorporation of operational constraints into the model has proved to be a problem area.

### 5.4 Linear Programming

Comparing with other mathematical optimization techniques directed towards economic dispatch applications, linear programming perhaps attracts by far the most intensive research effort[35,101,194,202,202,203]. LP is also widely applied to other optimization problems and hence a wealth of experience has been gained. The advantages of LP approaches are numerous including reliability, speed of solution, sufficient accuracy of linearized power system models, comparatively straightforward formulation, formalised solution technique and has the flexibility to incorporate most of the constraints affecting the economic dispatch solutions. Stott et al[202,203] gave a comprehensive review on the subject and Sterling[196] compared the computational efficiency of the main LP algorithms. In here, a brief review is offered.

Without loss of generality, the economic dispatch in LP formulation has a general mathematical expression as follows:

$$\text{Minimize} \quad C = \sum_{g=1}^{Ng} C_g P_g + \text{Constant} \tag{5.24}$$

subject to linear constraints.

One of the essential issue in LP formulation is the accuracy of linearized models. For the objective function, linearized incremental costs, $C_g$, are generally sufficiently accurate. This is particularly so because the generation shift requirements of the generators at dispatch target times a few minutes ahead are generally small. For inherently non-linear generation cost characteristics, the cost model can be linearized at the anticipated operating points instead of for the whole operating range. Furthermore, piece-wise linear model can also be employed if load variation is significantly large or if there are plant availability changes. This improved cost modeling however would involve more complex programming effort and longer computational time.

The important advantage of LP approaches over the equal incremental cost and gradient searches methods described earlier is their capability to consider network security. In many LP dispatch methods, an incremental linear P-$\theta$ relationship is used to model the line MW flow which is of the form:

Incremental flow in each line:

$$F_{ik} = h_{ik} (\Delta\theta_i - \Delta\theta_k) \tag{5.25}$$

and for power injection and phase angle relation:

$$[\Delta P] = [H] [\Delta\theta] \tag{5.26}$$

The accuracy of the model depends on the choice of matrix [H]. One possible choice of [H] is the Newton load flow Jacobian submatrix but it has the disadvantage being asymmetrical and therefore necessitate the storage of its upper and lower elements of both itself and its triangular factors. It is particularly inefficient for line outage simulation. One simplification is to assume that the line flow in a line is an average of the flow in both ends and a symmetrical [H] is formed with $h_{ij}=h_{ji}=V_iV_jB_{ij}$. Further simplification is to replace bus voltages with 1.0 p.u. and $B_{ij}=1/X_{ij}$ resulting in the classical D.C. load flow relationship:

$$[\Delta P] = [B][\Delta\theta] \tag{5.27}$$

A non-incremental form can also be used by including an error correction factor [K] due to the simplification, i.e.

$$[P] + [K] = [B] [\theta] \tag{5.28}$$

where $[K] = [B][\theta^0] - [P^0]$

and the line MW flow becomes

$$F_{ij} = (1/X_{ij})(\theta_i-\theta_j) \tag{5.29}$$

LP dispatch algorithms generally involved several iterations of trial dispatches. [K] therefore can be conveniently updated at the beginning of a new iteration.

### 5.4.1 Choice of LP Algorithms

The fundamental algorithm for the solution of a LP problem is the Simplex method which was introduced by Dantzig[48] in the early 1960's. Since then many enhancements were proposed. Sterling[196] compared the computational efficiencies of the predominant LP dispatch algorithms, including Simplex, Revised Simplex, Duoplex and Dual Revised Simplex, in terms of the number of exchange steps, computer core storage and CPU time. It was concluded that Revised Simplex takes the longest computation time and requires most storage words. The other algorithms are of similar orders but Dual Revised Simplex has the significant advantage that no initial feasible solution is required and that the computation time is the least for all test cases.

### 5.4.2 Transmission Losses

LP is not the ideal formulation to optimize transmission losses because of the inherent non-linear property of losses. Fortunately, the effect of losses on operating cost can be included in the dispatch calculation by modifying the incremental fuel cost of a unit with its transmission loss penalty factor. The total transmission losses may be estimated based on a load flow solution of a tentative dispatch solution. The total load to satisfy by the units in the system is then the summation of forecast load and the losses. This incremental cost modification approach, however, sometimes creates a *bi-stable* situation. For example, in an initial dispatch, a generator may be dispatched to its maximum output because of its low incremental cost. Based on the dispatch results, the penalty factor and hence the effective incremental cost of the unit may be calculated. It could happen that because of an increased in penalty factor, in the second dispatch the same generator will be assigned to generate at its minimum output, i.e. the initial conditions of the first dispatch. A third dispatch would then give an identical solution of the first one. One feasible technique to combat this situation is to artificially shrink the upper and lower limits of the generator in successive iterations until the solution converges to a defined accuracy. The artificial limits might excluded the true optimum in certain stages of the solution process; but in practice this is found to happen only occasionally and only displaced from the optimum by a very small percentage.

### 5.4.3 Non-sparse versus Sparse Formulation

In formulating the LP dispatch problem, there is always the question of whether the formulation should be in non-sparse or in sparse forms. For the non-sparse formulation, the control variables, primarily generator outputs, are used directly as the state variables and have the form :

$$\text{Minimize} \quad C = \sum_{g=1}^{Ng} C_g P_g \tag{5.30}$$

subject to line power flow, F, constraints:

$$[F_{min}] \leq [F]=[S][P_g] \leq [F_{max}]$$

As discussed earlier power flow of a line depends on power injections from all generating sources of a system. The sensitivity coefficients matrix [S] in Eq.(5.22) is therefore non-sparse and the formulation is hence commonly referred to as non-sparse.. In implementation, however, [S] is usually derived by using bus-branch distribution factors[63,112] which are stored in sparse triangular factors for storage economy and computational efficiency. The coefficients in [S] is generated only if a line is founded overloaded or nearly overloaded and included in the LP constraint set.

The sparse LP formulation is based on the approximate relationship of line power flow to phase angles at the nodes of a line leading to line flow constraints in the form of Eq.(5.29), i.e. $[F] = [H]\ [\theta]$. In this case, the objective function is:

$$\text{Minimize} \quad C = \sum_{g=1}^{Ng} [C^t][\theta] \tag{5.31}$$

where $[C^t]$ is a transformed incremental cost row vector. The advantage of this formulation is the sparsity of the [H] matrix. It can be shown, however, that a non-sparsity formulation can result in a more efficient algorithm[202]. There are many contributing factors to this conclusion. These include less indexing overhead for non-sparse formulation and the number of variables is also less as the number of generators is generally less than the number of nodes in a power system.

### 5.4.4 Spinning Reserve

A generating unit may have to be taken off-line because of a sudden failure of an auxiliary part. A proper amount of spinning reserve therefore should be maintained in the system so that the remaining units may make up the deficit without excessively depressing the system frequency. Low frequency is undesirable because it may damage frequency sensitive equipment connected to the system and may activate frequency actuated automatic load shedding devices thereby leading to widespread brown out or possibly black out. Widely adopted spinning reserve constraints that can be conveniently included in LP formulation are as followed:

$$R_g = P_g^{max} - P_g \tag{5.32}$$

$$\sum_{g=1}^{Ng} R_g \geq R_{sys}^{required} \tag{5.33}$$

where $R_g$ is the available spinning reserve from any generator g. As discussed in Chapter 2, the actual amount of spinning reserve of each unit depends on the operating point of a unit. Given that most advanced LP algorithms have a number of trial dispatches before converging to a final optimal solution, trial dispatch results can be used to assist the modeling of spinning reserve more accurately. For example, Eq.(5.32) can be modified to:

$$R_g = P_g^{max} - P_g \qquad \text{if } P_g^{max} - P_g^{trial} \leq R_g^{max} \tag{5.34a}$$

or, $$R_g = R_g^{max} \qquad \text{if } P_g^{max} - P_g^{trial} > R_g^{max} \tag{5.34b}$$

The flexibility and capability of LP consider spinning reserve requirement is clear. Indeed such flexibility can be generally applied to any constraint which can be modelled by linear functions. This is perhaps the most important contributing factor to the popularity of LP methods.

## 5.5 Quadratic Programming

Quadratic programming (QP) is the optimization of a quadratic cost function subject to linear constraints. The QP approach in economic dispatch problem offers several advantages over the conventional equal incremental cost and LP methods including:

1. Transmission losses as a quadratic function of generator active power outputs can be expressed explicitly in the problem formulation.
2. Second-order representation of generation cost curves is permitted.
3. A direct, non-iterative solution is possible even when losses are included.
4. All linear constraints such spinning reserve, station limits, transmission limits can be included.
5. Dispatch solution is obtained in a finite number of linear programming type basic exchange steps and hence avoiding arbitrary convergence criterion.

The first application of QP in economic dispatch was by Nicholson and Sterling[159] in 1972. The approach subsequently attracted further attention[4,15,103,128,161,177]. The following paragraphs described some of the significant algorithms developed to date.

### 5.5.1 Nicholson and Sterling

The algorithm presented by Nicholson and Sterling[159] represents the classical QP approach to economic dispatch problem: quadratic objective function using generator outputs as state variables and linear constraints capturing most of the essential operating system and component limits. The important contribution of their work is the explicit inclusion of transmission losses in the objective function. They derived that the real power losses on the network can be expressed as a direct function of the nodal generator active and reactive power outputs at different nodes of the system as shown in Eq.(5.10). The equation does not rely on a "base case" but can be calculated efficiently from the impedance matrix of system and an A.C. load flow solution. It is therefore responsive to topological changes of the network. By defining a connectivity matrix [K] such that the elements of K are 0 or 1 depending upon the nodes to which a generator is connected and assuming that active power dispatch does not alter significantly the nodal reactive power distribution, the loss equation can be modified to

$$P_L = [P_g]^T[K][\alpha][K][P_g]+2\{[D]^T[\alpha]-[Q]^T[\beta]\}[K][P_g] + \text{constant} \tag{5.35}$$

where

$[P_g]$ = generator active power output $([P]=[K][P_g])$

The transmission losses given by Eq.(5.35) is costed as an average cost of received power, $C_{loss}$, and estimated by

$$C_{loss} = \frac{\text{Total production cost}}{\text{Total generator active power output}} \tag{5.36}$$

The value of $C_{loss}$ can be updated after each generation scheduling to account for any change in system conditions.

Given that the generator cost curves are represented by $F_g(P_g)$, then the objective function of an economic dispatch problem can be written as:

$$\text{Min } C = \sum_{g=1}^{N_g} F_g(P_g) + C_{loss}\{[P_g]^T[K][\alpha][K][P_g]+2\{[D]^T[\alpha] - [Q]^T[\beta]\}[K][P_g] + \text{constant}\} \tag{5.37}$$

When the generation cost curve are approximated by linear or second order functions, Eq.(5.37) can be simplified to

$$\text{Min } C = [\Phi_1]^T[P_g] + [P_g]^T[\Phi_2][P_g] \tag{5.38}$$

where $[\Phi_1],[\Phi_2]$ = coefficient vector and matrix derived from the loss equation and the generator cost functions.

by omitting the constant term since it does not affect the optimal distribution of the active power generation. The objective function is subjected to the normal generator lower and upper limits and other security constraints. Sterling and Nicholson solved the dispatch problem by applying Beale's algorithm in which finite simplex type basis exchange procedures are followed to arrive at the optimum. The proposed approach avoids an arbitrary convergence criterion as in the gradient technique and replaces this with direct matrix manipulation. The active power dispatch is followed by a minimisation of transmission losses with respect to reactive power using a steepest descent method. This involves the calculation of a variable step length and the use of an averaging technique to overcome oscillation of the slack busbar reactive generation during the solution. The active and reactive power dispatch are iterated successively until changes between consecutive iterations are within tolerance.

Irving and Sterling[103] subsequently developed along a similar idea a sparse matrix QP formulation for active power dispatch based on a linear complimentary algorithm. The relationship between generation, load and nodal phase angles is approximated by DC load flow model. Total transmission loss is approximated by a quadratic form:

$$P_L = [\theta]^T [G_v][\theta] \tag{5.39}$$

where

$[\theta]$ = vector of nodal phase angles

$[G_v]$ = $N_n x N_n$ square matrix defined as:

$$G_{vij} = - V_i G_{ij} V_j \qquad (i \neq j)$$

$$G_{vij} = - \sum_{j=1}^{N_n} G_{vij} \qquad (i=j)$$

$G_{ij}$ = real part of nodal admittance matrix element *ij*

$V_i$ = voltage magnitude at node *i*

Similar to Nicholson and Sterling's approach, the objective function is defined as

$$\text{Min} \quad C = \sum_{g=1}^{Ng} F_g(P_g) + C_{loss} P_L \tag{5.40}$$

where $C_{loss}$ = marginal cost of received power.

Numerical results show that the approach is robust with respect to the loss cost factor, $C_{loss}$, and erroneous voltage magnitude estimates.

### 5.5.2 Nabona and Freris

Nabona and Freris[155] also applied Beale's algorithm to solve the economic dispatch formulated as a QP problem. Their approach is, however, distinctively different from Sterling's method. The real and reactive dispatches are not dealt with separately but treated within an unified algorithm. Total generation cost is minimized directly with losses imbedded in the constraint set. Any network variable can be considered as a control variable. Constraints on the control variables or a function of the control variables, including the real and reactive generation limits, voltage magnitude limits, flow limits and generation reserve, are then included in the problem formulation, through sensitivity coefficients, expressed as linearized relations. A simplified flow diagram of the optimization algorithm is shown in Fig.5.4. Let [U] represent the set of control variables and assume that generator cost curves are approximated by quadratic functions, then the total generation cost $C$ is given by a quadratic relation:

$$C = A_0+[A_1][P_g]+[P_g]^T[A_2][P_g] \tag{5.41}$$

A change $[\triangle U]$ will result in a new cost given by

$$C+\triangle C = A+[A_1][P_g+\triangle P_g]+[P_g+\triangle P_g]^T[A_2][P_g+\triangle P_g] \tag{5.42}$$

The incremental change in cost due to a change of $[\triangle U]$ is

$$\triangle C = (A_1+2[P_g][A_2])[\triangle P_g]+[\triangle P_g]^T[A_2][\triangle P_g] \tag{5.43}$$

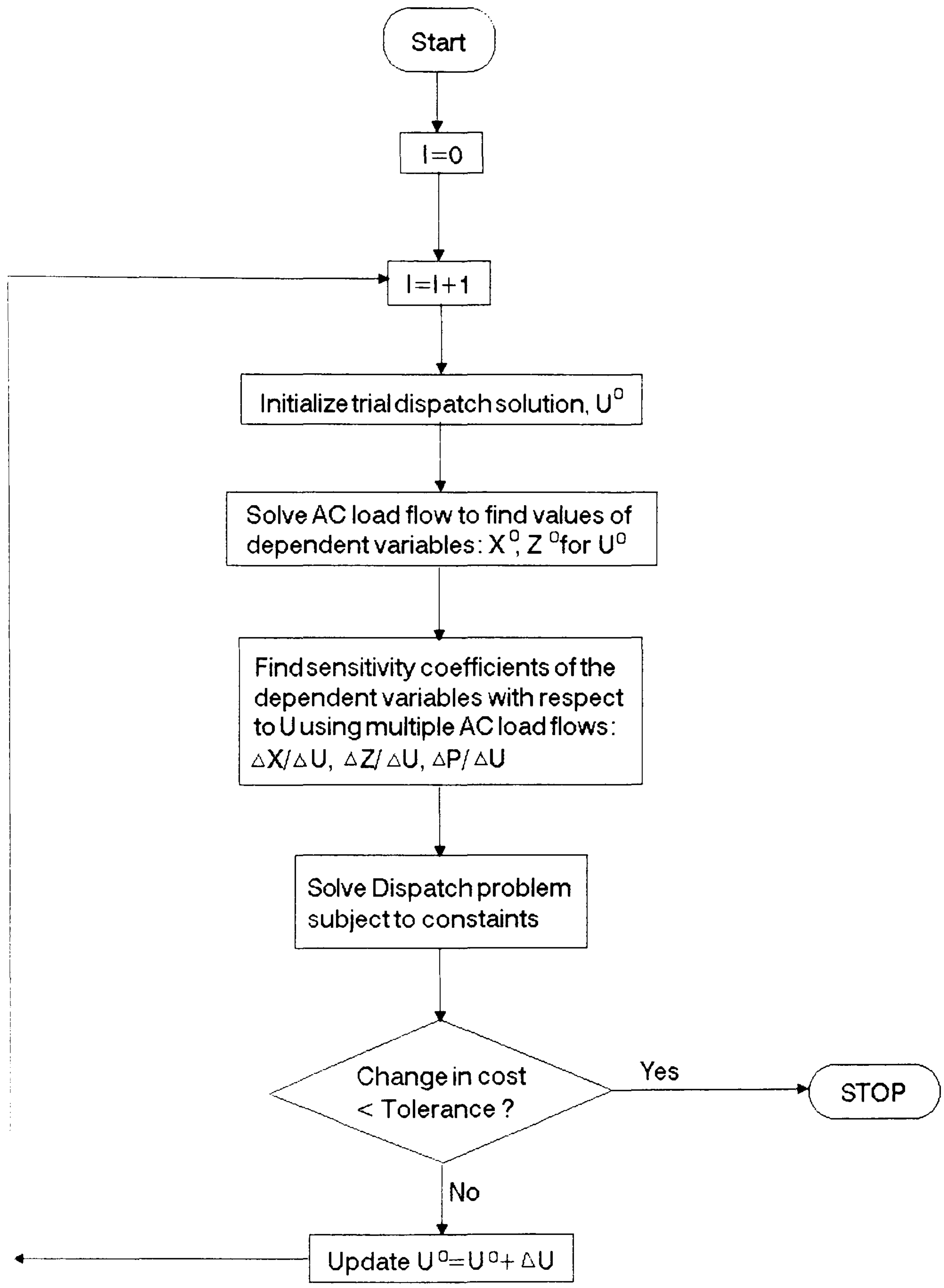



Fig. 5.4 Simplified flow chart of Nabona and Freris's quadratic programming dispatch technique utilizing sensitivity coefficients

Given the sensitivity of $P_g$ to a change in every control variable U, such that $[\Delta Pg]=[\Delta Pg/\Delta U][\Delta U]$, then Eq. (5.43) can be rewritten as

$$\Delta C = (A_1+2[Pg][A_2])[\Delta Pg/\Delta U][\Delta U] + [\Delta U]^T[\Delta Pg/\Delta U]^T[A_2][\Delta Pg/\Delta U][\Delta U] \tag{5.44}$$

The optimization of the objective function in Eq.(5.44) is subject to limitations of the incremental changes in the control variables $\Delta U$ and other dependent $\Delta X$ and functional dependent variables $\Delta Z$. These constraints can also be expressed as a linear relationship of $\Delta U$, i.e.

$$[\Delta X^{min}] \leq [\Delta X]=[\Delta X/\Delta U][\Delta U] \leq [\Delta X^{max}] \tag{5.45}$$

$$[\Delta Z^{min}] \leq [\Delta Z]=[\Delta Z/\Delta U][\Delta U] \leq [\Delta Z^{max}] \tag{5.46}$$

Transmission losses are included in the optimization based on a similar approach. The load balance constraints then becomes:

$$\sum_{g=1}^{N_g} \Delta Pg = \sum_{u=1}^{N_u} (\Delta P_L/\Delta U_u)\Delta U_u \tag{5.47}$$

The salient characteristic of the approach is the estimation of the required sensitivity coefficients, $[\Delta P/\Delta U]$, $[\Delta X/\Delta U]$, $[\Delta Z/\Delta U]$ and $[\Delta P_L/\Delta U]$ through numerical load flow solutions instead of the normal analytical approach. The advantages of this concept are the capability to utilize any network variable as a control variable and that the set of control variables may be changed within the solution process as some of them reach their limits.

Reid and Hasdorff[176] offered a similar QP formulation but Wolf's algorithm was employed to solve the dispatch problem.

### 5.5.3 Aoki and Satoh

The QP approaches described so far treated transmission losses either as additional generation cost in the objective function or as a linearized incremental change in the constraint set. Aoki and Satoh[4] proposed that transmission losses should be included in the optimization in quadratic form in the power balance equation, such that,

$$D + \sum_{g=1}^{N_g} P_g + [P_g]^T[B][P_g] = 0 \tag{5.48}$$

To deal with this nonlinear constraints, a Lagrangian multiplier $\mu$ associated with Eq.(5.48) was used to imbed this nonlinear constraint in the objective function. Let Eq.(5.48) be written as $G(P_g)=0$, the objective function of the economic dispatch problem becomes:

$$\text{Minimize} \quad \sum_{g=1}^{Ng} F_g(P_g) + \mu\, G(P_g) \tag{5.49}$$

subject to all linear system and unit constraints. The optimum solution is obtained when $\mu$ equals to $\mu^*$ such that $G(P_g)$ equal to zero. By considering the Kuhn-Tucker conditions of Eq.(5.49), Aoki and Satoh proved that $F_g(P_g)$ and $G(P_g)$ can be expressed as a function of $\mu$ and can be arranged in a Simplex tableau format and solved in a finite number of steps of basic and non-basic variable exchange to obtain the optimal $\mu^*$ and hence the optimal generation dispatch solution. The solution method proposed is shown to be efficient and comparable to other QP methods. To date, QP is generally is best approach in handling losses; but is computationally slower than LP methods.

## 5.6 Dynamic Programming

The techniques described so far assume a smooth, continuous and differentiable generation cost function. In reality, the specific heat rate of a power plant is of complex shape that involve sudden changes in slope and discontinuities caused by the throttling losses in steam admission or governor valves of a multi-valve machine at the valve intercept points. Equal incremental cost dispatching therefore may have no meaning when valve loops are include in the heat rate curves and linear and quadratic cost functions used in LP and QP approach may be poor approximations. Ringlee and Williams [146] estimated that a theoretical savings of 0.1 to 0.2% of total fuel consumption may be achieved by recognizing the valve throttling losses over those methods which do not. A rigourous algorithm has been developed by Happ et al[64] to give the functional relationship between fuel input and megewatt generation recognizing the throttling effects caused by multi-steam admission valves. A DP optimization technique is subsequently employed which is sufficiently general that arbitrary cost functions may be scheduled. The dispatch algorithm is centred on an optimum dispatch table constructed using a DP based recursive formula. The optimum dispatch table is built by combining the generators, one at a time.

This is similar to Lowery's algorithm for unit commitment as described in Chapter 3. For example, two machines are dispatched such that their total costs are minimum at various combined output levels. This table is then regard as an equivalent unit of the two which is then combined with a third machine to produce an optimum dispatch table for the three generators. This process is repeated for as many units as are available in the system. Only those units scheduled for synchronization by unit commitment for the target time need to be considered. When all available units have been considered, the optimum cost to supply a forecast load demand can then be read off from the final combined optimum table.

Ringlee and Williams[177] proposed that transmission losses can be included by the application of small perturbation model and solved again using a dynamic programming formulation. Using the [B] matrix coefficient approach, total transmission loss is :

$$P_L = [P_g][B][P_g]+[B_0][P_g]+B_{00}$$

where [B], $[B_0]$ are suitably modified to take into account the connectivity of the generators. When transmission losses are included, the load balance constraint becomes

$$\sum_{g=1}^{Ng} P_g = D+[P_g][B][P_g]+[B_0][P_g]+B_{00} \qquad (5.50)$$

where D = total system load demand. Suppose the generation for each unit is changed by $\Delta P_g$, Eq.(5.50) becomes

$$\sum_{g=1}^{N_g}(P_g+\triangle P_g) = D+[P_g+\triangle P_g][B][P_g+\triangle P_g]+[B_0][P_g+\triangle P_g]+B_{00} \tag{5.51}$$

Subtract Eq.(5.50) from Eq.(5.51) and neglect the second order $\triangle P_i$ terms yields

$$\sum_{g=1}^{N_g} \triangle P_g(1-B_{0g}-2\sum_{i=1}^{N_g} B_{ig}P_i) = 0 \tag{5.52}$$

$$=> \sum_{g=1}^{N_g} \tau_g \triangle P_g = 0$$

where $\tau_g=1-B_{0g}-2\Sigma B_{ig}P_i$ and represents the coefficient of effective output of generation injection for a small deviation from its initial scheduled $P_g$ to satisfy the load demand and shall be updated in each iteration when new $P_g=P_g^{old}+\triangle P_g$ is determined.

The optimal shifts, $\triangle P_g$, in Eq.(5.52) can be determined by the application of a conventional dynamic programming forward status cost calculation and back tracking procedures outlined as followed.

**Forward Status Cost Calculation**

Following the normal DP solution procedures, the problem is broken into a number of subproblems or stages. As each generating unit is treated as a stage, there are as many stages as the number of units available in the system. Define a residual value, $R_k$, such that

$$\sum_{g=1}^{k} C_g \triangle P_g = R_k \tag{5.53}$$

subject to constraint

$$P_g^{min} \le P_g + \triangle P_g \le P_g^{max} \qquad \text{for } j=1,2..N_g$$

representing the total effective change in the power generation up to stage $k$, then the possible range of $R_k$, suitably discretized forms the states in each stage. The feasible range of $\triangle P_k$ for unit $k$ together with the feasible states of stage $(k-1)$ will define the possible range of $R_k$ of stage $k$. At each stage $k$, the status cost of state $R_k$ is the minimum total operational cost of supplying $\Sigma(P_g+\triangle P_g)$ to the system for $g=1,2,3...k$. Consider stage 1, the possible states are:

$$R_1 = C_1 \triangle P_1 \qquad (5.54)$$

The status cost of states $R_1$ are

$$f_1(R_1) = \min_{\triangle P_1} F_1(P_1+\triangle P_1) \qquad \text{for all feasible } R_1$$

which are equal to the fuel cost of unit 1 at the corresponding output $(P_1+\triangle P_1)$. In stage 2, the status cost of states $R_2$ are then

$$f_2(R_2) = \min_{\triangle P_2} \{F_2(P_2+\triangle P_2)+f_1(R_2-C_2\triangle P_2)\} \qquad (5.55)$$

where $F_2(P_2+\triangle P_2)$ is the fuel cost of unit 2 with its output modified to $(P_2+\triangle P_2)$ and $f_1(R_2-C_2\triangle P_2)$ is the status cost of state $(R_2-C_2\triangle P_2)$ in stage 1. In DP terminology, $F_2(P_2+\triangle P_2)$ is the transition cost from state $(R_2-C_2\triangle P_2)$ of stage 1 to state $R_2$ of stage 2. In general, the following recursive formula can be applied iteratively to obtain the status cost of the feasible states of stage $k$.

$$f_k(R_k) = \min_{\Delta P_k} \{F_k(P_k+\Delta P_k)+f_{k-1}(R_k-C_k\Delta P_k)\} \qquad (5.56)$$

**Back Tracking**

To satisfy load balance constraint of Eq.(5.52), the only state needed to be consider in stage $N_g$ is $R_{Ng}=0$. The status cost of $f_{Ng}(0)$ is the minimum cost of supplying the load D including transmission losses. The optimal shift, $\Delta P_g$, for all units in the system leading to this revised optimal cost can be determined by backtracking from the final stage to the first stage of the DP process. When $\Delta P$ for all available units are determined, the new operating point $P_g^{new} = P_g^{old} + \Delta P_g$ may be used to update the effective generation coefficients in Eq.(5.52) and a new iteration to refine the optimal generator outputs may be started. The iteration process is terminated when the difference in optimal production cost between two successive iteration is within a pre-specified tolerance.

Shoults, Venkatesh[191] et al also applied the DP technique to overcome the non-monotonously increasing characteristic of some generating units in Texas Utilities Generating Company. The DP recursive formula used is similar to Ringlee's. The optimal output level of the individual unit against the total system generation is then smoothed and piece-wise linearized. The resultant piece-wise linearized model is then used in a conventional dispatch algorithm. The average error introduced by the linearization process is found to be about 0.04%. In comparison, the error by using the conventional quadratic input/output curve of a unit is 0.423%

which is ten times the error of the DP derived linearized model.

The advantage of the DP approach to economic dispatch is its unique applicability to any shape of fuel cost function in which input may not be a monotone function of output and not necessarily differentiable and continuous. The disadvantages are that it suffers from the same symptoms as Lowery's DP approach to the unit commitment problem such that it may not be able to deal with large scale problems inherent to dynamic programming methods and that, like the equal incremental cost method, it cannot easily handle transmission line limitations. Shoults, Venkatesh[191] et al's approach overcame these limitations by combining the DP fuel cost modelling capability with a conventional dispatch technique; but no appreciable fundamental improvement of the capability of the DP technique has been proposed. In the next chapter, a new DP algorithm is presented which addresses directly the dimensionality and transmission constraint issues.

## 5.7 Summary

Since the emergence of larger power system from the early 50's in the U.S.A and European countries, economic dispatch has been an essential subject for the energy efficiency minded electricity supply utilities. This chapter has outlined some of the prominent methodologies employed. The techniques reviewed included the fundamental equal incremental cost methods, gradient search approaches, linear programming, quadratic programming and dynamic programming. Among these a variant of equal incremental cost method, merit order, a variant of incremental cost method, is widely used either on its own right or incorporated as a rapid trial dispatch in the initialization stage of more sophisticated algorithms. Gradient search methods are generally regarded as computationally slow and have limitations to consider many operational constraints. LP techniques are by far the most flexible, reliable and computationally efficient. An LP formulation is also capable of incorporating many constraints which may not be easily tackled by other methods. LP is therefore by far the most popular of all methods. However, QP algorithms offer the capability to model the quadratic characteristic of transmission losses more accurately and has attracted much attention recently. On the other hand, DP has the unique advantage of considering any form of cost curve but its inherent requirement of enormous computer storage and long computation time has limited the scope of its applications. There has been success in combining the DP technique with other dispatch techniques to achieve a comprehensive algorithm to include operational constraints.

# CHAPTER 6

# LARGE SCALE DYNAMIC PROGRAMMING BASED DISPATCH INCLUDING TRANSMISSION LOSSES

The complex optimization problem associated with the economic allocation of generator real power outputs to meet a load demand has been the subject of considerable research. In the last chapter, the available techniques reported in the literature have been reviewed in some detail. In general these recorded techniques work satisfactorily; but for large systems, linearization of non-linear fuel cost models, simplification of line flow limits using area import/export constraints or disregard of network losses have to be introduced either to reduce the problem size or to conform to a particular problem formulation. Any of these approximations will probably introduce sub-optimality to the final solution. In this chapter an original method, Dynamic Programming with Loss Minimisation (DPLM)[37], is described. The technique is based on the principle of dynamic programming (DP) and includes both transmission limits and accurate loss representations in the overall optimization strategy. This thesis reports the theoretical derivation of the method and gives an objective evaluation of its performance in comparison with some existing techniques.

## 6.1 Limitations of the Existing DP Algorithms

There are two fundamental advantageous properties inherent with DP approaches. The first originates from its unique discrete input-output representation of generation cost model which leads to its complete flexibility to deal with complex generator cost characteristics. Non-linearity, discontinuity or non-differentiability of the generator operating cost functions relevant to multi-steam valves or multi-fuel machines can all be accommodated without complication[82]. The second is its intrinsic ability to decide the on-off status of the units while allocating the optimal load sharing among the available units. In most existing dispatch techniques, it is assumed that a subset of the available units are assigned in the unit commitment phase to participate in the economic sharing of the system load. There are occasions, such as sudden failure of generating units or large load forecast error in which it will be useful if the dispatch method employed has the in-built ability to decide the starting up/shutting down of one or more quick start units such as gas turbine plants in order to maintain the system regulating margin while considering the economic consequences. DP approaches provide such an inherent capability.

Despite these two distinctive advantages, DP has not attracted much attention for economic dispatch applications in either on-line or off-line mode. This lack of interest is probably due to the fact that DP methods are inherently also much more CPU intensive and normally require enormous memory storage. The earliest work on DP approach by Ringlee and Williams[177] in the beginning of 1960's, in addition, suffered from neglecting transmission limitations of a system. The recent algorithm proposed by Shoults, Venkatesh et al[191] made use of the DP generation cost modelling capability but have to resolve to a conventional dispatch method for overall cost minimisation and inclusion of operational constraints. There was no significant improvement of DP technique introduced to deal directly with the dispatch problem. The new method described in the following paragraphs succeeded in overcoming the inherent disadvantages DP approaches. Numerical results on applications to various networks including a data set from the CEGB indicate that the technique is potentially suitable for on-line large system applications.

## 6.2 Problem Formulation

The structure of the proposed solution scheme is organised in such way so that maximum flexibility and computational efficiency may be realised.

(a) The objective function is assumed nonlinear. It represents the total operation cost of all synchronized generators which may be linear or quadratic or any other non-linear complex cost function. This complete non-restrictive generating cost characteristic reflects the indigenous advantage of DP based methods. The unique capability of DPLM technique is obtained primarily through the correct choice of cost functions for each generating unit.

(b) The constraints are assumed linear for computational efficiency.

(c) Dynamic programming is then employed to compute the resultant optimal solution.

The implementation of this conceptual algorithm is described as follows.

### 6.2.1 Objective Function

The objective function for the DPLM approach is straightforward and is simply the minimisation of the total production cost of all on-line units whose total active power generation equals to the forecast demand plus any transmission loss that may occur.

$$\text{Minimize} \quad C(D^{total}) = \sum_{g=1}^{N_g} F_g(P_g) \tag{6.1}$$

where

$C$ = total production cost of all on-line units,

$D^{total}$ = total forecast load demand including losses for the system,

$F_g$ = fuel cost as function of active power output of generator g,

$P_g$ = active power output of generator g,

$N_g$ = number of available generating plants, including off-line gas turbine and pumped storage units which are allowed to start up rapidly.

The generator fuel cost functions, $F_g(P_g)$, in the above equation are completely general, restricted neither to linearity, convexity nor differentiability requirement. Any analytical or empirical cost to generation output relationship may be used as long as the generation cost at any active power output level of a unit can be readily calculated. The cost optimization is subject to a large number of constraints derived from operational limitations. The most frequently referenced constraints are treated in the following paragraphs and the method to incorporate these in DPLM approach is described.

### 6.2.2 Network Limitations

These constraints are essentially the current carrying limits of the transmission lines. Current in a network depends on both the distribution of the load and of generation in a non-linear fashion. However an approximate linear relationship between line flows, load distribution, transmission losses and generation injections at different buses of the network can be derived as follows.

#### 6.2.2.1 Power Flow

Consider a line *k* with nodes *i* and *j* at its sending and receiving ends respectively. Let the voltages at these nodes be $V_i/\underline{\theta_i}$, $V_j/\underline{\theta_j}$ and the impedance of the line be $(R_{ij}+jX_{ij})$. Then the current flow in the line is:

$$I_k = (V_i/\underline{\theta_i}-V_j/\underline{\theta_j})/(R_{ij}+j\,X_{ij})$$
$$= \{(V_i\text{Cos}\theta_i-V_j\text{Cos}\theta_j)+j(V_i\text{Sin}\theta_i-V_j\text{Sin}\theta_j)\}/(R_{ij}+jX_{ij})$$

Assuming $V_i,V_j \approx 1.0$ p.u. and $\theta_i,\theta_j \ll 1.0$, then

$$I_k = j(\theta_i-\theta_j)/(R_{ij}+jX_{ij})$$
$$\text{Real}(I_k) = F_k = \{X_{ij}/(R_{ij}^2+X_{ij}^2)\}(\theta_i-\theta_j)$$

In matrix form,

$$[F] = [H][\theta] \qquad (6.2)$$

where

[F] = line real current flows, a column vector having $N_L$

elements

[H] = $N_L x N_n$ matrix, with elements

at $(k,i)$ = $X_{ij}/(R_{ij}^2+X_{ij}^2)$

at $(k,j)$ = $-X_{ij}/(R_{ij}^2+X_{ij}^2)$

at $(k,m)$ = 0.0 for $m \neq i$ or $j$.

$[\bar{\theta}]$ = nodal voltage phase angles, a column vector having $N_n$ elements

$N_L$ = number of lines in the network

$N_n$ = number of nodes in the network.

#### 6.2.2.2 Nodal Injections

Let the voltage at node $i$ equal to $\mathbf{V}_i = V_i \underline{/\theta_i}$ and the current injection due to a generator be $\mathbf{I}_g = I_g \underline{/\theta_g}$, then

complex power injection = $\mathbf{V}_i \mathbf{I}_g^* = V_i I_g \underline{/\theta_i - \theta_g}$

active power injection, $P_g = V_i I_g \mathrm{Cos}(\theta_i - \theta_g)$

If it is now assumed that $V_i \approx 1.0$ p.u., then active power, $P_g \approx I_g \mathrm{Cos}(\theta_i - \theta_g) = F_g$. The active current injection due to active power generation is therefore approximately equal to the active power output of a unit, i.e. $F_g = P_g$.

As for generation, the real current injection of a load demand at a node may be approximated by the active load value except that a negative value is needed to signify that it draws current from a node, i.e. $F_d = -D_d$.

Current injection due to a shunt element or susceptance of a line is similar to the current flow in a line except that the receiving node is grounded. Thus,

$I_s = V_i \underline{/\theta_i} / jX_s$

$= (V_i \mathrm{Cos}\theta_i + jV_i \mathrm{Sin}\theta_i)/jX_s$

$$\text{Real}(I_s) = F_s \approx V_i \sin\theta_i / X_s \approx (1/X_s)\theta_i$$

Now, the summation of real current injection into a node is zero and thus for node $i$,

$$\sum_{g\epsilon i} P_g - \sum_{d\epsilon i} D_d = \sum_{j\epsilon i} F_{ij} + \sum_{s\epsilon i} F_s$$

$$= \{ \sum_{j\epsilon i} X_{ij}/(R_{ij}^2+X_{ij}^2) + \sum_{s\epsilon i} 1/X_s \}\theta_i - \sum_{j\epsilon i} X_{ij}/(R_{ij}^2+X_{ij}^2)\theta_j \qquad (6.3)$$

where g, d, j and s are set of generators, loads and lines and shunts connected directly to node i. In matrix form,

$$[K_g][P_g] - [K_d][D_d] = [A][\theta]$$

or, $$[\theta] = [A]^{-1} \{[K_g][P_g] - [K_d][D_d]\} \qquad (6.4)$$

where

$[\theta]$ = a $N_n$ column vector of nodal voltage phase angle,

$[A]$ = a $N_n x N_n$ admittance matrix as defined by Eq.(6.3),

$[P_g]$ = a $N_g$ column vector of generation for each generating unit,

$[D_d]$ = a $N_d$ column vector of load demand,

$[K_g]$ = a $N_n x N_g$ connection matrix between nodes and generators,

$[K_d]$ = a $N_n x N_d$ connection matrix between nodes and loads,

$N_d$ = number of loads in the network.

Substituting Eq.(6.4) to Eq.(6.2) to give the functional relationship of line flow [F] in terms of generator outputs $[P_g]$ and load distribution $[D_d]$.

$$[F] = [H][A]^{-1} \{[K_g][P_g] - [K_d][D_d] - [U][M]\}$$
$$= [S][P_g]-[K] \qquad (6.5)$$

where

[M] = nodal load, converted from transmission line losses, (see Transmission Losses section below)

[U] = a unity matrix of $N_n$ order,

[S] = $[H][A]^{-1}[K_g]$ = sensitivity matrix,

$[\bar{K}]$ = $[H][A]^{-1}\{[K_d][D]+[U][M]\}$ = constant column vector.

In the above equations, [A] is a $N_n$ square matrix as against $(N_n-1)$ square that is normally used in a DC load flow. Here, there is no slack or swing bus. Singularity of [A] is avoided by the fact that shunts or line charging exists in the system. Since [S] is a constant for any particular network topology, and [K] is a constant for a forecast load distribution and a given set of line loss values, [F] can be calculated directly for a given generator outputs $[P_g]$. The proposed method DPLM is used to determine $[P_g]$ so that line flows monitored using Eq.(6.5) do not violate any current rating limit while the total fuel cost described by the objective function is a minimum, with further conditions and constraints described below. It should be noted that the line flow calculated using Eq.(6.5) is the active current flow only. Since the current rating of a line is the magnitude of a complex current value, an inequality constraint in the form of Eq.(6.6) may be used to reflect this.

$$[SQRT(F^2+E^2)] \leq [\text{Current limit}] \tag{6.6}$$

where [E] = estimated reactive current in the lines. It is generally recognised that the re-distribution of active power generation does not significantly affect the reactive current flow in a line. [E] can be treated as a constant in the active power dispatch process and can be calculated from an on-line state estimator. Given the reactive current flow E, active current flow limitation can be simplified to:

$$F \le [\text{Active current flow limit}] \tag{6.7}$$

The power transfer stability limits of the lines may be added to the analysis by imposing additional flow limits to the lines:

$$[F] \le [\text{Active Power Transfer Limits}] \tag{6.8}$$

Note that active current and active power flow are interchangeable in the above equations since all voltages are approximated by unity. Matrix [A] is sparse and symmetrical. To save memory space and to increase speed, $[A]^{-1}$ may be obtained using a sparse matrix factorizing technique such as Zollenkopf's algorithm[231].

### 6.2.3 Generator Output Constraints

In general, a generating unit has lower and higher output limits, such that

$$[P_g^{min}] \le [P_g] \le [P_g^{max}] \tag{6.9}$$

In dispatching the generators for a future load, the ramping rates for the generators from their present outputs must also be included. Further limitations on generator outputs therefore apply.

$$[P_g^{present}]-[R_g^{d}]t \le [P_g] \le [P_g^{present}]+[R_g^{i}]t \tag{6.10}$$

where

$[P_g^{present}]$ = generator output at the time of executing the active power dispatch program obtained from an on-line State Estimator.

$[R_g^{d}],[R_g^{i}]$ = ramping rate to decrease and increase respectively which can be a constant or $[P^{present}]$ dependent,

$t$ = look ahead time, typically 5 to 30 minutes.

To improve system security or to represent approximately the station or boiler limitations, lower and upper limits can be imposed on a group of generators which may or may not be on the same station. Thus,

$$[P_{group\_i}{}^{min}] \le [\sum_{g\in group\_i} P_g] \le [P_{group\_i}{}^{max}] \qquad (6.11)$$

Similar to unit ramping rate limits, group ramping limits can also be imposed on a group of generators which will affect the group capacity limits.

### 6.2.4 Area Import/Export and Tie-line Constraints

To further enhance the security of the system, import and export limitations can be applied to certain areas in the system and are frequently employed in many existing dispatch algorithms to ensure certain regulating margin reserved for the complete or regions of a power network. The proposed DPLM algorithm has no difficulty to incorporate such limitations into the problem formulation. Mathematically, these constraints can be written in the form of group line flow limits as depicted in the Eq.(6.11). A graphical representation of the constraints is shown in Fig.6.1.

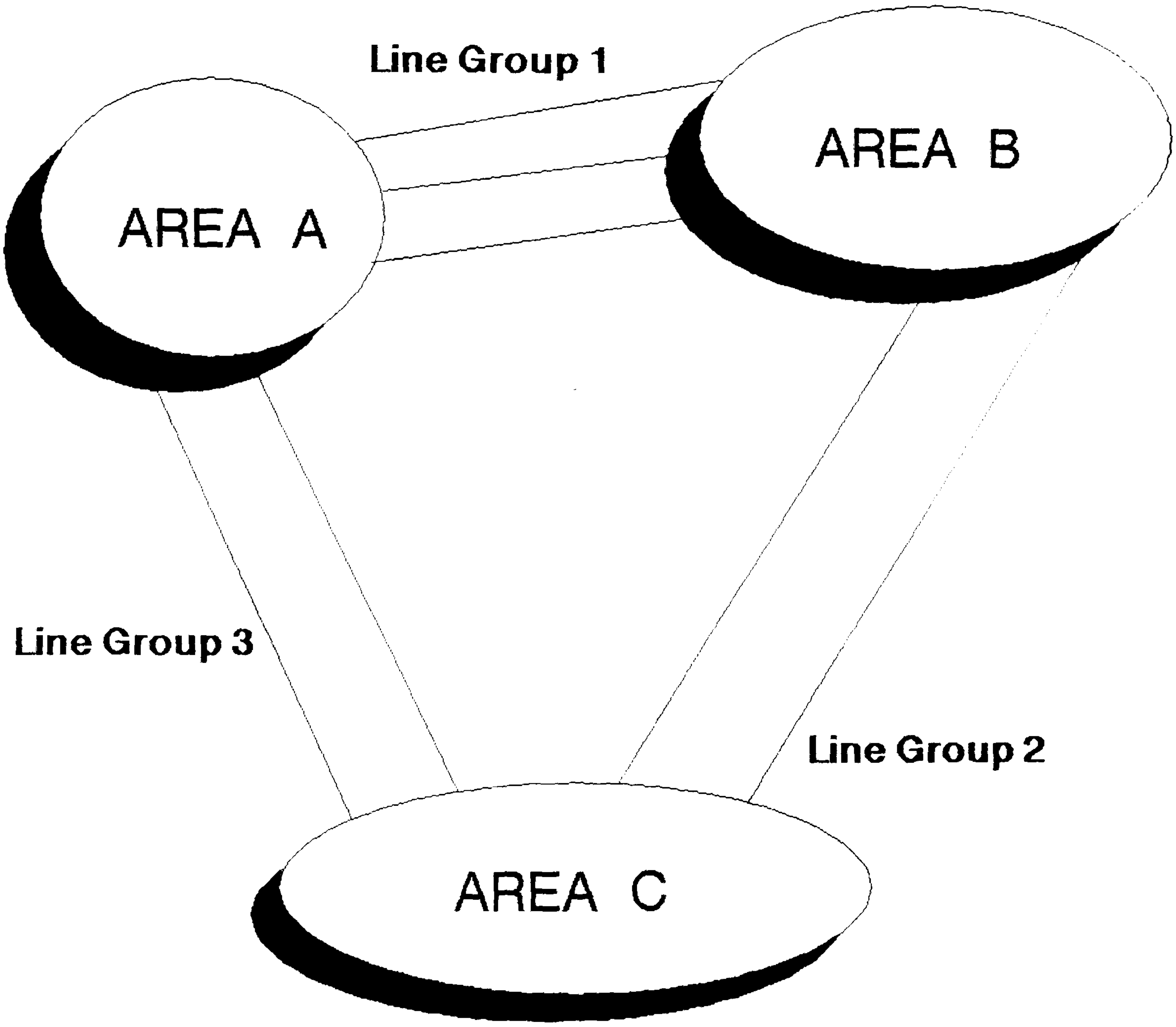



Fig. 6.1 Area Import/Export Constraints (Line Group Constraints)

$$[F_{min}^{group\_i}] \leq [\sum_{k \epsilon group\_i} F_k] \leq [F_{max}^{group\_i}] \qquad (6.12)$$

Tie line power transfer constraints between systems can also be represented accurately in a similar fashion.

### 6.2.5 Transmission Losses

The importance of inclusion of transmission losses in economic dispatch was discussed in the last chapter and various formulae for transmission losses estimation were reviewed. While some of the methods, especially the conventional [B] coefficients loss formulae, are simple to use but suffered from the inability to response to rapid changes in system topology and load distribution. The method of Nicholson and Sterling has the advantages of responsive to changes in system condition but the loss to active power generation relationship is complex to established and not exactly compatible with the design of the proposed dispatch method in that DPLM does not use any impedance matrix in any stage of the solution process. To facilitate the DPLM algorithm and for high accuracy, a new formula for evaluating transmission losses is derived here. It has been shown that for minimum total operation cost, the incremental cost of all contributing units including losses should be equal. Thus,

$$dF_g(P_g)/dP_g \cdot pf_g = \mu = \text{net incremental cost} \qquad (6.13)$$

where

$pf_g$= penalty factor of unit g = $1/[1-d(\text{Loss}^{total})/d(P_g)]$

g = generating units = $1,2,3,\ldots,N_g$

Now, consider a line $k$ with impedance $Z_k=R_k+jX_k$ and current $I_k=F_k+jE_k$, then loss in the line is :

$$\mathrm{Loss}_k = I_k I_k^* Z_k = (F_k^2+E_k^2)(R_k+jX_k)$$

$$\text{Real Power Loss}_k = (F_k^2+E_k^2)R_k \tag{6.14}$$

The total real power loss for all lines in the system is then

$$\mathrm{Loss}^{\mathrm{total}} = \sum_{k=1}^{N_L} \mathrm{Loss}_k = \sum_{k=1}^{N_L} (F_k^2+E_k^2)R_k \tag{6.15}$$

Hence,

$$d(\mathrm{Loss}^{\mathrm{total}})/dF_k = 2F_kR_k$$

From Eq.(6.5),

$dF_k/dP_g = S(k,g)$ where $S(k,g)$ is the $(k,g)$ element of $[S]$

and since

$$d(\mathrm{Loss}^{\mathrm{total}})/dP_g = \sum_{k=1}^{N_L} [(d\mathrm{Loss}^{\mathrm{total}}/dF_k)(dF_k/dP_g)]$$

therefore, the penalty factor for unit g is

$$pf_g = 1/[1 - \sum_{k=1}^{N_L} 2F_kR_kS(k,g)] \tag{6.16}$$

where $F_k$ = real current flow in line $k$ with given load and generation distributions. The proposed DPLM technique dispatches the generator outputs iteratively. In each iteration the optimal generator outputs to meet the forecast load and losses are calculated. Using Eq.(6.5), the active current flow in each line for the estimated optimal generation pattern can be determined. Revised loss in each line is found by substituting the line flows in Eq.(6.14) and is then distributed equally at the two end nodes of the line as additional nodal loads to the system. Penalty factors of the generating units can also be updated using Eq.(6.16) before starting a new iteration. At the first iteration, the penalty factors of the available units may be initialized to unity or the values given by the last dispatch can be used. Line losses may be initially set to a certain percentage of the forecast load or utilizing an A.C load flow to determine the exact losses of the system at its present conditions.

The advantage of using the above formulae for the calculation of losses and penalty factors over the conventional [B] coefficients approach is three fold:

1. There is no need for pre-calculation of any penalty factors before executing the dispatch program.
2. There is no need for a "base" case which can only give an approximation to system losses. The base case approach cannot readily reflect the rapid changes in system topology, load distribution and generation pattern.
3. The sensitivity coefficients $S(k,g)$ and line flow $F_k$ in Eq.(6.14) and (6.16) are an integrated part of the DPLM algorithm.

Furthermore, there is no significant additional computation involved to update system losses and generator penalty factors at each iteration. It is particularly useful that the technique is not only responsive to the rapid system topology changes, but also to the predicted load level, its distribution, and optimal distribution of generation for the predicted load.

### 6.2.6 Power Balance

As mentioned in the last section, transmission loss in each line is distributed equally at the two end nodes of the line as additional loads to the system. The summation of generator outputs must therefore satisfy the summation of loads including losses at each node, or,

$$\sum_{g}^{N_g} P_g = \sum_{i}^{N_d} (\text{Forecast load demand}) + \sum_{i}^{N_n} (\text{Estimated nodal load due to line losses}) \qquad (6.17)$$

## 6.3 Computational Algorithm

With the objective function and its relation to the power balance constraints, line flow, generation distribution and transmission losses defined in the above paragraphs, the method of computation, Successive Dynamic Programming technique (SDP), utilized to determine the optimal generator outputs is presented in this section. It has been shown in Chapter 4 that a DP technique can be successfully applied to the unit commitment problem. The SDP method represents an extension of the previous work designed to further reduce the computation time, storage requirements and improve accuracy. In essence, the proposed SDP calculation mechanism retains the same basic recursive formula, but is applied iteratively so that the number of stages in each DP iteration is reduced. Accuracy of the solution is improved by progressively approaching the exact optimal generation outputs (within tolerance) using the estimated optimal generation outputs obtained in the previous iteration.

### 6.3.1 Generation Production Cost Model

The generation fuel cost model adopted is designed to have three important characteristics:

1. The cost to generation output relation, $F_g(P_g)$, is not restricted to any particular type of analytical function. Much research on economic dispatch uses linearized or piece-wise linearized fuel cost functions, but such representations are a poor approximation for many types of turbine/generator plant and can introduce over 0.4%[159] error to the total operating costs.
2. It is used to minimize the transmission losses in the system. In each iteration the cost function is modified by the penalty factors calculated with the latest results obtained in the previous iteration to reflect the effective production cost of each available unit for the given load distribution.
3. In each iteration, the optimal operating point of a unit is estimated. With this estimated operating point available, the capacity of a unit can be artificially reduced to a pseudo maximum and a pseudo minimum limit. This capacity range is then further reduced in each successive iteration. The production cost model of a unit is therefore also used to progressively improve the accuracy of the dispatch solution. Any convergence criterion can be set on the generator outputs but 0.1 MW

might be a typical figure. Closer tolerances have little effect on the overall solution time.

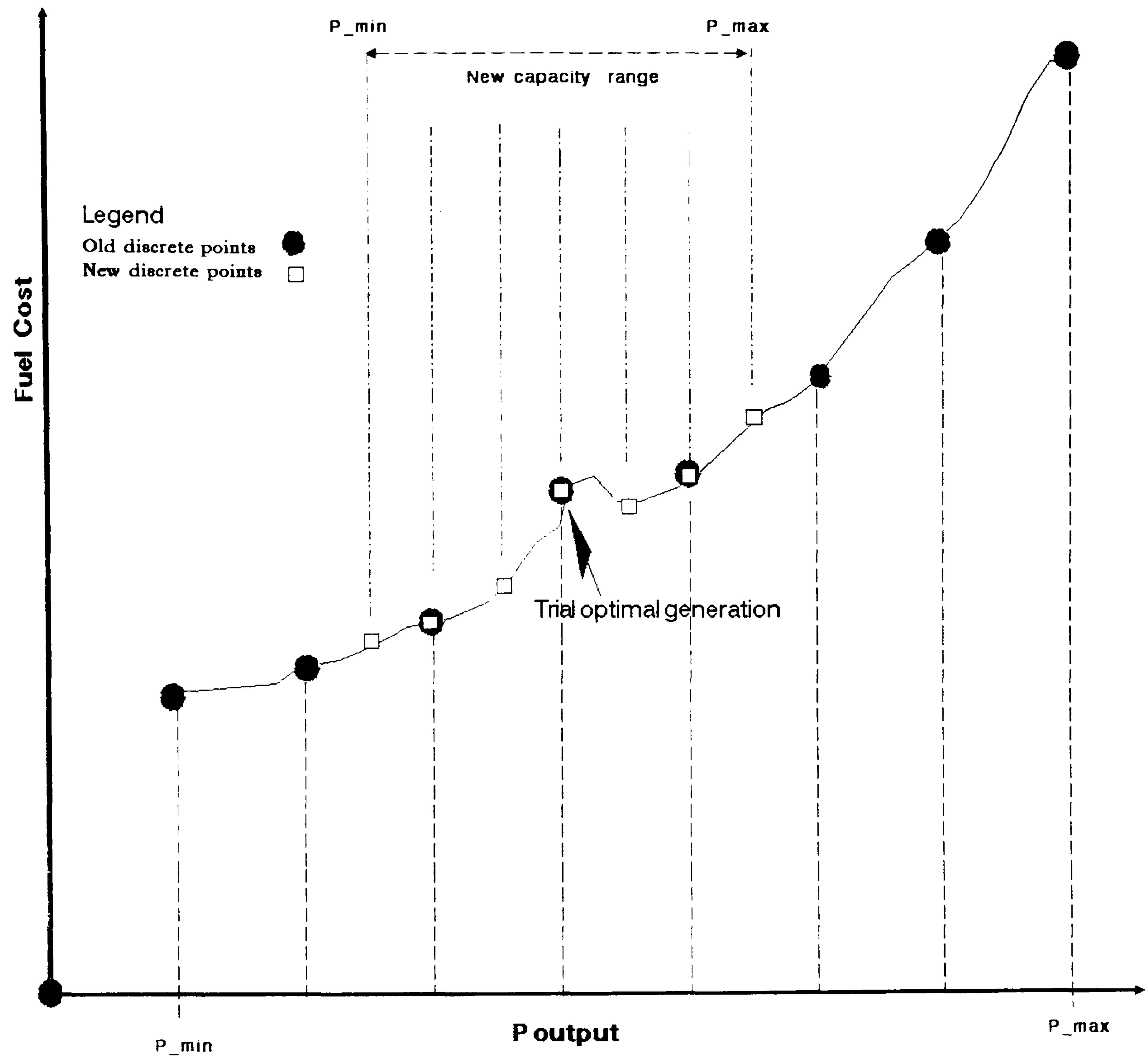
In the DPLM implementation, the production cost functions of the available units are discretized in each iteration and the same accuracy criterion is applied to each unit with respect to their pseudo capacity range. The pseudo maximum to pseudo minimum range can be different for each unit and for successive iterations. Fig.6.2 depicts the fuel cost model of a unit. Pseudo maximums and pseudo minimums and hence the location of the discrete points are updated at each iteration. From the figure, it is easy to see that the shape of the fuel cost function, be it linear, quadratic, or other forms, is not critical for the technique to work. The model only recognises the operational costs at the different generation levels. The convexity or otherwise, nor the differentiability of the cost function is not of any consequence. Furthermore, for discontinuous cost characteristic, such as those for multi-steam valve turbine generators, the technique will guide the unit to an operating point such that the high thermal losses regions will be avoided.

### 6.3.2 Successive Dynamic Programming

Equation 6.18 is the fundamental recursive relationship in the proposed DP approach. It describes how an optimal total operation cost for a system load, D, can be obtained by using the known generator fuel cost functions.

$$C(D) = \text{Min}\ \{C(D-\triangle P_g)+\triangle F_g(\triangle P_g)\} \qquad (6.18)$$

where




***Three important characteristics:***

*1. Generator production cost is translated into a discrete function*

*⇒ can be used for non-linear cost functions.*

*⇒ can be used to decide starting up and shut ing down of rapid response units if needed*

*2. Modified with penalty factors to minimize losses*

*3. P_min & P_max capacity ranges are reduced in each successive iteration for accuracy improvement.*

Fig.6.2 Generator Production Cost Model

$C(D)$ = optimal total fuel cost for a total generation D of all on-line units,

$\triangle P_g$ = additional output for generator g from its optimal loading point at $(D-\triangle P_g)$ level,

$\triangle F_g(\triangle P_g)$ = additional fuel cost. To minimize transmission losses, the effective fuel cost function should be used.

The recursive process is started with $C(D^0)$:

$$C(D^0) = \sum_{g}^{Ng} F_g(P_g^{min}) \qquad \text{where } D^0 = \sum_{g}^{Ng} P_g^{min}$$

The optimal total production cost at any total load level D can then be obtained by repetitive use of Eq.(6.18). It is important to check that the increased output of a unit $P_g$ shall not lead to an overloading of any line in the system. Eq.(6.5) is used to determine the incremental change in line flows due to an incremental change in one or more of the generation units. If any optimal generation output combination causes violation of any line flow limit, the next least expensive combination of generator output $[P_g]$ satisfying all line flow limits will be stored to allow the DP process to continue.

When D equals the total forecast load plus losses, one dispatch iteration is completed. The loading of the generators corresponding to $C(D^{total})$ is the estimated optimal generator outputs. With these generator loading points calculated, the pseudo_max and pseudo_min of the units may then be adjusted to a shorter range. The fuel cost function between these pseudo limits is then discretized with a new step size. The line flows and transmission losses corresponding to the newly estimated generation pattern can then be calculated and penalty factors also updated. A second iteration to give a more accurate operating point of the units may then proceed. The number of iterations required depends on the size of the units in the system and the desired accuracy but typically 5 to 6 iterations are sufficient for a large network with unit output accuracy set to 0.1 MW tolerance. A graphical representation of the recursive formula is shown in Fig.6.3 and the optimization scheme is outline in the flow chart shown in Fig.6.4.

## 6.4 Computational Examples

The economic dispatch models and algorithm described above have been implemented in **FORTRAN 77** on a DEC *VAX-8600* computer using single precision 32 bit floating point storage and arithmetic. The performance of the proposed method was evaluated using various test systems. Results on two of these systems, one of 22 units and another of 115 units are included in this thesis for illustration purposes. The data for the smaller network is extracted from Sterling's[196] book. The generator cost functions, unit and station group operating limits, line resistance, reactance and current limits together with the single line diagram of the network is reproduced in Appendix D for completion. The data for the larger system was provided by courtesy of the Central Electricity Research Laboratory(CERL) of the pre-privatized Central Electricity Generating Board(CEGB) whose details with a schematic diagram are included in Appendix E. Programs utilizing linear programming (LP) and quadratic programming (QP) techniques

*Recursive Formula:* $C(D) = \mathrm{Min}\left\{ C(D - \Delta P_g) + \Delta F_g(\Delta P_g) \right\}$

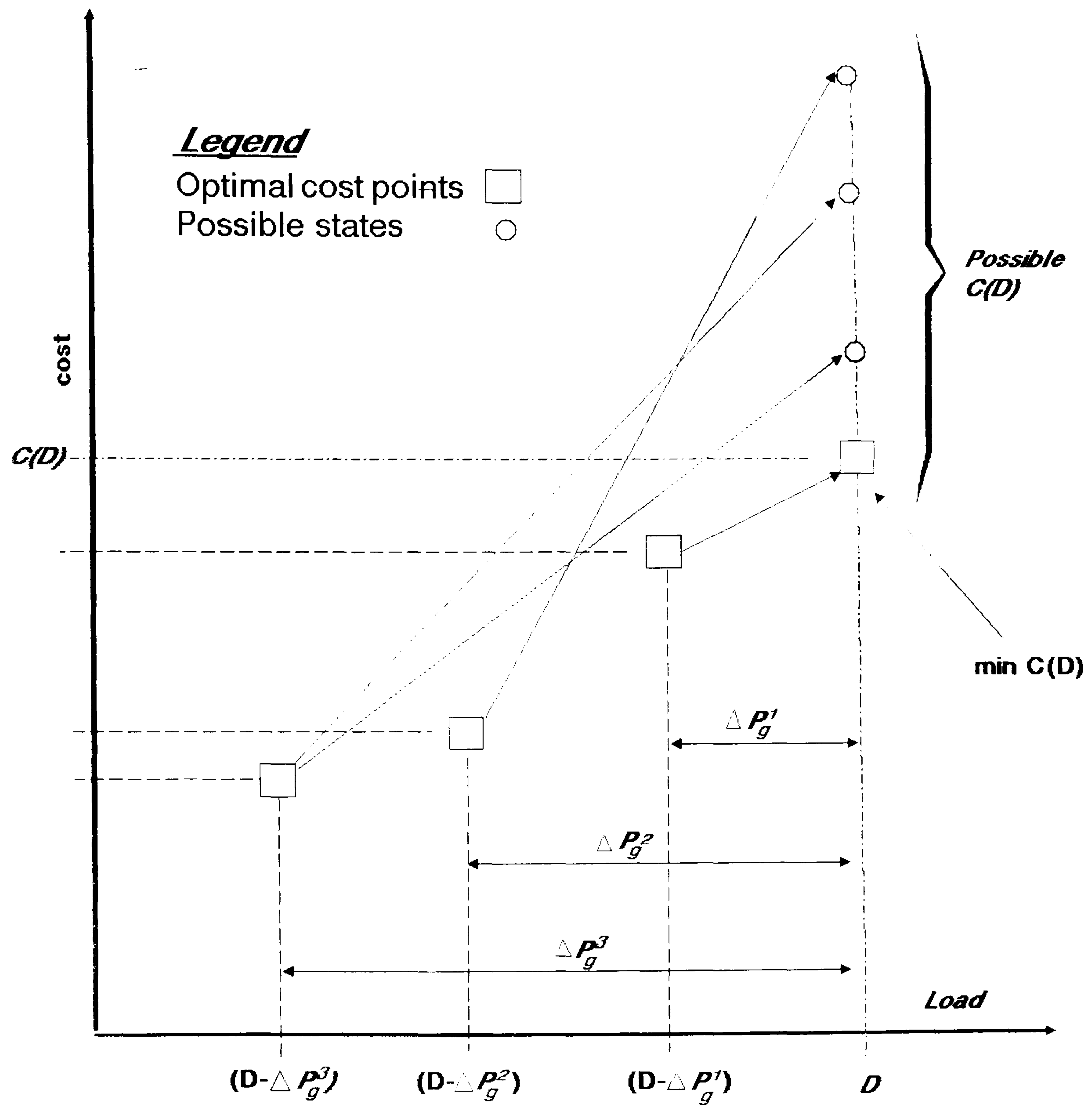



Fig. 6.3 Graphical Representation of Successive Dynamic Programming Recursive Formula

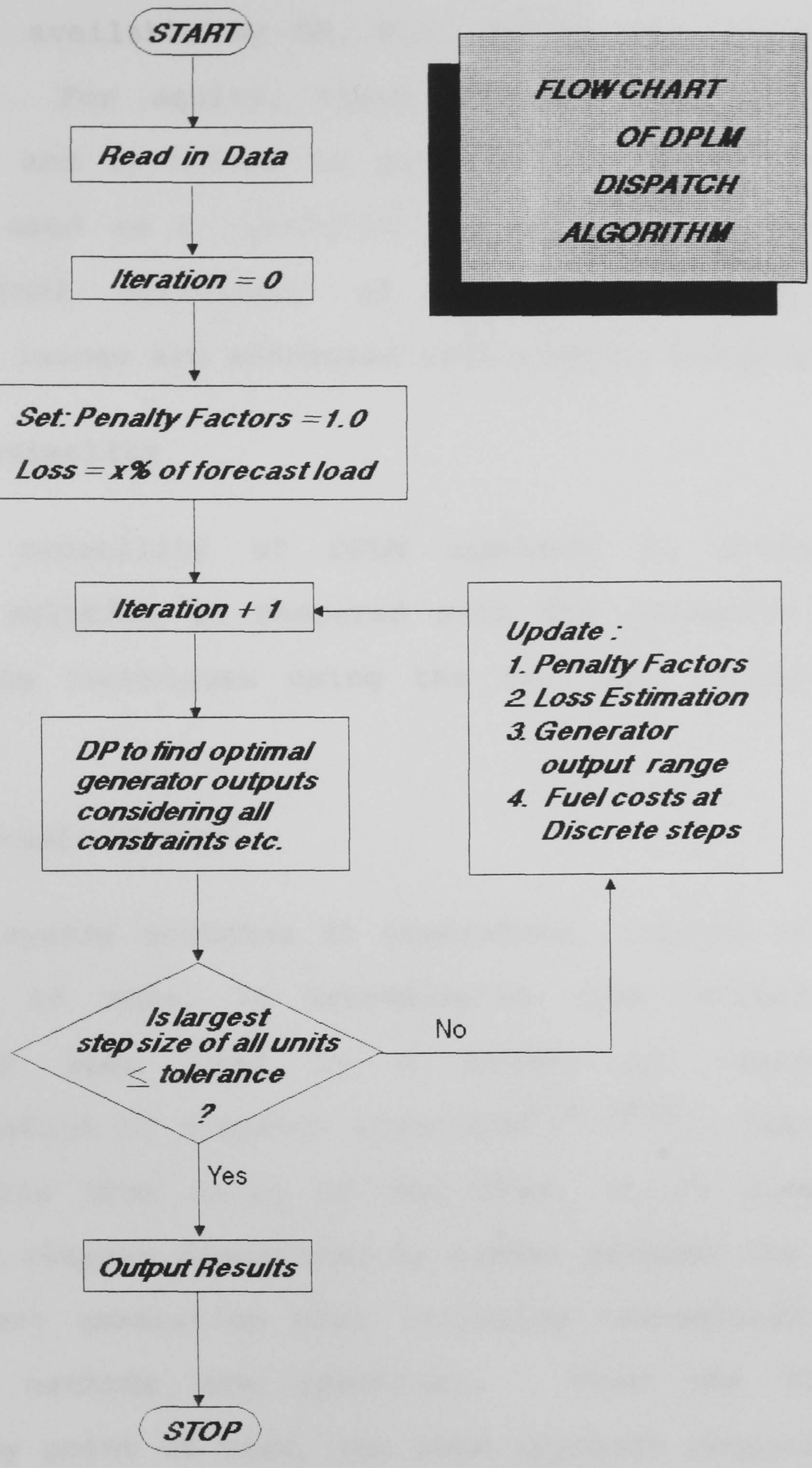



Fig. 6.4 Flow chart of DPLM economic dispatch algorithm

were made available by DR. M.R. Irving and Professor M.J.H. Sterling. For equity, these programs were modified where necessary and optimized to give optimal performance. These are then used as a yardstick to measure the optimality and computational efficiency of the DPLM method. Several important issues are addressed with results detailed below.

### 6.4.1 Optimality

The capability of DPLM approach to achieve optimal dispatch solution is compared with the quadratic and linear programming techniques using the two test systems mentioned above.

#### a) A 22-unit System

The system contains 22 generators, located in 7 stations within a 10 node, 14 transmission line network. It has previously been used by a number of researchers for implementation of dispatch algorithms[4,159]. Table 6.1 shows the results from using QP and DPLM. It is clear that The generator outputs dispatched by either methods are similar and the optimal generation cost including transmission losses by the two methods are identical. From the computational efficiency point of view, the DPLM approach requires less than half of the CPU time needed for the QP technique. The table also shows an important characteristic of DPLM. It tends not to schedule the units to their maximum output limits if there is another unit of similar cost which can share the load. This is a highly desirable feature as this has the advantage of providing more spinning reserve ready for an emergency than would be available otherwise.

**Table 6.1** Generator Optimal Dispatch by DPLM and Quadratic Programming

| Gen No. | Initial (MW) | QP Dispatch (MW) | DPLM Dispatch (MW) |
|---|---|---|---|
| 1 | 60. | 30. | 29.5 |
| 2 | 60. | 30. | 29.5 |
| 3 | 60. | 25. | 24.5 |
| 4 | 60. | 25. | 24.5 |
| 5 | 60. | 20. | 19.6 |
| 6 | 60. | 20. | 19.7 |
| 7 | 80. | 100. | 71.3 |
| 8 | 80. | 100. | 71.8 |
| 9 | 80. | 100. | 78.4 |
| 10 | 80. | 100. | 99.3 |
| 11 | 80. | 50. | 87.3 |
| 12 | 80. | 50. | 92.0 |
| 13 | 30. | 24. | 23.7 |
| 14 | 30. | 24. | 23.7 |
| 15 | 20. | 50. | 50.0 |
| 16 | 20. | 50. | 50.0 |
| 17 | 20. | 50. | 50.0 |
| 18 | 10. | 18. | 17.8 |
| 19 | 10. | 18. | 17.8 |
| 20 | 10. | 18. | 17.8 |
| 21 | 30. | 56. | 55.6 |
| 22 | 30. | 56. | 55.5 |

| | | |
|---|---|---|
| Load= 1000.0 MW. | **QP Dispatch:**<br>Loss=10.1MW | **DPLM Dispatch:**<br>Loss=10.1MW |
| Time in advance = 30 mins. | Cost=2135 units/hr<br>CPU time=2.0 s | Cost=2135 units/hr<br>CPU time=0.9 s |

**b) A CEGB Test Network.**

The test network data provided by the Central Electricity Research Laboratory of CEGB has 145 nodes, 115 generating units and 275 branches. Four loading patterns were also given which were dispatched with the proposed DPLM technique. A comparison of results with those obtained using LP and QP is shown in Table 6.2. It is evident from Tables 6.1 and 6.2

that the proposed dynamic programming approach is capable of solving economic dispatch problems with optimal solutions similar to those obtained by other familiar technique such as LP and QP as demonstrated.

**Table 6.2** Comparison of LP/DP/QP Dispatch Results (Transmission Losses Neglected)

| Load Condition | LP Cost | QP Cost | DP Cost | Difference % (LP-QP)/LP or (LP-DP)/LP |
|---|---|---|---|---|
| Winter Plateau | £914292 | £914279 | £914269 | negligible |
| Winter Trough | £479269 | £479245 | £479240 | negligible |
| Summer Plateau | £471334 | £471318 | £471313 | negligible |
| Summer Trough | £124240 | £124229 | £124229 | negligible |
| CPU time | 5 Sec. | 126 Sec. | 21 Sec. | - |

### 6.4.2 Computational Efficiency

The CPU time requirements, including all data inputs and solution outputs, for the test cases using LP, QP and DP methods are indicated in Tables 6.1 and 6.2. The computation time for the four loading conditions of the CEGB test network required by the DPLM approach in fact varies slightly. Generally, in the summer, the loads in southern England supplied by the relatively economic generators in the north activate more line overloading constraints than the evenly spread heavy load conditions in the winter and hence requires slightly more computer time for constraint checking and to converge to the required accuracy. One useful application of the DPLM technique is therefore to identify the small number of lines which restrict the flow of power preventing the system from operating more economically. As clearly shown in the tables, the computational efficiency of the DP approach is indeed very good. Although it is slower than the LP method, it is much faster than the QP technique. These results also confirm that the 'curse of dimensionality' problem commonly believed to be associated with the DP approach has been overcome by the proposed computational procedures.

### 6.4.3 Transmission Losses Optimization

In Section 6.2.5 above, a detail derivation of penalty factors and losses estimation utilizing a sensitivity matrix [S] is given. This concept is also applied to the LP and QP dispatch approach with penalty factors and losses updated iteratively similar to the DP scheme, resulting in two further computer programs: RLP (recursive LP) and RQP (recursive QP). The objective of this exercise is to establish firmly whether DPLM approach gives a comparable optimal solution when transmission losses are included in the analysis. In Table 6.1, it has been shown that both DPLM and QP give the same optimal operation cost and same transmission losses for the load demand at the target time. This particular case however cannot be regarded as an equitable comparison because the 22-unit system has been widely studied and both the QP and DPLM methods could be tuned to produce the best optimum. It would be interesting therefore to see the competition between the RLP, RQP and DPLM approaches to give the best dispatch solutions to a new data set. The CEGB test network was used in this exercise to investigate the performance of the three fundamentally different computational algorithms in losses optimization.

#### 6.4.3.1 Recursive LP approach for loss optimization

In the recursive LP formulation, after an initial estimation of the optimal solution, the dispatch problem is switched to an incremental model. The objective function of Eq.(5.24) is modified to:

$$\text{Minimize } \Delta C = [C_g \; pf_g][\Delta P_g] \tag{6.19}$$

where $[\Delta P]$ = generation shift from last LP iteration.

Subject to:

1. Load Balance:

$$\sum_{g}^{N_g} (P_g+\Delta P_g) = D + \text{latest estimation of Losses}$$

2. Generation limits: $[\Delta P_g^{min}] \le [\Delta P_g] \le [\Delta P_g^{max}]$

3. Group generation limits:

$$P_{station}^{min} \le \sum_{g \epsilon station} (P_g+\Delta P_g) \le P_{station}^{max}$$

4. Area import/export limits:

$$F_{group}^{min} \le \sum_{k \epsilon Group} (F_k+\Delta F_k) \le F_{group}^{max}$$

5. Line power flow limits: $[F^{min}] \le [F+\Delta F] \le [F^{max}]$

#### 6.4.3.2 Recursive QP approach for loss optimization

The original QP implementation available from the School of Engineering and Applied Science of Durham University optimizes losses by inclusion of a quadratic loss function[103] in the objective function. The total loss is costed at a fixed rate determined off-line based on many trial results or modified in between the iterations according to changes in the latest optimal solution. Either technique minimizes the total operating cost including losses satisfactory but the best results for the test system are obtained with iterative modification of penalty factors of the generators in the objective function as applied to the RLP and DPLM. Table 6.3 summarised the best optimal generation cost considering transmission losses using the DPLM approach and the recursive LP/QP methods.

**Table 6.3** Production Cost Including Transmission Losses using RLP/RQP/DPLM Optimization Technique

| Load Condition | "Control" | RLP Cost | RQP Cost | DPLM Cost |
|---|---|---|---|---|
| Winter Plateau | £932145 | £933502 (+0.15%) | £929929 (-0.24%) | £929621 (-0.27%) |
| Winter Trough | £491854 | £491535 (-0.06%) | £489216 (-0.54%) | £489070 (-0.57%) |
| Summer Plateau | £482760 | £482321 (-0.09%) | £480619 (-0.44%) | £480353 (-0.50%) |
| Summer Trough | £131672 | £131841 (+0.13%) | £131285 (-0.29%) | £131074 (-0.45%) |
| CPU time | | 40 sec | 882 sec. | 25 sec. |

In the above table, "control" is the generation schedule which include the transmission losses estimation in the power balance equation but not optimized. The results of Table 6.3 were quite unexpected. While the penalty factor approach works well with the QP and DP formulation to reduce the overall production cost, it does not have the same beneficial effect on LP approach. Other pseudo-forms of penalty factors also tried to modified the objective function in Eq.(6.19) but without successfully improving the LP solutions. The increase in production costs of the LP approach is probably due to the bi-stable situation created by the penalty factors special to LP methods, as explained in the Section 5.4.2. The table clearly shows a substantial economic benefit by including transmission losses in the optimization if the technique employed can handle losses adequately. DPLM approach indicates a 0.45% average cost saving for the CEGB system. Since the system generally spends more time in the medium load range i.e. winter trough and summer plateau than the extremity of winter plateau and summer trough conditions, an even greater average percentage saving is realizable.

Table 6.3 also shows the CPU time requirements for all the three approaches with the DPLM having the best timing. This however is of no consequence since the objective of the exercise is solely to examine which method will give the best optimal solution when losses are considered. Furthermore, the CPU times quoted are tentative values only and they depend on the stopping criteria which affect the number of the iterations of each method. There is however one important aspect, convergence characteristic, which is not shown in the table. Tests indicate that while DPLM objective function converges smoothly to the optimal value the RLP and RQP results for the four different load cases of the CEGB network do not generally converge. The optimal solutions shown for the RLP and RQP methods are the best results among ten iterations with the best tuned parameters for each load condition.

### 6.4.4 Accuracy of the Sensitivity Matrix

One of the essential element in the DPLM approach is the use of a DC load flow type sensitivity matrix [S]. It performs three important tasks which are fundamental to the speed and accuracy of the DPLM approach:

1. It is used for estimating the line flow in relation to generation and load pattern of the system: Eq.(6.5).
2. It is used for establishing the linear coefficients of the area import/export and tie line transfer limitations: Eq.(6.12).
3. It is used for updating the penalty factors and the effective generating cost of a unit: Eq.(6.16).

DC load flow is generally regarded by the electricity supply industry as an acceptably accurate technique for fast evaluation of active power flow in a line. Table 6.4 below gives a reduced set of line flow and transmission losses determined by the proposed sensitivity matrix and a Newton Raphson A.C. load flow. The table shows the close match of line flows and total losses calculated using an A.C. load flow and those using Eqs.(6.2) and (6.8). It demonstrates the validity of the approximate linear relationship between line flows and active power injections and that the line flow and losses derived using the sensitivity matrix [S] is of high accuracy. A complete economic dispatch results for the winter plateau load condition is included in Appendix D.

**Table 6.4** Comparison of Line Current Flow by DPLM approximation and accurate A.C. Load Flow

(Winter Plateau Load Condition of CEGB Test Network)

| Line No. | Send Node | Recv Node | Flow Limit | Line Flow by AC LF | Line Flow by DPLM |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 9.35 | -2.4490 | -2.4534 |
| 2 | 1 | 3 | 9.35 | 1.2288 | 1.2317 |
| 3 | 4 | 2 | 9.35 | -1.7346 | -1.7400 |
| 4 | 4 | 5 | 9.35 | 0.5144 | 0.5177 |
| 5 | 6 | 7 | 42.50 | 3.2730 | 3.2988 |
| 6 | 6 | 8 | 42.50 | 3.3856 | 3.3605 |
| 7 | 6 | 9 | 42.50 | 6.4106 | 6.3427 |
| 8 | 10 | 5 | 13.00 | 1.4204 | 1.4158 |
| 9 | 10 | 11 | 13.94 | -2.7652 | -2.7691 |
| 10 | 12 | 13 | 11.51 | -5.5237 | -5.4764 |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| 266 | 62 | 113 | 4.25 | 0.1084 | 0.1010 |
| 267 | 62 | 113 | 4.25 | 0.1083 | 0.1009 |
| 268 | 145 | 73 | 42.50 | -2.9339 | -3.0341 |
| 269 | 145 | 131 | 42.50 | -3.0798 | -2.9479 |
| 270 | 143 | 132 | 6.37 | 2.2344 | 2.1869 |
| 271 | 143 | 132 | 6.37 | 2.2265 | 2.1792 |
| 272 | 19 | 40 | 42.50 | -6.3799 | -6.2986 |
| 273 | 142 | 77 | 18.70 | -7.7997 | -7.7143 |
| 274 | 142 | 77 | 18.70 | -7.7997 | -7.7143 |
| 275 | 142 | 29 | 23.03 | 6.1610 | 5.8701 |
| Total Loss (per unit) = | | | | 4.4597 | 4.5256 |

Difference in total loss = 1.5%

All line limits and current flows are in P.U.
(1 P.U. = 100 MVA.)

### 6.4.5 Effect of Step Size

Another important element in designing the overall scheme of the DPLM approach is to overcome the CPU time and huge storage requirement normally associated DP methods. A combination of strategies have been employed. These include:

a) A new DP recursive formula is introduced in which a two-dimensional problem (generator number and generation level) is converted into a single dimensional problem (generation level).

b) Successive iterative procedure is used so that the discretization for each generator may be related to its capacity range and accuracy can be improved with each successive iteration.

As a result of implementing these strategies, step size is chosen automatically within the computer program and the users are needed only to specify the accuracy tolerance which is defined as the largest step size for any unit in the system at the last dispatch iteration. The resultant effect is that accuracy tolerance has little effect on the CPU time requirements. For example a 10 fold increase in accuracy generally means one further iteration; but because many smaller units are already discretized in very small step sizes in the later iterations, the number of capacity states representing these units will be two or three only. Therefore the addition computational requirements for the addition iteration, although system dependent, is minimal.

### 6.5 Summary

The chapter has described a new algorithm, DPLM, for active power dispatch. As illustrated using the study examples, it is apparent that the proposed method is a viable alternative to the existing popular techniques for economic dispatch problems and is applicable to both small and large systems. The advantages of the proposed method can be enumerated as follows:

1. Nonlinear representation of generator fuel cost models. The indigenous DP advantage of complete flexibility in handling practically any form of generation cost functions is retained.
2. Units connected to the system at the same busbar can be modelled separately. This is a very desirable feature particularly when the units in the same station have different production cost characteristics.
3. Uses a new transmission losses formula which is responsive to any change in system topology, load and generation distributions.
4. Minimizes transmission losses while monitoring individual line flow limits which is generally not achievable in some of existing algorithms for large systems.
5. Robust. It gives an optimal generation pattern in each iteration. It produces a best relaxed solution when there is no feasible solution.
6. Precise. Resolution of 0.1 MW for generator outputs is easily obtainable.
7. Speed. To dispatch a test network with 145-nodes, 115-generators and 275-lines requires only 25 seconds on average using a VAX 8600.
8. Unlike most dynamic programming approaches, the technique does not incur large computational penalties as the system size grows.

# CHAPTER 7

# SECURITY CONSTRAINED DISPATCH

In the last two chapters, economic dispatch algorithms were described which schedule the optimal generator outputs to meet a predicted load demand subject to constraints related to a defined system configuration. Operational constraints as a result of unplanned topological change such as line failures or generator forced outages were ignored. This class of dispatch may be referred to as "pure" economic dispatch since post disturbance system conditions are ignored. It has been recognized for some years that system component failures can cause the system to transfer from a normal state into alert or even emergency states. For example, when a transmission line is switched off-line by the automatic protection devices upon detecting a fault condition, the remaining transmission circuits in the system will have to take up the power that was originally flowing in the now opened line. One or more of the remaining lines may now be overloaded and tripped open leading to further line overloading and tripping. It is therefore advantageous that in the solution of economic dispatch, the effect of post-contingency system states should be considered to ensure that plausible initial failure will not lead to overloading in the remaining components of the system. The implementation of such a secure and prudent dispatch, generally referred to as security constrained dispatch, is gradually being adopted by many electricity supply utilities, notably in New York Power Pool[133]. In the United Kingdom, the electricity suppliers, formerly the CEGB and presently the twelve distribution companies, have a statutory duty to 'develop and maintain an efficient, coordinated and economical supply of electricity in bulk continuously except in case of emergency' (Electricity Act, 1957). Security constrained dispatch can play an important part in achieving the optimal compromise between the economy and security of power system operation in compliance with the letter and spirit of the statutory obligations.

This chapter outlines the complications introduced by the inclusion of contingency consideration in an optimal generation dispatch and explores the available techniques to solve this expanded problem. A new technique called Current Injection Method (CIM) is proposed which can be used to simulate line outages. The derivation of the CIM concept is presented. Its application in security constrained dispatch is illustrated by employing the technique to expand the capability of a LP based "pure" economic dispatch algorithm originally developed by Dr. M.R. Irving and Professor M.J.H. Sterling. The network data set provided by the Central Electricity Research Laboratory of the former Central Electricity Generating Board is used as the test system. Study results indicate that the computational requirement of the proposed methodology is comparable to a pure economic dispatch and can be realistically included in a power system EMS control package for real time operation.

## 7.1 Problem Description

The objective of a security constrained dispatch is identical to a pure economic dispatch problem, i.e. minimizing the total operational cost:

$$\text{Minimise} \quad C = \sum_{g=1}^{N_g} F_g(P_g^0) \tag{7.1}$$

where $Pg^0$ is the optimal generator output prior to the occurrence of any contingency. The extra complications introduced by considering post contingency states lie in the requirements that operational constraints for both pre-contingency and all credible post-contingency system states must be satisfied in a concerted manner as illustrated in Fig.7.1. The post contingency constraints may be separated into two distinctive groups according to the cause of disturbance: generator outage and transmission line outage. The available techniques to incorporate these two classes of contingencies in a security constrained dispatch are described as follows.

### 7.1.1 Generator failure Contingencies

One of the primary objective of considering generator contingency is to ensure the availability of enough reserve to cover the lose of a generator. This can be achieved by the use of an additional load balance constraint for each contingency such that

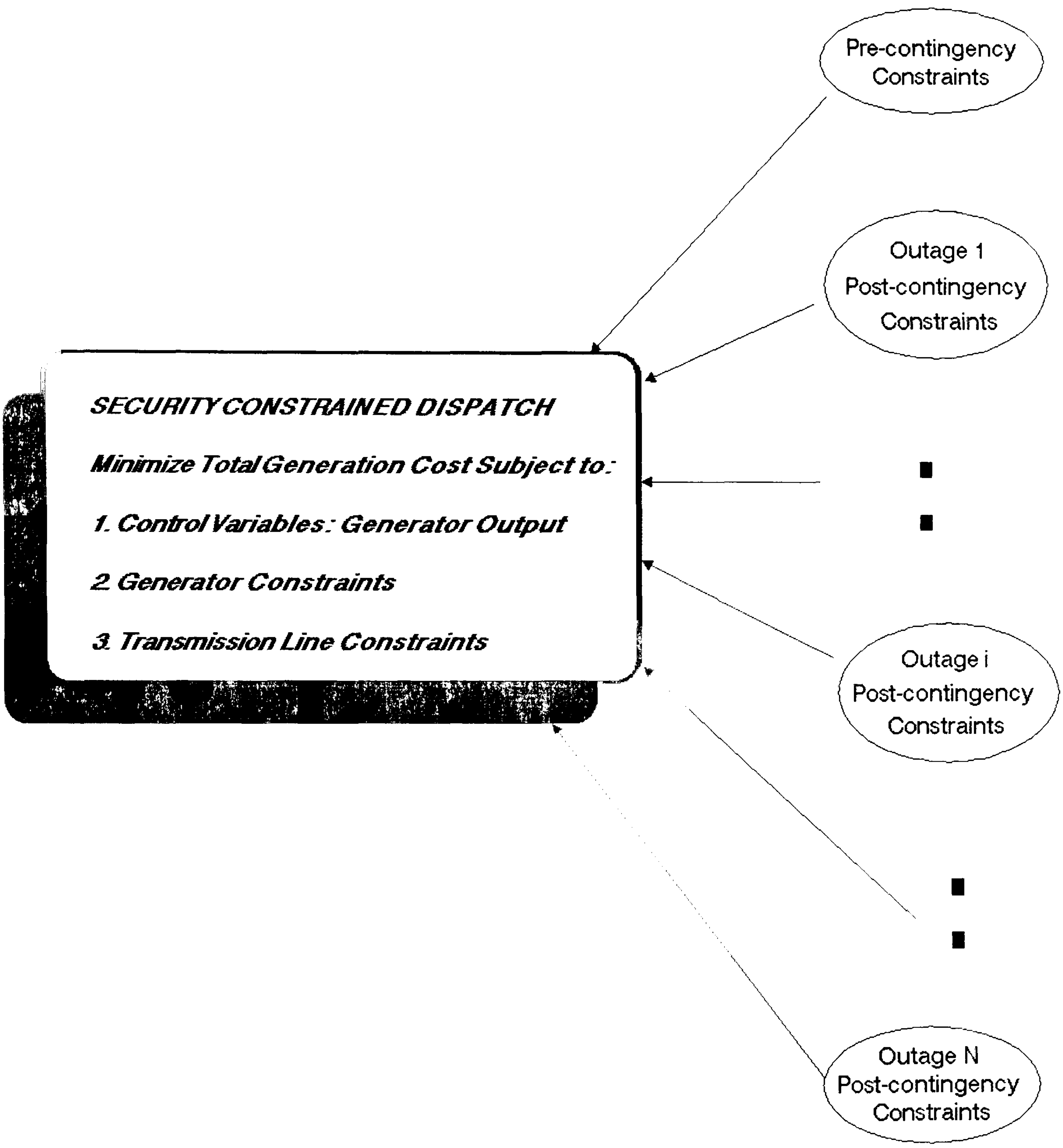



Fig.7.1 Security Constrained Economic Dispatch with Consideration to Post Contingency System States

$$\sum_{g\neq k}^{N_g} P_g^{\,k} = D + \text{Loss} \qquad (7.2)$$

and, $P_g^{\,k} = P_g^{\,0} + \Delta P_g^{\,k}$

where $P_g^{\,k}$ is the output of generator g following the outage of generator k and $\Delta P_g^{\,k}$ is the estimated output change in order to cover the generation depression. The amount of generation change of a unit depends on its reserve availability and its response rate, which in turn depend on its governor droop, boiler, turbine and condenser conditions. Given that the actual amount of additional generation shared between the units can be approximated by simple models and that the transmission configuration remains intact, generator failure contingencies can be encompassed in a dispatch algorithm in a fairly straightforward manner. Assume that generator k is forced outage and that the original power output of the now disconnected unit is divided among the remaining units according to a linear function as shown in Eq.(7.3), then the post contingency operational constraints of the system can be modelled by Eqs.(7.4) and (7.5):

$$[\Delta P_g^{\,k}] = [\phi_{gk}]\, P_k^{\,0} \qquad (7.3)$$

subject to:

(1) Generation limit:

$$[P_g^{\,min}] \le [P_g^{\,k}]=[P_g^{\,0}]+[\Delta P_g^{\,k}] \le [P_g^{\,emergency\_max}] \qquad (7.4)$$

(2) Line flow constraints:

$$[F^k] = [S^0][P_g^{\,k}] \le [F^{emergency\_max}] \qquad (7.5)$$

where

$[\phi_{gk}]$ = pre-specified participation factors for the sharing of generation deficiency for generator k failure.

$[F^k]$ = resultant line power flow after generator k failure and the remaining units shift to their new output level $[P_g^k]$;

$[S^0]$ = Sensitivity matrix which relates the line power flow to generator output in an intact system;

$[P_g^{emergency\_max}]$ = emergency maximum output limits of the generators;

$[F^{emergency\_max}]$ = emergency maximum power transfer limits of the transmission lines.

A common adopted approximation for the coefficients $[\phi_{gk}]$ is to have the values linearly proportional to the relative rated capacities of the remaining[227] units such that:

$$\phi_{gk} = \frac{P_g^{max}}{\sum_{g \neq k}^{Ng} P_g^{max}} \tag{7.6}$$

The appropriate model for the participation factor depends on how critical the effect of a generator failure to the system. For small systems with relatively large units, a more exact model for the reserve capability of the units, together with load shedding coordination and rapid unit start up capability, will be required in order to minimize the spinning reserve requirement. For large systems, a simplification such as Eq.(7.6) is probably adequate. Not all units on the system necessarily participate in the dynamic pick-up of generation deficiency; but given the set of units which are under the direct on-line control of the system operator or EMS for emergency back-up, a similar expression may be derived for each plausible generator outage.

Another frequently utilized technique to model the effect of generator failure on power transmission is to employ the generation shift factor[27,207,227], designated $\alpha_{ji}$, which is defined as:

$$\alpha_{ji} = \Delta F_j / \Delta P_i \tag{7.7}$$

where

$j$ = line index;

$i$ = bus index;

$\Delta F_j$ = change in power flow on line j when a change in generation, $\Delta P_i$, occurs at bus i;

The post-contingency power flow on line j is:

$$F_j^k = F_j^0 + \sum_{i=1}^{Nn} \alpha_{ji} \, \Delta P_i^k \tag{7.8}$$

where

$\Delta P_i^k$ = change in generation at bus i when generator k fails and is equal to the summation of all changes of generators connected directly to bus i;

$F_j^k$ = post contingency power flow in line j;

$F_j^0$ = pre-contingency power flow in line j.

The generation shift factors are obtained from the standard D.C. load flow equation:

$$[P] = [B][\theta]$$
$$[\theta] = [B]^{-1}[P] = [X][P] \tag{7.9}$$

where [B] is the susceptance matrix.

and the approximate power flow equation for a line j with sending end s and receiving end r:

$$F_j = \frac{1}{x_j} (\theta_r - \theta_s) \tag{7.10}$$

For a change of $\triangle P_i$, the effect on the change in phase angle $\theta$ is:

$$[\triangle\theta] = [X][0 \quad 0 \quad 0 \quad 0 \quad . \; . \; \underset{\substack{| \\ \text{position } i}}{\triangle P_i} \; . \; . \; 0]^T \tag{7.11}$$

and the change in power flow in line $j$ is:

$$\triangle F_j = \frac{1}{x_j} (\triangle\theta_s - \triangle\theta_r) = \frac{1}{x_j} (X_{si} - X_{ri}) \triangle P_i$$

where $x_j$ is the reactance of line j ,and $x_{si}$ and $x_{ri}$ are the elements in [X]. This implies,

$$\alpha_{ji} = \frac{\triangle F_j}{\triangle P_i} = \frac{1}{x_j} (X_{si} - X_{si}) \tag{7.12}$$

The post-contingency transmission line power transfer limits in Eq.(7.5) becomes:

$$[F^k] = [F^0]+[\alpha][\triangle P^k] \quad [F^{emergency\_max}] \tag{7.13}$$

where $[\triangle P^k]$ is the generation shift vector for generator k outage contingency.

It is apparent from Eqs.(7.2) to (7.5) and (7.13) that for a system with $N_g$ generators, there will be $N_g$ similar sets of constraints. For a typical system with $N_g$=100 and the number of transmission lines $N_L$=200, there would be approximately 30,000 additional constraints. This large number of constraints would exceed the capacity of most mathematical optimization techniques known today. When multiple generator outages are considered, the possible number of contingency combinations will lead to an even larger number of post-contingency constraints. Although the model for the generation shift shared between the remaining units can be simplified by assuming a fixed coefficient $[\phi_g]$ applicable for all generator failures, this would not reduce the number of power flow constraints in Eqs.(7.5) and (7.13). Fortunately, unlike transmission lines which are subjected to random arduous climatic, environmental and system interference, many generator failures have advance warning of several minutes or much longer. Furthermore, multiple generator failure rarely happened within a very short time span. Single and multiple generator outages are therefore considered only if regarded as plausible.

### 7.1.2 Line Outage Contingencies

The inclusion of transmission line outage contingencies in an economic dispatch is more complicated than the case for generator contingencies because of the change in the network topology. Even assuming the generators in the system remain intact, any line outage will alter the power flow pattern in the network. The change in power flow can be approached again using a sensitivity technique:

$$[F^k] = [S^k][P_g{}^0] \quad [F^{emergency\_max}] \tag{7.14}$$

where

$[F^k]$ = Power flow in the remaining lines of the system after line k outage;

$[S^k]$ = Sensitivity matrix which relates the line power flow to the optimal generator output for line k outage.

For the same typical system having $N_b$=200 or more transmission branches, if all possible single line failures are considered, there would be 40,000(=$N_L^2$) transmission power flows to be monitored. Consideration of multiple line contingencies again will push the possible number of line constraints into astronomical figures.

In security constrained dispatch implementation, one of the crucial issues to deal with is therefore the handling of this large number of constraints. Intuitively, a decomposition algorithm can be derived so that each contingency of interest may be treated separately and hence reduce the problem size. Using a decomposition technique such as master-slave formulation[100], a final solution to the complete problem may be accomplished iteratively. Decomposition, however, is not always practical because adjustments made in the control variables to achieve optimality due to one contingency may have effects on the other contingencies. Convergence can be slow and cannot be guaranteed.

The other obvious solution is to cut down the number of contingencies included in the optimization. A survey of the literature indicates that solution algorithms which monitor all pre- and post-contingency constraints and solve the generation dispatch as one large problem are preferred[4,104,110,124,146,150,170,202]. To tackle the dimensionality problem, a two-stage approach is generally adopted. The first stage is essentially known as contingency selection in which all credible contingencies are evaluated and ranked in their descending order of constraint violation severity. A subset of the most severe contingencies and constraints are then taken as input data in the second solution stage, the security constrained dispatch. This 2-stage approach is shown schematically in Fig.7.2. There are, however, several shortcomings using this solution scheme, including:

1. Dividing the problem into two sub-problems instead of solving the complete problem in one step could resulting in longer CPU time and may require twice the effort to code.
2. The contingency evaluation is based on a base case which is generally taken as the present generation and load conditions. Since generation dispatch is required for future generation and future load conditions, the contingency ranking obtained may not reflect the same constraint violation severity in the optimal generation solution.
3. In the two stage solution scheme, a subset of possible contingencies is pre-determined in advance of the dispatch optimization process. This subset may exclude some post-contingency violations from consideration. The solution obtained therefore may not be the true optimum nor satisfy the security requirements.

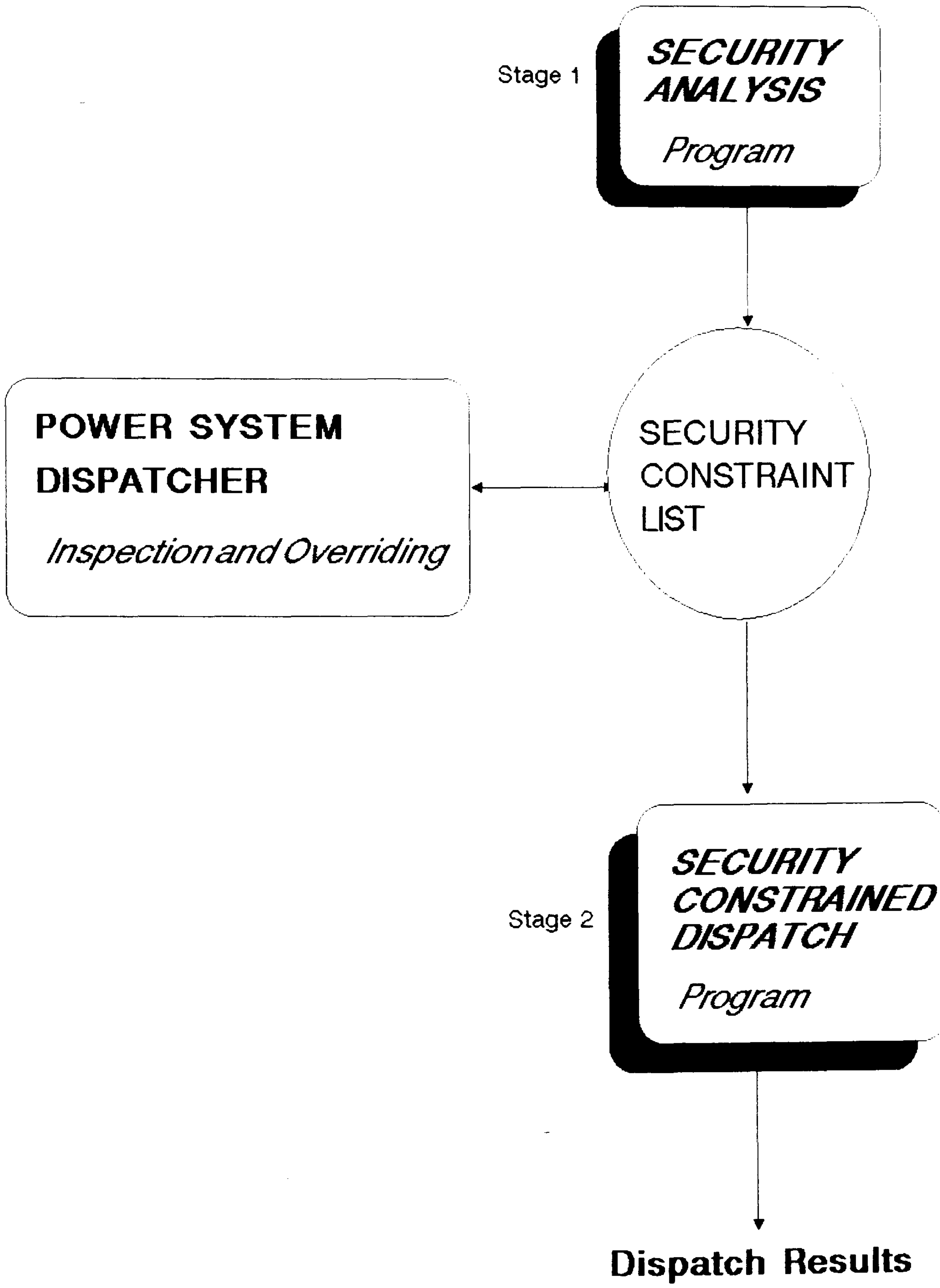



Fig.7.2 Two-stage Solution Method for Security Constrained Economic Dispatch

Despite these deficiencies, this class of approach is generally adopted in many existing and new control centres[56,64]. One of the significant advantage in the new algorithm proposed in this chapter is to eliminate the need for a contingency screening stage and merge the two solution steps into a single unified process. This is made possible by the use of an original post-contingency line flow evaluation technique, Current Injection Method (CIM), which is described later in the chapter.

## 7.2 Solution Design

In this thesis, single line outages are treated. This is generally known as *the (N-1)* security constrained dispatch problem. From the description in the previous section, it is clear that algorithm implementation can be achieved, in principle, by adding post-disturbance system constraints as further constraints to a pure economic dispatch problem. For generator contingency consideration, the formulation is more straight forward since the post-contingency line flow has similar functional relationship as the intact case except that the generator outputs will be shifted to some new values. For line contingencies, not only are there even more operational constraints needing to be satisfied but because of the change in network configuration, the functional relationships between power flow and generation patterns also need to be determined for each contingency. For any viable solution scheme, the mathematical expression between the post-contingency power flow and generator outputs must be described in terms of the intact system variables to allow convenient checking of line overload. The formulae derived must also be simple and preferably linear so that the constraints can be easily incorporated in a linear programming algorithm which is commonly regarded as the most computationally efficient technique available for the economic dispatch problem and hence giving the algorithm so developed the best chance to be applicable to real time operation. There are basically two distinct approaches reported in the literature for calculating the effect of line outage on the remaining lines in the system. These are outlined as followed:

### 7.2.1 Matrix Inversion Lemma

The first popular approach of line outage simulation is based on, or derived from, the matrix inversion lemma[87]. In this approach, the post-contingency topological change and its associated matrices and inverses are derived mathematically as a function of the matrices and inverses of the intact system. Recall Eq.(6.5) in which active power flow in an intact system is approximated by a D.C. load flow formula:

$$[F^0] = [H][B]^{-1}[P] = [H][X][P] \tag{7.15}$$

where

[H] = zero matrix excepts two elements in each row:

$H_{km}=x_k$; $H_{kn}=-x_k$

where $x_k$ is the reactance of line k and, m and n are its two ends;

[P] = net nodal active power injection.

For a change in the susceptance of branch k, $\Delta b_k$, the change in the inverse of the system susceptance matrix, [X], is:

$$[\Delta X^k] = [B^k]^{-1} - [B]^{-1} = [[B]+\Delta b_k[M_k][M_k]^T]^{-1} - [X]$$

$[M_k]$ is a $N_n$x1 incidence vector of line $k$, having values 1 and -1 at the two elements corresponding to the sending and receiving buses of the line, and zeros for all other elements in the vector. By the matrix inversion lemma,

$$[\Delta X^k] = \phi_k \, [\alpha_k][\alpha_k]^T \tag{7.16}$$

where

$$\phi_k = -\Delta b_k / (1+\Delta b_k [M_k]^T[\alpha_k])$$

$$[\alpha_k] = [X][M_k]$$

which implies,

$$[X^k] = [B^k]^{-1} = [X] + \phi_k \, [\alpha_k][\alpha_k]^T \tag{7.17}$$

This can be substituted into Eq.(7.15) and set $\Delta b_k = -x_k$ for line $k$ outage to give Eq.(7.18):

$$[F^k] = [H][X^k][P] = [H] \, [[X]+\phi_k[\alpha_k][\alpha_k]]^T \, [P] \tag{7.18}$$

in which the $k^{th}$ row of $[F^k]$ and $[H]$ are zero reflecting line k being disconnected.

### 7.2.2 Line Flow Distribution Factors

The second method may be classed as sensitivity approach in which linear line flow distribution factors, $\mu_{zk}$, are utilized to relate the change in power flow of a monitored line $z$ to a unit of pre-fault power flow of a tripped line $k$. The development of the techniques have been derived by several authors[54,228]. Using power injection at the two ends of the faulty line to simulate line outage, the expression for the coefficients developed by Wood and Wollenburg[228] is amazingly simple:

$$\mu_{zk} = \frac{(X_{in} - X_{jn} - X_{im} + X_{jm})\, x_k/x_z}{x_k - (X_{nn} + X_{mm} - 2X_{nm})} \qquad (7.19)$$

where

$\mu_{zk}$ = $\triangle F_z{}^k/F_k{}^0$ = change in active power flow in line $z$ for a unit change of pre-fault power flow in the outage line $k$;

$x_k$, $x_z$ = line reactance of lines $k$ and $z$;

m,n = sending end and receiving end of faulty line $k$;

$X_{in}$ = element in the row i and column n of the inverse of pre-contingency system susceptance matrix.

Given the line flow distribution factor vector for line $k$ outage $[\mu^k]$, it can be combined with the D.C. power flow formula of Eq.(7.15) such that the post-contingency power flow may be expressed as pre-contingency network parameters.

$$[F^k] = [F^0] + [\mu^k]\, F_k{}^0 \qquad (7.20)$$

In practice, line flow distribution factors for all credible line contingencies are calculated once at the beginning of dispatch computer program execution and stored for later used in the program.

Both techniques are employed in various published constrained dispatch algorithms. In general, the second class of approach is computationally more efficient as it may involve less matrix manipulation. The two classes of approach also mixed to suit particular implementations. In the following, a new technique termed Current Injection Method which can also be grouped under the sensitivity matrix approach is described. The proposed technique makes use of the basic superposition theorem for post-contingency power flow calculations. As will be shown below, the proposed technique is easy to understand, convenient to use and extremely efficient in terms of computational time.

## 7.3 Current Injection Method (CIM)

### 7.3.1 A Simple Example

The proposed algorithm can best be introduced by way of a simple example. Consider a linear network which has one current source, one sink and two resistive branches connected in parallel as depicted in Fig.7.3a(1). By the current divider theorem, currents of 8A and 2A are flowing in branches 1 and 2 respectively. When branch 1 is taken out of the network then branch 2 will be carrying the full load of the system as shown in Fig.7.3a(2). By applying the superposition theorem, the solution in Fig.7.3a(2) can be obtained in two steps. First, line 1 and all active sources are disconnected from the network. Inject the pre-outage current of the outage line into the system, but with opposite direction, at the two ports of line 1 and calculate the current flow in all other parts of the system. The resultant flow in the system is then superposed on the original network to obtained the final solution. These are shown in Figs.7.3b(1), (2) and (3). In the process described, line 1 is taken out of the network in Fig.7.3b(2) and then the currents in the remaining lines are

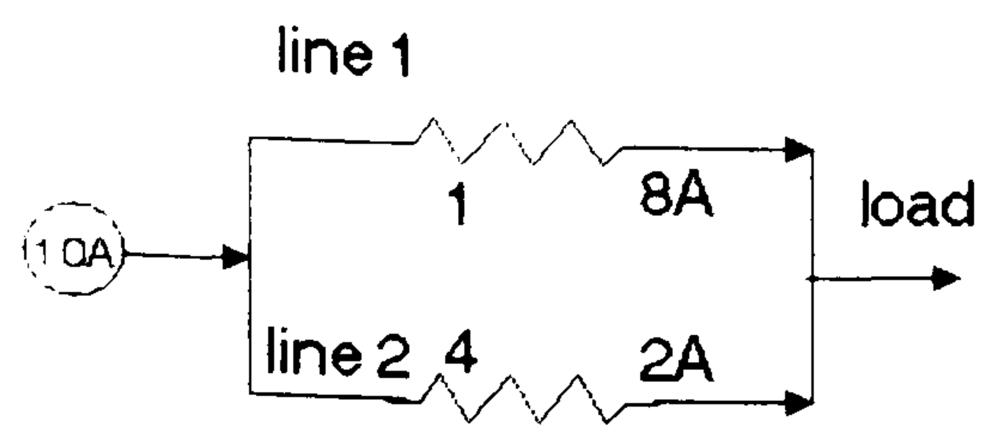



**Fig.7.3a(1) Intact system**

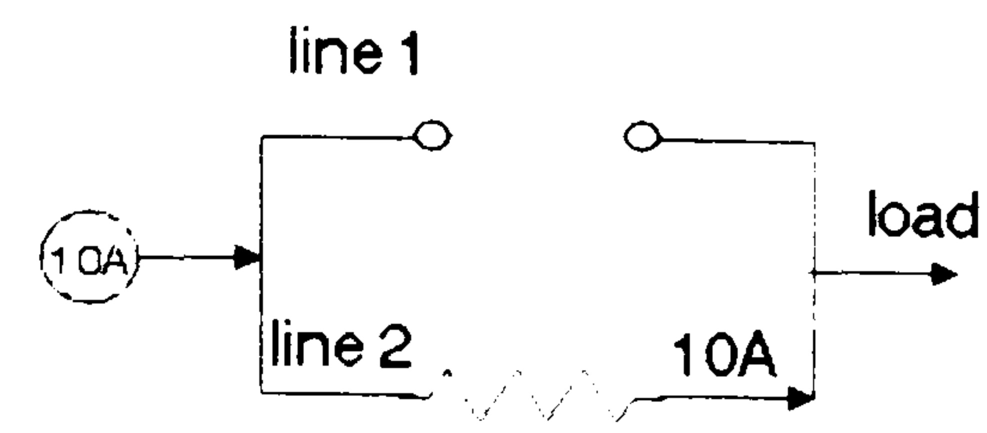



**Fig.7.3a(2) System condition after line 1 switched out**

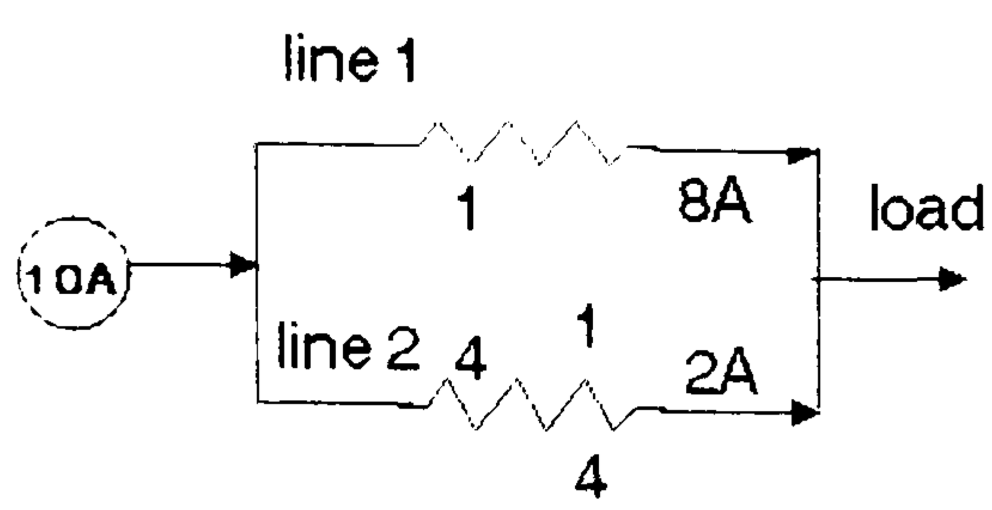



**Fig.7.3b(1) Intact system**

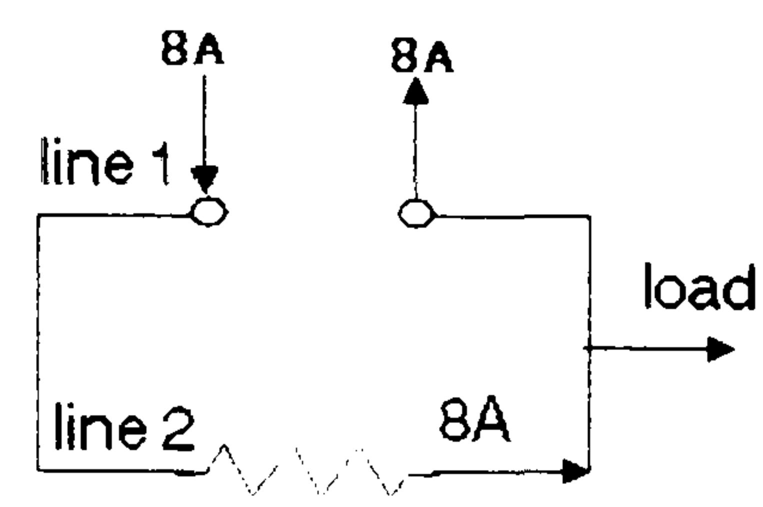



**Fig.7.3b(2)**
**Line 1 switched out with all active sources eliminated from the system**

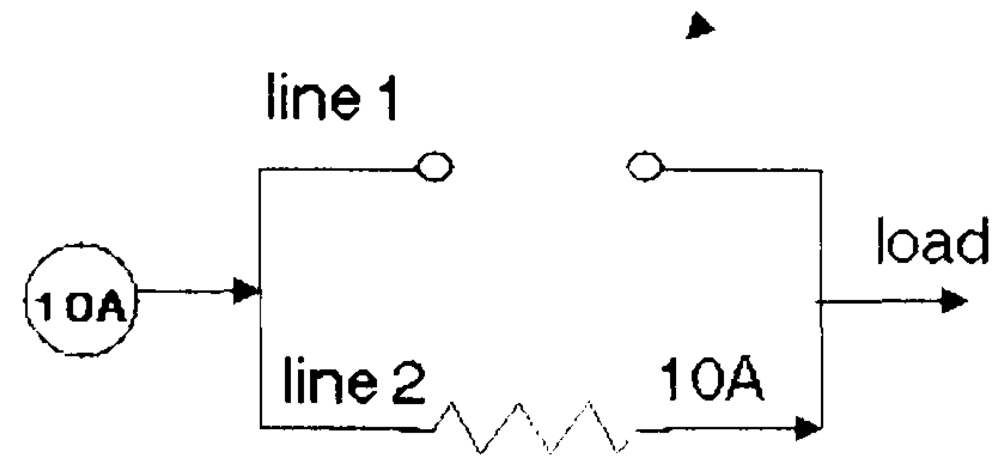



**Fig.7.3b(3)**
**Post-contingency system condition using Superposition theorem**

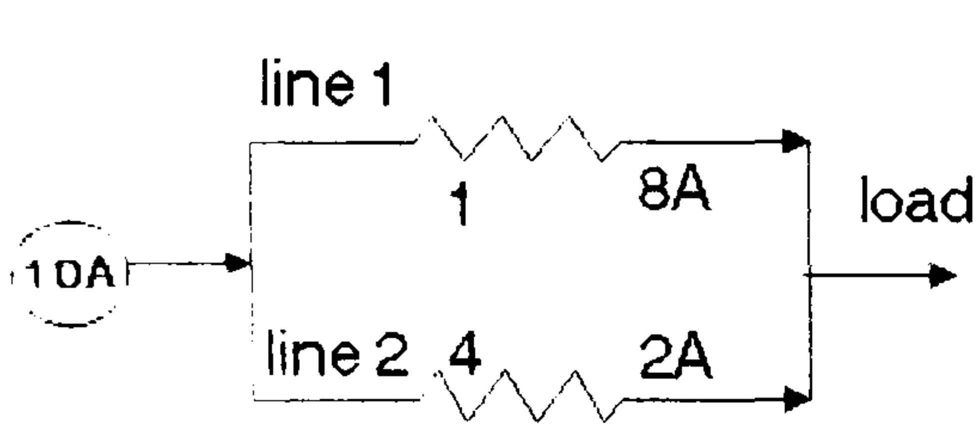



**Fig.7.3c(1) Intact system**

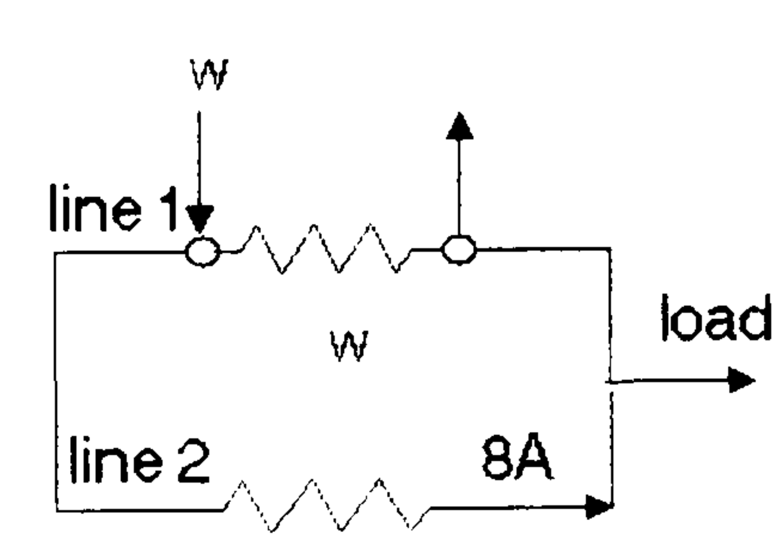



**Fig.7.3c(2)**
**Line 1 outage simulated by external injection**

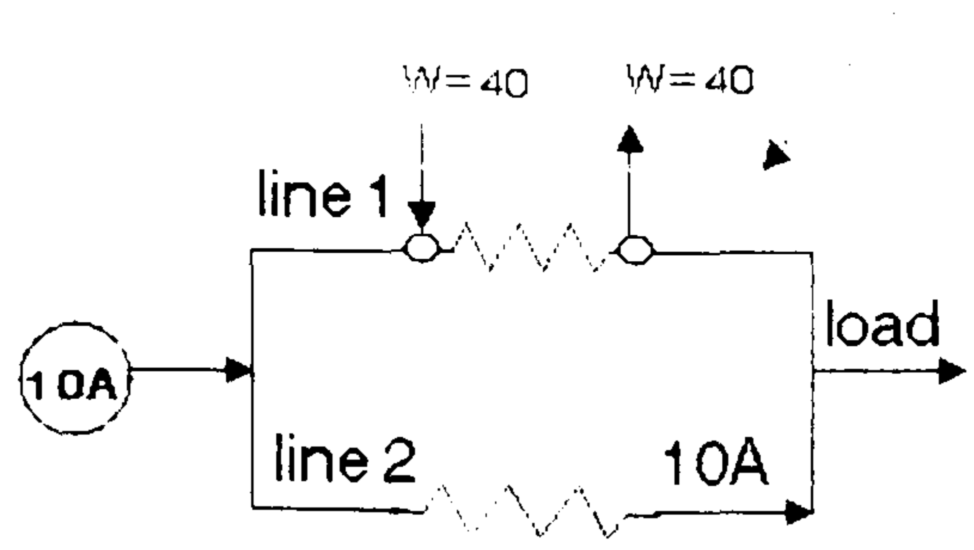



**Fig.7.3c(3)**
**Post-contingency system condition using CIM technique**

Fig.7.3 Post-contingency Power Flow Simulation calculated. This is an undesirable procedure because when line 1 is taken out, the topology of the system is changed and so are the admittance and impedance matrices of the system. The published techniques based on the inverse matrix lemma utilize the susceptance matrix and its inverse of the original network to obtain the required matrices for the line outage cases. Although such techniques avoid a direct matrix inversion for the modified network, substantial computation is still required. It would be ideal if it were not necessary to modify the network, its admittance or its inverse in any way and yet arrive at the same solution.

For the above example network, the key question to answer is, without changing the network, what are the required injections into the two nodes of branch 1 which would result in currents in the remaining branches of the network, as if branch 1 had been removed. The solution is achieved in two stages and is depicted in Figs.7.3c(1), (2) and (3).

**Stage 1:**

Because the electrical network is linear with respect to current, a sensitivity matrix for the example network can be formed which relates the current flow in all lines to the injections at different nodes of the system. By inspection,

$$F_1 = 0.4\ I_1 + (-0.4)\ I_2 \tag{7.21}$$

$$F_2 = 0.1\ I_1 + (-0.1)\ I_2 \tag{7.22}$$

where $F_1$ is current in branch 1, and $I_1$ is the current injection into node 1.

**Stage 2:**

Let the required current injections into the two ends of line 1, namely node 1 and node 2, be W and -W (W is positive) respectively in Fig.7.3c(2) to simulate the condition of Fig.7.3b(2) but with line 1 remaining in the system. Since the net current injected into the system external to branch 1 must be 8A and -8A as in Fig.7.3b(2), then at node 1,

$$W - F_1' = W - (0.4\ W + (-0.4)(-W)) = 8$$

where $F_1'$ is the current in line 1 due to W and -W at nodes 1 and 2 respectively. This implies:

$$W - 0.8\ W = 8$$

$$\Rightarrow \qquad W = 40$$

Checking the solution, substitute W=40A in Eq.(7.22),

$$F_2' = 0.1\ (40) - 0.1\ (-40) = 8A$$

This is identical to Fig.7.3b(2) although for this case, $F_1'$ is now 32A. This, however, is not of any consequence because branch 1 in reality is switched out.

Likewise, the solution of Fig.7.3c(2) is superposed on the original network state, Fig.7.3c(1), in which $F_2$=2A. The resultant current in branch 2 is therefore equal to 10A shown in Fig.7.3c(3) which is identical to Fig.7.3a(2) and Fig.7.3b(3). It should be pointed out that using the simulation technique described, the current in the outage line is <u>not</u> equal to <u>zero</u> mathematically as one might presume intuitively. The key to line outage simulation is not the apparent current in the faulty line but the effect of its outage on the remaining lines of the system. Another vital characteristic of the simulation technique is that the artificial external injections is equal but of opposite direction to the post-contingency "current" in the outage line; in this example is 40A. The importance of the example is clear. It demonstrates that the current flow in a network after the occurrence of a line outage can be calculated without resort to any topological change of the original network.

### 7.3.2 CIM for a Multi-node Power System

The above concept can be generalized to consider a multi-node power system with multiple generation sources and multiple load demand points. Consider the approximate power flow equation developed in Chapter 6 Eq.(6.5) which gives the approximate linear relationship between line flow and net nodal power injection:

$$[F] = [S]\,[P] \tag{7.23}$$

$$[\Delta F] = [S][\Delta P] \tag{7.24}$$

The current flow constant [K] in Eq.(6.5) due to load demand, estimated losses and inaccuracy correction factor is imbedded in the above equations; but this does not affect the development of the algorithm that follows.

Let a line k which has its sending and receiving ends at nodes $m$ and $n$ respectively carries a pre-outage current of $F_k$. Using the same principle as in the above simple example, let the power injections to nodes m and $n$ be $W$ and $-W$ to simulate line k outage, then at node $m$,

$$W - \{ S(k,m)W + S(k,n)(-W) \} = F_k$$

$$¢\ W \{ 1 - (S(k,m) - S(k,n)) \} = F_k$$

$$¢\ W = F_k/\{ 1 - (S(k,m) - S(k,n)) \} \qquad (7.25)$$

Substituting $\triangle P_m = W$, $\triangle P_n = - W$ and other P's equal to zero in Eq.(7.24), the effect of line k outage on the remaining lines of the system can be determined, i.e.

$$[\triangle F] = [S]\ [0\ 0\ 0\ 0\ 0 \ldots \underset{m}{W} \ldots 0 \ldots \underset{n}{-W} \ldots 0 \ldots .]^T \qquad (7.26)$$

$[\triangle F]$ is then superposed to the pre-outage line flow $[F]$ to obtain the power flow in the remaining lines of the system, i.e.

$$[F^k] = [F]+[\triangle F] \qquad \text{for the remaining lines}$$

$$F_k{}^k = 0.0 \qquad \text{for the outage line k} \qquad (7.27)$$

Any line that becomes overloaded as a result of the contingency for a given generation pattern can therefore be determined. In economic dispatch problem, the generation pattern [P] is unknown. However, many generation allocation algorithms solved the dispatch problem iteratively. For a given trial optimal generation pattern, the power flow in the remaining lines of a system for any given line outage contingency can be checked using Eqs.(7.23) to (7.27). Iterative approaches also have the indispensable advantage that all functional constraints can be relaxed initially. When a trial optimal generation has obtained, those constraints which are close to or exceed their normal/ emergency loading limits can then be added to the constraint set in the problem formulation ready for the next optimization iteration. This special feature is generally known as "relaxation and iterative constraint selection". It is the central tactic to cut down the number of line constraints needed to be considered in the optimization processes. Otherwise, the enormous number of possible constraints will overwhelm any currently known mathematical optimization technique. In the following paragraphs, a linear relationship between power flow in line j for a given line k outage and a given generation pattern of the system is derived.

Substitute pre-outage current of line k from Eq.(7.23) in Eq.(7.25),

$$F_k = S(k,1)P_1+S(k,2)P_2+S(k,3)P_3+....+S(k,N_n)P_{Nn}$$

$$W = \{S(k,1)P_1+S(k,2)P_2+S(k,3)P_3+...+S(k,N_n)P_{Nn})\} /\{1-(S(k,m)-S(k,n))\} \quad (7.28)$$

and let

$S(k,1)/\{1-(S(k,m)-S(k,n))\}$ be $\beta_1$

$S(k,2)/\{1-(S(k,m)-S(k,n))\}$ be $\beta_2$

.

.

$S(k,N_n)/\{1-(S(k,m)-S(k,n))\}$ be $\beta_{Nn}$

Eq.(7.28) becomes,

$$W = \beta_1P_1 + \beta_2P_2 + \beta_3P_3 + ... + \beta_{Nn}P_{Nn} \quad (7.29)$$

This can then be substituted in Eq.(7.26) to obtain the change in power flow of a monitored line j,

$$\triangle F_j^{k} = (S(j,m)-S(j,n))\ (\beta_1 P_1+\beta_2 P_2+\ldots+\beta_{Nn} P_{Nn}) \tag{7.30}$$

Substituting this in Eq.(7.27), we have for contingency k,

$$\begin{aligned} F_j^{k} = {} & \{S(j,1)+(S(j,m)-S(j,n))\beta_1\}P_1 + \\ & \{S(j,2)+(S(j,m)-S(j,n))\beta_2\}P_2 +\ldots \\ & \{S(j,Nn)+(S(j,m)-S(j,n))\beta_{Nn}\}P_{Nn} \end{aligned} \tag{7.31}$$

Note that the $\beta$ factors are constants for any contingency and do not depend on the generation schedule, but depend on the sensitivity matrix coefficient of the original network which is also a constant for a given intact network topology. With Eq.(7.31), the post contingency power flow for any proposed generation schedule can therefore be determined. Furthermore, by simplifying Eq.(7.31), a linear relationship for the post-contingency power flow to any trial generation pattern may be established.

$$F_j^{k} = S^k(j,1)P_1+S^k(j,2)P_2+\ldots+S^k(j,N_n)P_{Nn} \tag{7.32}$$

The simple expression of Eq.(7.32) is similar in structure to the approximate load flow equation of Eq.(7.23) for the intact system. Any economic dispatch algorithms which can monitor the power flow for the intact system using a similar sensitivity matrix approach may therefore include the post-contingency line flow constraints in the optimization process.

### 7.3.3 System Split

The proposed CIM technique has a natural way of identifying any line outage which causes a system split. In Eq.(7.25), for any line outage k causing a system split, the factor §S(k,m)-S(k,n)† will be equal to unity making $F_k$/{1-(S(k,m)-S(k,n))} infinite. There is a logical physical interpretation for this condition. Any external injections of opposite signs at the two nodes of a line whose failure would cause a system split will cause power flow in that line only. Therefore an infinite power injection would be needed to supply any power external to this line.

### 7.3.4 Advantages of CIM

The advantages of the proposed approach can be enumerated as follows:

1. No modification of the topology, admittance matrix nor the inverse of the original network for any line failure contingency is needed.
2. The sensitivity matrix of the intact system, [S], is stored in sparse factorized form. It is calculated only once at the beginning of the computer program execution.
3. The linear relationship between post-contingency power flow and generation injections at the buses of the system is a general formula. It can be applied to any optimization algorithm which capable of handling linear constraints.
4. Fast. Because there is no matrix inversion involved and all major calculations utilize sparse matrix methods, the additional computation time compare to ordinary economic dispatch is minimal.

## 7.4 Application of CIM to Security Constrained Dispatch

The practicality of the proposed CIM technique for the solution of security constrained economic dispatch problem can be illustrated by incorporating the concept to a pure economic dispatch solution. An economic dispatch LP program utilizing Sparse Dual Revised Simplex (SDRS) algorithm is available from the School of Engineering and Applied Science of University of Durham, which was originally developed by Dr. M. R. Irving and Professor M.J.H. Sterling. An LP algorithm is chosen as the most suitable vehicle to validate the capability of the new approach because LP based methods are generally regarded as the most computationally efficient mathematical optimization techniques for economic dispatch solution. This general perception on LP's computability was also confirmed by the test results summarized in Table 6.2 of Chapter 6. In the present project, the existing implementation is extensively modified so that full network model is incorporated in the problem formulation. This would have released the generator group constraints requirements in the existing implementation which was designed to approximate the line flow limitations between areas of a network. Generator group output limits, however, are retained in the new version to model the constraints pertaining to the operational constraints of a group of generators in a station primarily to account for boiler or station transformer limitations etc. The line group constraint capability is also incorporated. With this line group constraint feature, the tie line power flow contracts between utilities or import/export agreements between areas of the same network can be modelled accurately. For detail derivation of the SDRS computational algorithm, interested readers should consult reference[101]. The LP formulation of the security constrained economic dispatch problem is described in the following paragraphs.

### 7.4.1 Problem Formulation

The objective of security constrained economic dispatch is to minimize the cost of electrical power/energy production satisfying a forecast load. This is to minimize the summation of fuel cost of all generators in the system for the intact state since the contingency state, if it occurs at all, only last for a short transitional period. Generator/line failure consideration therefore should not change the fundamental objective of the generation dispatch but may affect the optimal generation pattern. In a LP implementation, the fuel cost of a generating unit is approximated by a linear or piece-wise linear function.

$$\text{Minimise} \quad C = [C_1] + [C_2]^T [P_g] \tag{7.33}$$

where

$[C_1]$ = generator fixed costs

$[C_2]$ = fuel cost coefficients or piece-wise linear fuel cost coefficients of the generators

The minimisation is subject to operational constraints which reflect the capacity limits of the plants and system security requirements for both the intact and contingency system states.

a) Power balance: The sum of generator outputs must equal to total power demand of the system plus transmission losses.

$$\sum_{g}^{N_g} P_g = \sum_{i}^{N_n} D_i + \text{Loss} \tag{7.34}$$

b) Generator limits: Each generator must operates within its design limits.

$$[P_g^{min}] \leq [P_g] \leq [P_g^{max}] \tag{7.35}$$

c) Ramping limits: In order to satisfy load at an immediate future of lead time t ahead, then the lower and upper limits of the generator should be modified to reflect the rate of change limitations.

$$[P_g^0] - [P_g^d]t \quad [P_g] \leq [P_g^0] + [P_g^i]t \tag{7.36}$$

where

$[P_g^d]$ = vector of rate limits for decreasing output

$[P_g^i]$ = vector of rate limits for increasing output

t = elapsed time between present generation $[P_g^0]$ and implementation of new dispatch $[P_g]$

Obviously, the greater of the two lower limits and the lesser of the two upper limits in Eq.(7.35) and (7.36) are the effective lower and the upper limits of the generators.

d) Group generator limits: Station limits are modelled by group generator output limits.

$$\sum_{g \epsilon station} P_g \leq P^{station\_max} \tag{7.37}$$

e) Area import/export limits: Exchange contracts between utilities or areas of the same system is modelled by group line constraints. The group line limits are particularly useful when there are a number of tie lines between utilities or areas and only the total power exchange is specified. These constraints help to allocate the generator output optimality satisfying the total power exchange contract. If the contractual power transfer is fixed on any line, then such tie line may be modelled as a generator with fixed power injection at the two ends of the line.

$$[F^{group\_min}] \leq \sum_{j \epsilon group} F_j \leq [F^{group\_max}] \tag{7.35}$$

f) Line power flow limits for intact system: The existing implementation uses area import/export constraints to approximate the network constraints in the system. A more satisfactory solution is to model the power flow for individual lines. A detail derivation of the approximate linear relationship between line flow and generation injection was given in Chapter 6 and re-presented in Eq.(7.23) and (7.24).

g) Line power flow limits for *(N-1)* contingency: For line outage contingencies, Eq.(7.25) to (7.32) apply which give the functional relationship between post-contingency power flow and a generation pattern. All possible single line outages are considered in the present implementation to ensure sustainable continuous electricity supply if any forced outage of any line occurred. The post-contingency line flow constraints are:

$$[F^k] = [S^k]\ [G]\ [P_g] \le [F^{emergency\_max}] \qquad (7.36)$$

where

$[F^k]$ = line power flow vector for line k outage contingency;

$[G]$ = $N_n x N_g$ incidence matrix which relates generator to its bus connection. All elements in the matrix are zero except at row *i* and column *g* which signifies generator *g* connected directly to bus *i*.

For generator failure contingency, a similar set of expressions can also be formulated as indicated by Eq.(7.2) to (7.13). Generator failures are not within the scope of the present project; but further work on generation rescheduling is presented in the next chapter.

### 7.4.2 Computational Strategy

It is apparent from the above formulation that the number of operational constraints included in the optimization process is enormous. In most published solution methods, a contingency pre-selection step is utilized to minimize the number of contingencies needed to be considered in the security constrained dispatch phase as a means to control the problem size. While automatic contingency selection coupled with security assessment fulfils a useful role of giving early warning to the system operator any potential system insecurity, pre-selection of contingencies can exclude constraints that may be violated from consideration in the dispatch phase as explained earlier. In the present proposed approach, contingency screening is <u>not</u> assumed. All single line failures are simulated and checked for constraint violation within the dispatch solution process. The computation efficiency is achieved by the adoption of the following strategies:

1. Iterative constraint relaxation: To overcome the dimensionality problem caused by the enormous possible number of line overloading constraints, an iterative constraint relaxation procedure is adopted. This is realised by firstly omitting all line power flow constraints in the initial dispatch. When a trial solution is available, all power flows are checked against their respective limits using Eq.(7.23) and (7.31). Those lines whose power flow exceeds or is near to their limits will then be included in the constraint set in the next dispatch iteration. In this way, the active line constraints in any LP iteration is controlled to a limited number. The additional significant advantage is that the actual number of power flow constraints in the optimization process now depends less on the number of physical transmission lines in the system but more on the inherent transmission capability of the system, and the geographic locations of load demand and generation sources. For a well designed strongly connected system, the number of active line constraints is very small, perhaps in the region of 0.5%

or less of all possible pre- and post-contingency line constraints.

2. Sparse matrix calculation: As far as possible, sparse matrix calculations are utilized. The sensitivity matrix [S] of the intact system is stored in factorized form using Zollenkorf[195] technique. This factorized form is used for all power flow limit checking. The sensitivity coefficients for power flow calculation are generated and stored only when such functional constraints need to be included in the constraints set in the LP dispatch iteration.

The flow chart for the LP security constrained economic dispatch is shown in Fig.7.4.

### 7.5 Computational Example

The (N-1) security constrained economic dispatch models and CIM technique described above have been implemented in FORTRAN 77 on a DEC VAX 8600 computer using single precision 32 bit floating point storage and calculation. The additional computer execution time to consider the post contingency security feature is investigated. The data set provided by the Central Electricity Research Laboratory of the former CEGB is chosen as the test system, consistent with the tests carried out in the previous chapters. The dispatch results for four seasonal loading conditions of the test network are used for comparison purposes. Table 7.1 shows the LP dispatch performance with and without considering (N-1) security constraints.

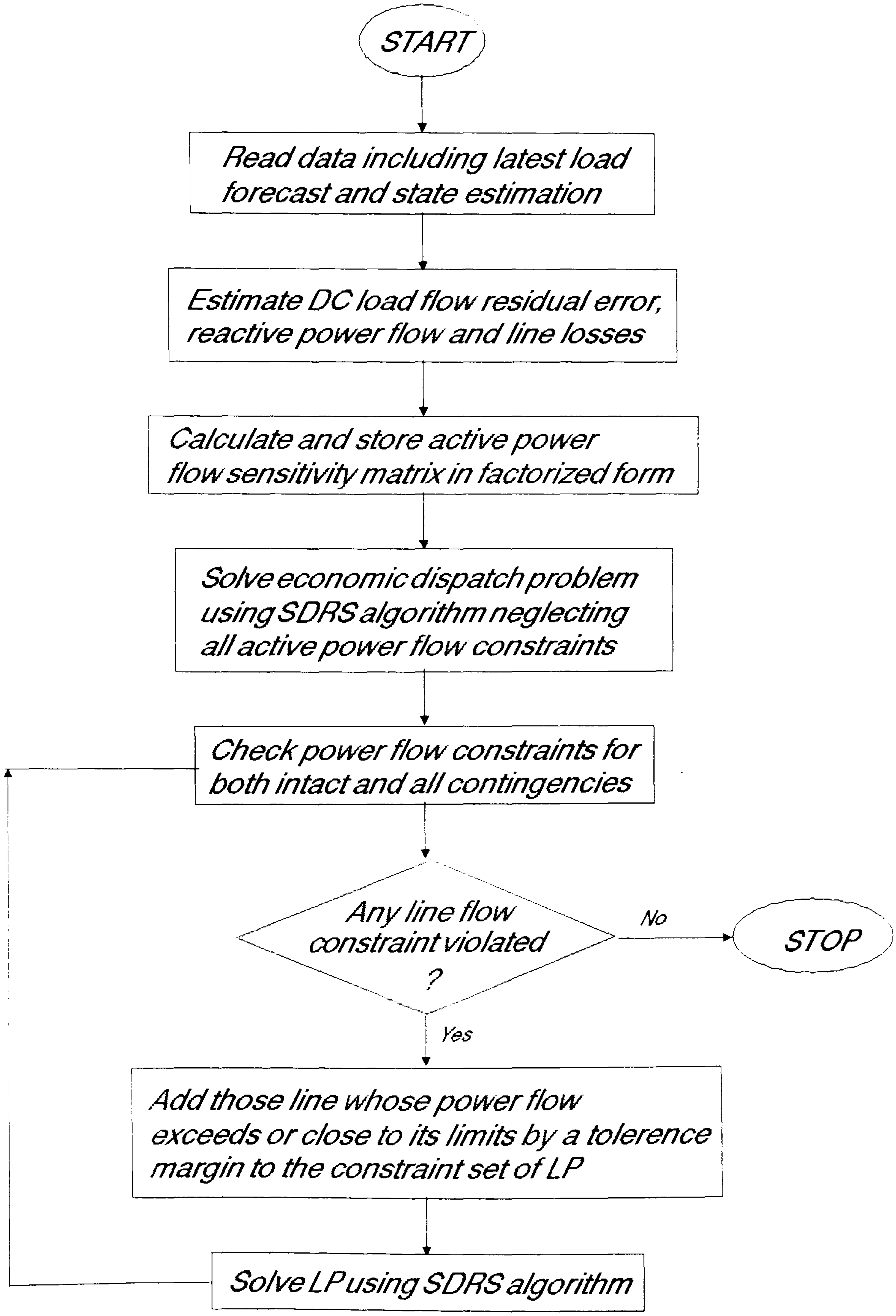



Fig.7.4 Flow Chart of (N-1) Security Constrained Dispatch using a Sparse Dual Revised Simplex Algorithm

### 7.5.1 Computational Efficiency

From the table, it is clear that for the CEGB system, the difference in CPU time requirements between a 'pure' economic dispatch and a security constrained dispatch is small. The additional computation time is system dependent and is a consequence of the system transmission capability and relative locations of load and generation sources. The test results indicate strongly that the proposed algorithm is applicable for real time operation.

**Table 7.1** Comparison of LP results with and without (N-1) security constraints

(Transmission Losses Neglected)

| Load Condition | LP Cost w/ security constraints | LP Cost w/o security constraints | Cost Penalty |
|---|---|---|---|
| Winter Plateau | £916255 | £914292 | +0.2 % |
| Winter Trough | £495842 | £479269 | +3.5 % |
| Summer Plateau | £491187 | £471334 | +4.2 % |
| Summer Trough | £153764 | £124240 | +23.8 % |
| Average CPU time | 8 s. | 5 s. | |

### 7.5.2 System Operational Cost Penalty

From an operational economics point of view, the generation schedule to include contingency constraints is clearly much more expensive. For the summer trough load condition, the increase in generation cost can be as much as almost a quarter of the total fuel bill. Closer examination of the power flow in the system shows that there are several lines in the system running from the northeast to the southeast of the country are of critical importance; they are loaded to their emergency maximum when lines in the same general locality are switched out. Because of the limited capacity of these north-to-south transmission lines, out of merit generating units in the south east have to dispatch at a higher output than would be necessary otherwise. The transmission capabilities of these critical lines therefore may be regarded as the bottle neck of the system restricting economic transmission of energy between areas of the system. Security constrained dispatch therefore can also be used as an analytic tool to identify the critical lines. It can also be used for power station site selection and transmission reinforcement studies in order to relieve the transmission bottle neck, improve the system security and to reduce generation cost.

## 7.6 Summary

This chapter has presented a (N-1) security constrained dispatch methodology. The implementation is based on a new line outage simulation technique called Current Injection Method. In the new dispatch algorithm, a full network model is incorporated and all single line outage contingencies are considered. The contingency pre-screening required in most existing algorithms is eliminated. Test results indicate that contingency constraints can be efficiently included in the economic dispatch, if so desired, with little computational penalty; but the generation costs might be significantly increased for some load conditions. With full network model capability, the program may be used to identify those critical lines in a network, to facilitate any transmission expansion study and thereby to improve the security and economic operation of the system.

# CHAPTER 8

## SECURITY CONSTRAINED DISPATCH WITH POST-CONTINGENCY CORRECTIVE RESCHEDULING

Security is one of the essential considerations in the operational control of an electric power system. Not only is it a statutory requirement but it also makes economic sense since without a secure system operation any economic gain obtained by breaching the physical and operational limitations of the plant will soon be negated by expensive plant failures, loss of supply and consequently loss of revenue for the utility and loss of production for industrial consumers. The economic benefit of optimal sharing of the system demand among the synchronised generating units has long been recognised. Since the introduction of the classical equal incremental cost concept by Ward[22], Kirchmayer[107] and others[72,150,211] in the 50's, the economic dispatch solution has gone through many significant stages of improvement. From the security point of view, the equal incremental cost approach normally includes only the generator output limits in the problem formulation and neglects the transmission system limitations[133,189]. By applying the mathematical optimization techniques such as linear programming[35,101], quadratic programming[103], dynamic programming[37] and others[4], the transmission network may be modelled and incorporated in the solution schemes. With this enhancement, the optimal cost solution ensures that the transmission line thermal capacities and tie line power transfer agreements between the utilities are not violated. However, the continuously changing conditions in which a power system operates mean that sudden failure of a vital plant is probable. This consideration has led to the implementation of many security constrained dispatch methods[38,124,133], which minimize the operational cost with consideration to the limitations of both the normal and post-disturbance system states, from the late 60's and this work is still growing in volume. Such enhancement is a big step forward in security improvement since the adoption of this dispatch solution would eliminate the possibility of any remaining components being overloaded should any transmission line or generator be forced off line. In the OCEPS research group of Durham University, intense study on the security constrained dispatch problem has been carried on for some time. The new (N-1) security constrained dispatch algorithm proposed in the last chapter is one of the end products for such effort. It has been apparent, however, for some years that the solution obtained by such a contingency bounded optimal solution is pessimistic[94,110,149,183]. The possible response capabilities of the system such as post-contingency generation rescheduling[71,183] or transmission line switching, initiated either automatically by the automatic generator controllers or manually by the operators, have been ignored. The exclusion of these recovery measures might imply that the restrictions imposed on the solution space by such contingencies may not be there in practice. With a strict application of the security constrained dispatch, the system is operated probably in an unnecessary expensive region to prevent system insecurity which might never happen or could be easily rectified. Furthermore, for some weakly connected networks, such an approach may even lead to an inoperable system[94], as a result forcing the operators to adopt a less stringent security requirement, primarily based on their knowledge of the system concerned, instead of a well defined and consistent security criterion.

This chapter investigates the techniques employed by existing solution methods to account for the post-contingency generation rescheduling capability in the context of economic dispatch and proposes that the well proved linear programming approach can also be applied to this computationally demanding but most interesting challenge. A detailed LP formulation of the problem is presented and an Iterative Constraint Selection[203] process utilized to reduce the dimensionality difficulty is described. The key issue of modelling the post-contingency system conditions is accomplished by extending the current injection concept introduced in Chapter 7. Tests results obtained using a 115-unit example system indicated that the proposed method is potentially applicable to real time operation. The question of improved fuel economy as a result of the additional simulation consideration and system security is also explored.

## 8.1 Existing Methods

In the last two decades, various possible post-disturbance system response capabilities such as load shedding[34,142,203], short-term transmission capacity[100], network switching[8,70,141,147] and generation rescheduling [134,192,207] have been studied in detail. Mathematical optimization techniques as well as problem specific algorithms are applied to the determination of the economically optimal and operationally practical actions should any contingency or system insecurity occur. Emergency control schemes based on scenario studies are being widely adopted in the electricity supply industry[71,93]. Survey of the literature, however, indicates that the amount of publications on the subject of incorporating these post-contingency system rectification capabilities in the economic dispatch solution stage is very limited. The inclusion of generation rescheduling was recently explored by Monticelli, Pereira, Pinto and Granville[149,166]. On the other hand, Schnyder and Glavitsch[183] extended their optimal switching concept to encompass the effect of post-contingency generation rescheduling in an economic dispatch algorithm. These two implementations are examined in greater detail in the next two sections.

### 8.1.1 Monticelli, Pereira, Pinto and Granville

The work of Monticelli and co-workers[149,166] is probably the first comprehensive attempt to formulate the constrained economic dispatch problem considering the post-contingency generation rescheduling capability in a rigourous mathematical framework. Monticelli at. el. interpret the problem as a two-stage decision process and employ a Benders decomposition technique to solve this compounded situation as follows:

**Stage 1: Operational cost optimization:**

Find an optimal generator operating point $P^0$ for the intact system satisfying the pre-contingency and pseudo post-contingency constraints; i.e.

$$\text{Minimize} \quad C = f(P^0) \tag{8.1}$$

Subject to:

a) Constraints pertaining to the intact system:

$$G^0(P^0) \le B^0 \tag{8.2}$$

b) Pseudo constraints pertaining to each contingency k:

$$W^k(P^0) \le 0 \tag{8.3}$$

Intact system constraints include the normal generator output limits, group generator output limits and transmission line power transfer limits. Eq.(8.3) is an approximate penalty function derived from stage 2 for each contingency *k* whose constraints are to be satisfied. This function is designed to steer the optimal generation schedule away from any post-contingency constraint violations progressively. In the initial trial dispatch, the penalty functions can be set to zero representing a "pure" economic dispatch problem.

**Stage 2: Penalty function estimation for each contingency k**

For a given optimal operating point $P^0$ determined in stage 1, the post-contingency rescheduled generator operating point $P^k$ which minimizes penalty cost, $W^k(P^k)$, is determined, i.e.

$$\text{Minimize} \quad W^k(P^k) = f(Z_1, Z_2) \tag{8.4}$$

Subject to :

a) Operational constraints:

$$G^k(P^k) - Z_1 \le B^k \tag{8.5}$$

b) Generation shift limits:

$$|P^k - P^0| - Z_2 \le \Delta^k \tag{8.6}$$

where

$Z_1, Z_2 \ge 0$ are slack variables corresponding to violation of operating constraints and coupling constraints of the system with outage;

$W^k(P^k)$ is the cost penalty for not satisfying the post-contingency constraints.

$\Delta^k$ in Eq.(8.6), limiting the possible range of $P^k$ for the given optimal solution $P^0$, depends on the time available to reschedule the generators and equipment. For any contingency, k, whose constraints are satisfied, $Z_1$ and $Z_2$ will be naturally equal to zero and $W^k(P^k)$ will also be zero. In general, not all of the contingencies considered will be satisfied in the early stage of the solution process and therefore non-zero $W^k(P^k)$ results.

Benders decomposition approach is an iterative solution scheme. After the solution of each post-disturbance optimization, i.e. Eq.(8.4) to Eq.(8.6), an updated approximation penalty cost function $W^k(P^0)$ for each violated contingency, known as Benders cut, will be generated from the $W^k(P^k)$ value. Monticelli suggests that a linear approximation of the form:

$$W^k(P^0) = W'(P^k) + \alpha^k (P^0 - P^{0\prime}) \tag{8.7}$$

where

$W'(P^k)$ = optimal solution of Eq.(8.4)

$P^{0\prime}$ = trial optimal solution of Eq.(8.1)

$\alpha^k$ = vector of Lagrangian multipliers associated with the constraint violation of Eq.(8.5) and Eq.(8.6).

can be used to provide a functional relationship between the change in post-contingency infeasibility and the changes in the operating point $P^0$ of the intact system in Eq.(8.1). Stage 1 and 2 described are then iterated successively until all post-contingency constraints are satisfied, i.e $W^k(P^k)$ in Eq.(8.4) equal zero for all contingencies. The decomposition algorithm is summarized in the flow chart of Fig.8.1.

### 8.1.2 Schnyder and Glavitsch

The formulation by Schnyder and Glavitsch[183] perhaps represents another school of approach. Schnyder and Glavitsch incorporate the post-contingency generation rescheduling by extending an optimal switching concept. In their approach an optimal *(N-1)* security constrained power dispatch solution is first obtained utilising a distribution factor matrix for post-contingency state estimation. In order to account for the possible economic improvement due to the rescheduling capability of the system, a second economic dispatch solution is then performed. In this step, only the intact system operating constraints need to be considered and the generator operating regions are redefined by relaxing the optimal operating point obtained in the security constrained dispatch, such that

$$P^* - \triangle P^{-T} \leq P^0 \leq P^* + \triangle P^{+T} \qquad (8.8)$$

where

$P^*$ = *(N-1)* security constrained optimal power flow solution

$P^0$ = revised generator output to be determined, i.e. optimal generation scheduling considering post contingency generation rescheduling capabilities

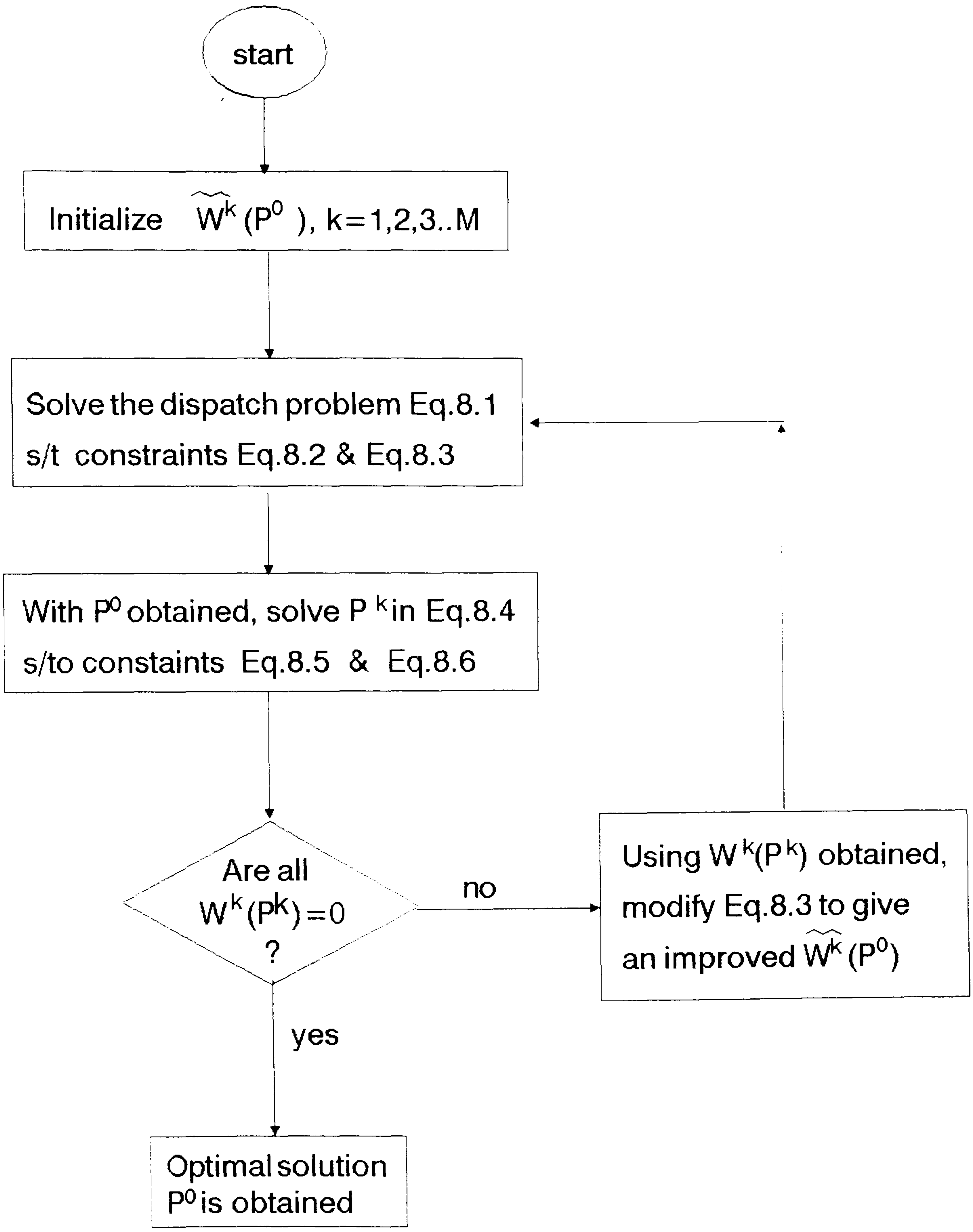



**Fig. 8.1**

Monticelli's Decomposition Scheme for Security Constrained Economic Dispatch with Post-contingency Rescheduling

$\triangle P^{-T}$ = allowable generation shift to reduce within time T after a contingency

$\triangle P^{+T}$ = allowable generation shift to increase within time T after a contingency

The keynote of the approach is to exploit the fact that when a contingency occurs, the system security is assured by shifting the generator outputs to $\bar{P}^*$ as originally determined by the security constrained dispatch. Beale's algorithm was employed to find the dispatch solutions of both problems which are imbedded in an optimal power flow formulation.

### 8.1.3 Comparison of the Existing Approaches

The methodologies devised by Monticelli el al and, Schnyder and Glavitsch, represent two conceptually different approaches to the problem. Besides using different mathematical optimization techniques, there are other significant differences. In this section a comparison of their approaches is made. This would also help to provide further insight into the complexity of post-contingency rescheduling problem.

Consider, for the sake of simplicity, a system with two generators and two contingencies. Let us assume that the feasible regions can be described graphically as shown in Fig. 8.2(a). If the usual convexity conditions are satisfied, the point $P^*$ representing the *(N-1)* security constrained dispatch solution lies at the boundary of the intersection of the feasible regions of all contingency cases. If the solution of a 'pure' economic dispatch is not 'naturally' *(N-1)* secure, then the point $P^P$ representing this solution would lie outside this intersection and on the boundary of its feasible region. In the figure, the objective function is represented by a series of straight lines.

As shown in Fig. 8.2(b), Schnyder el. al. relax the operating range of the generators in the post-contingency rescheduling stage by defining a 'box' centred on the *(N-1)* security optimal solution $P^*$. Any point inside this box will guarantee that the system has the option of shifting the generator outputs to point $P^*$ if either of the two contingencies occurs. By such an approach, the minimum operating cost considering post-contingency security has improved from the cost corresponding to $P^*$ to $P^S$.

In Monticelli's approach, shown in Fig. 8.2(c), a 'box' is also defined but is centred on the post-contingency rescheduling solution $P^M$. The box touches or overlaps the feasible regions of all contingencies indicating that if any contingency occurs, the generator can be shifted to the feasible region of this contingency. It is important to note that in this approach, there is no requirement that this box shall contain the *(N-1)* security optimal point $P^*$. In Schnyder's approach, if such box were drawn around the optimal operating point $P^S$, $P^*$ is always inside the box. The generator operating region in Monticelli's approach is therefore less constrained than Schnyder's formulation and hence is likely to arrived at a more economic operating point as indicated by the difference in operating costs $\Delta C$ shown in Fig. 8.2(d). It may therefore be concluded that Schnyder's approach does not always give the global optimal solution.

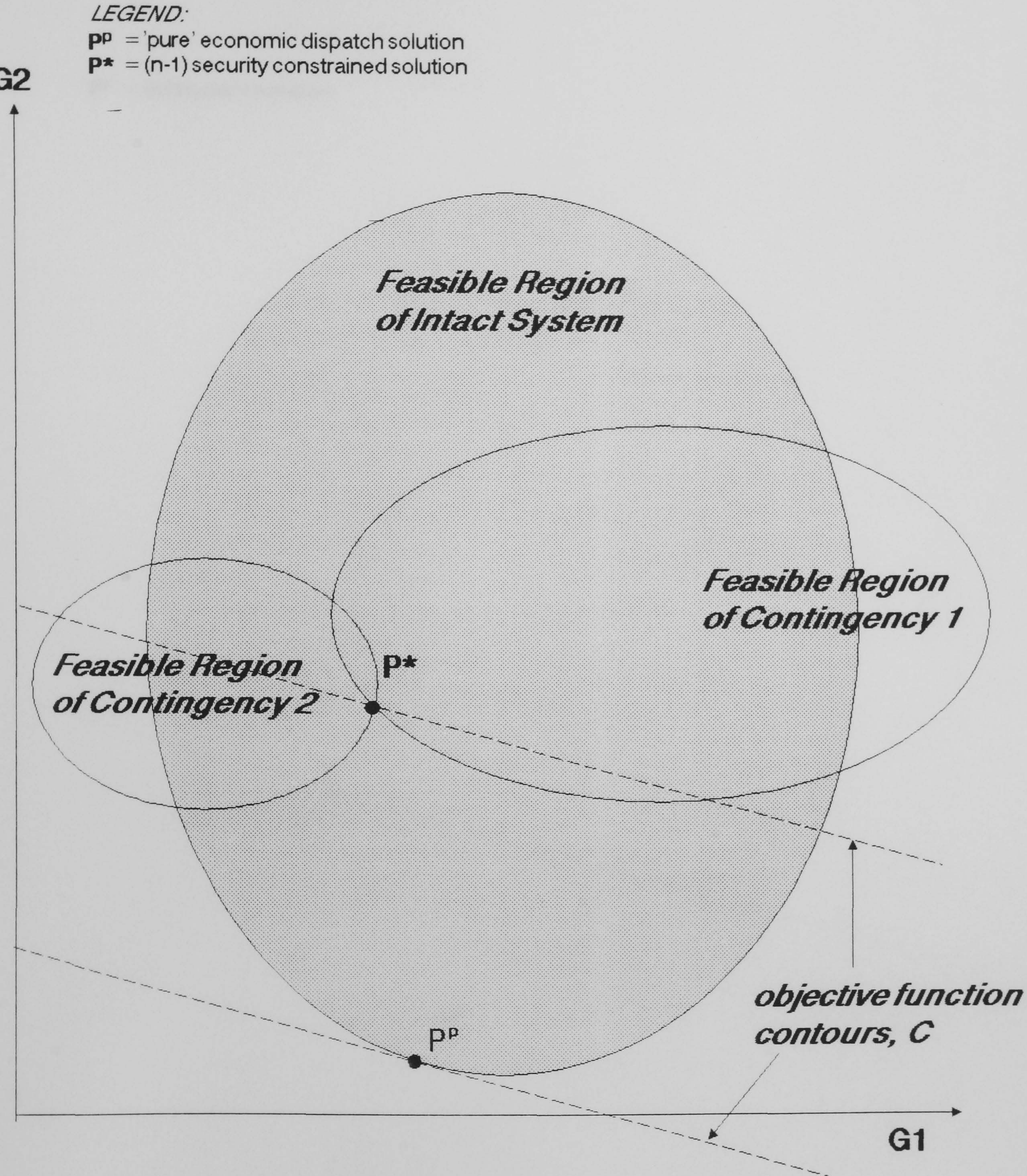



Fig.8.2(a) Graphical Representation of Feasible Regions for a 2-generator, 2-contingency System

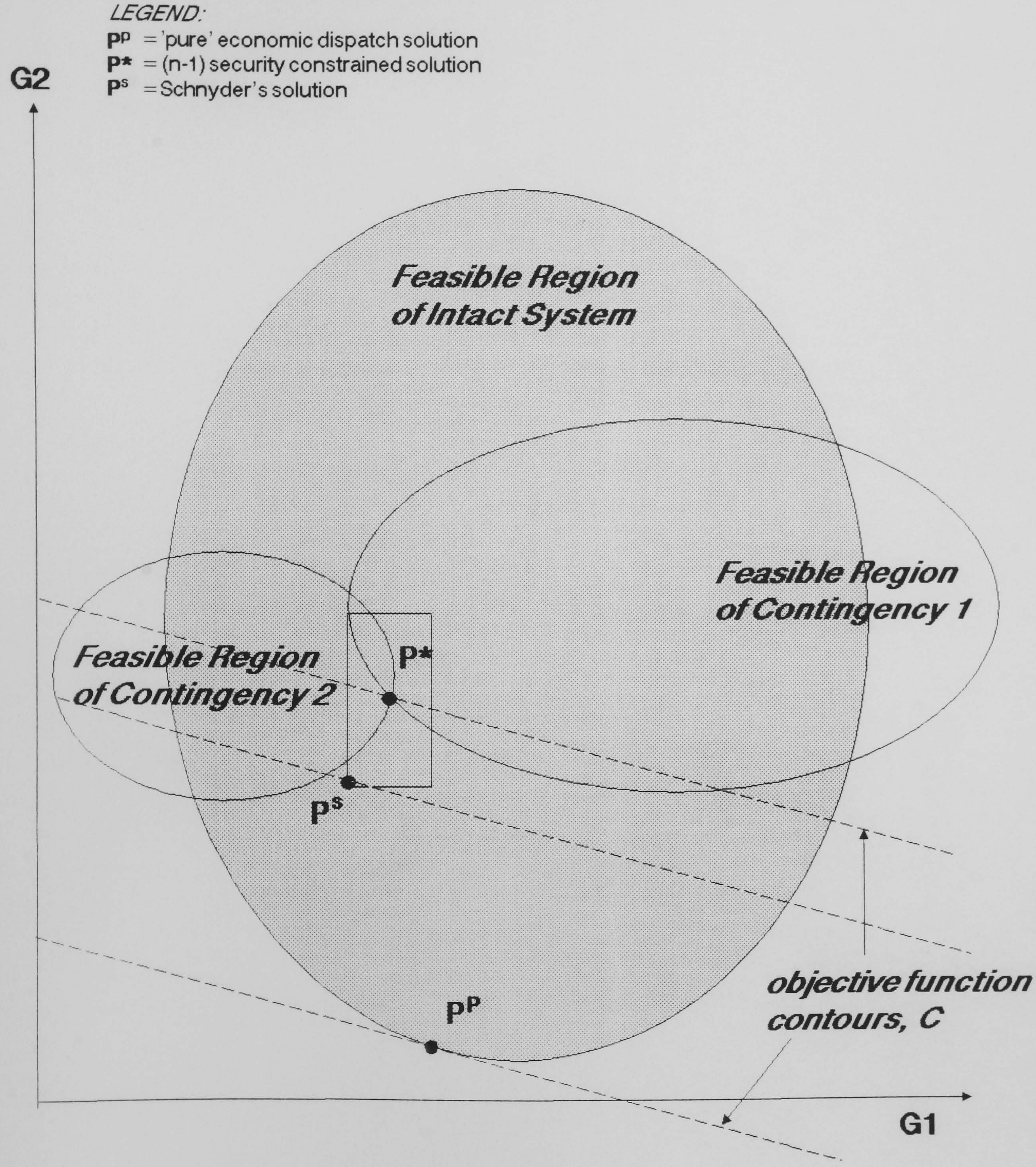



Fig.8.2(b)

Graphical Representation of Schnyder's Method for (N-1) Security Constrained Dispatch with Post-contingency Rescheduling

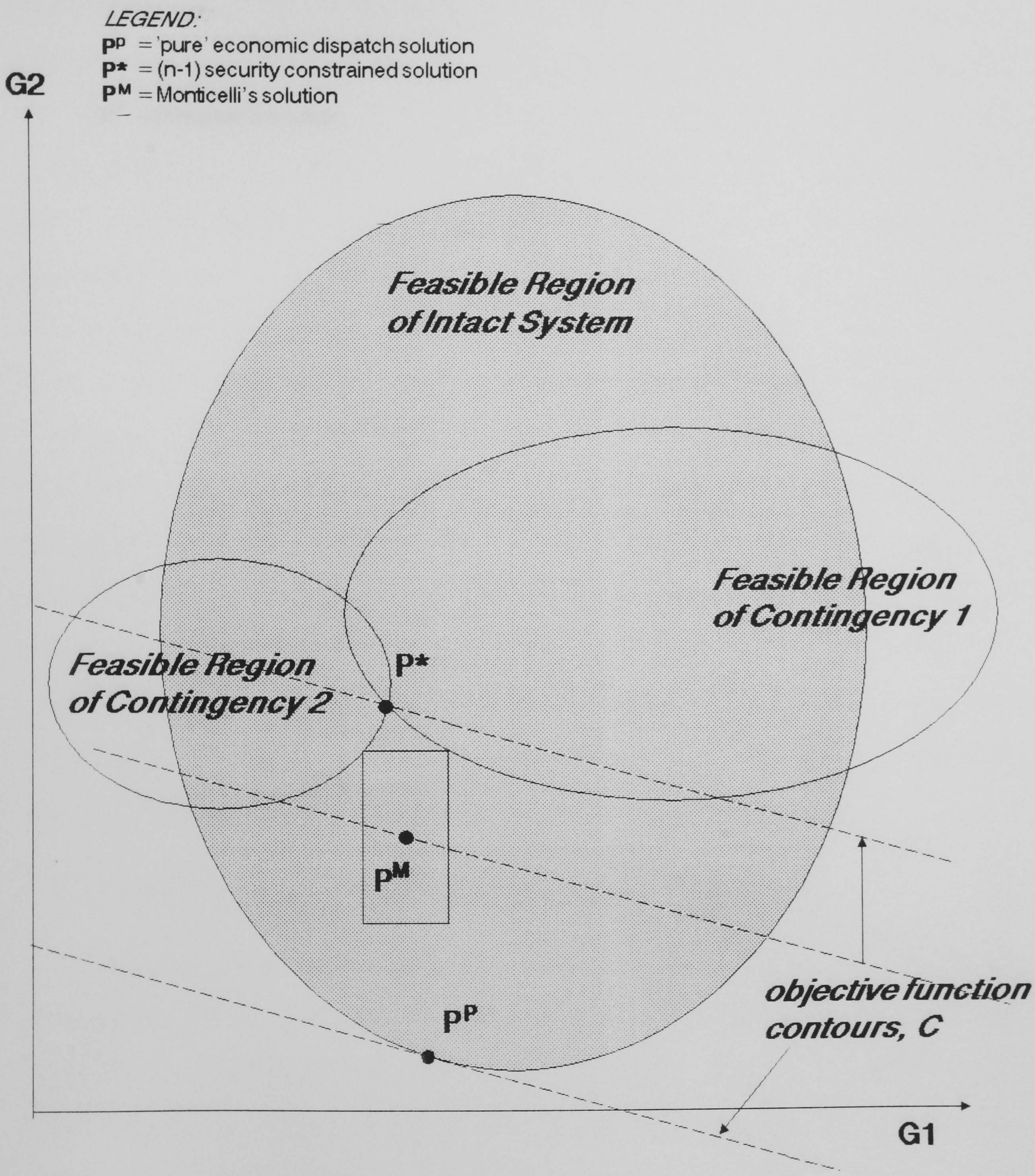



Fig.8.2(c)

Graphical Representation of Monticelli's Method for (N-1) Security Constrained Dispatch with Post-contingency Rescheduling

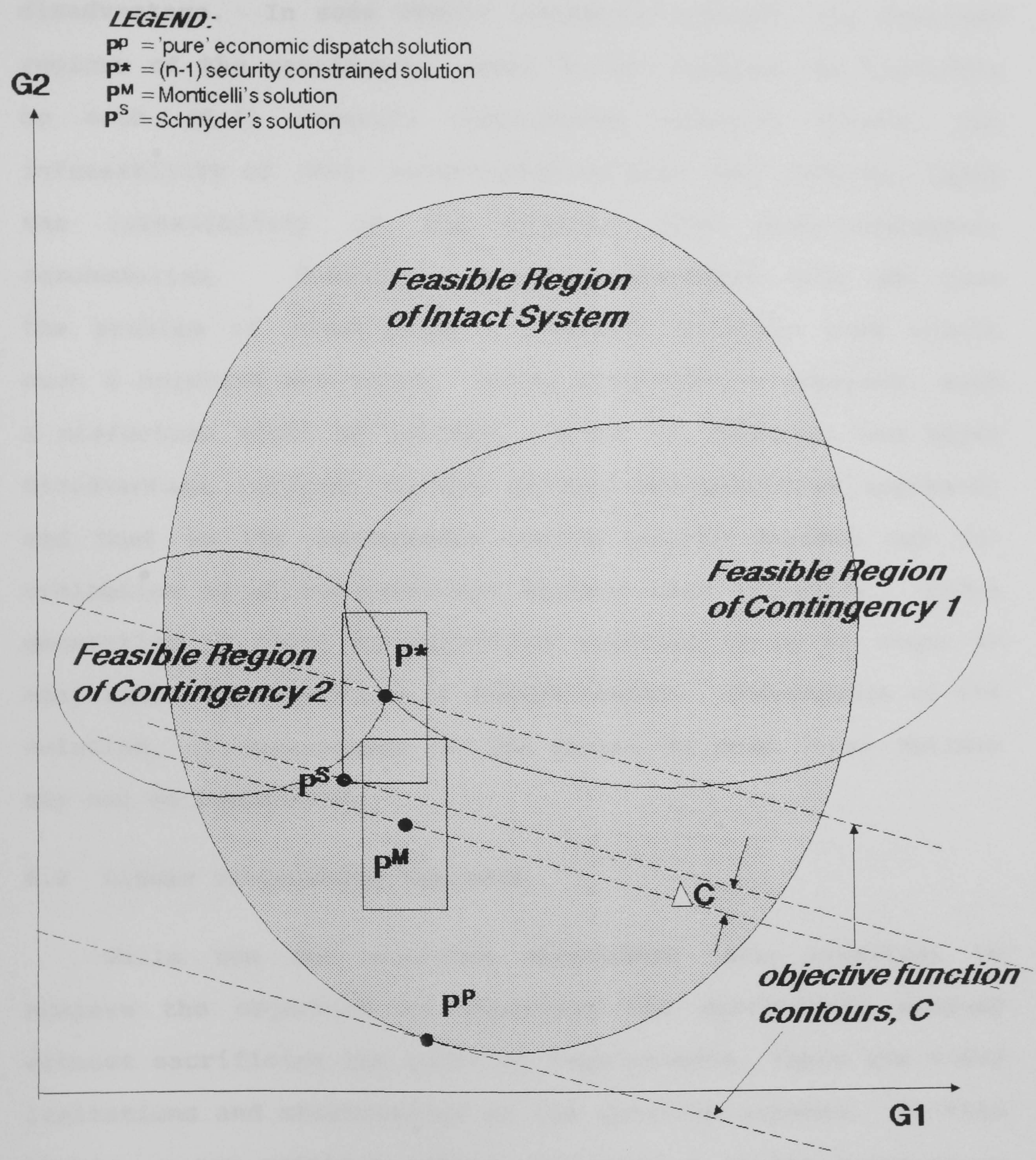



Fig.8.2(d)

Graphical Representation of Possible Optimal Cost Difference between Monticelli and Schnyder's Approaches

It is also apparent that the requirement to establish the *(N-1)* security solution in Schnyder's technique is a major disadvantage. In some weakly connected systems, the feasible regions of the contingency cases do not overlap and therefore no such *(N-1)* security constrained solution exists. The infeasibility of *(N-1)* secure problem does not, however, imply the infeasibility of the problem with post-contingency rescheduling. Schnyder's approach therefore also may have the problem of identifying a feasible solution even though such a solution may exist. Using Monticelli's approach, such a misfortune might be avoided. There is, however, one major disadvantage inherent to the Benders decomposition approach; and that is its convergence relies on the Benders cut co-ordination equations generated between each iteration. Since generating an exact co-ordination equation is by no means an easy task, approximation is always needed. Convergence of the solution, or convergence to the global or even local optimum may not be guarantied.

## 8.2 Linear Programming Approach

While the two existing approaches have potential to achieve the objective of improving the operational economy without sacrificing the security requirements, there are clear limitations and shortcomings of the solution schemes. In this thesis, a new solution method utilising a linear programming (LP) formulation is proposed. LP formulation of the economic dispatch problem has long been recognised as an efficient and flexible approach. The new *(N-1)* security constrained dispatch algorithm described in the last chapter is also LP based and has been demonstrated to be capable of including the post-contingency operational constraints in the overall cost optimization very effectively. The effort presented in this chapter represents the extension of the work described in the last chapter to encompass the post-contingency rescheduling capability in a unified approach to the economic dispatch problem.

### 8.2.1 Problem Formulation

The question of incorporating post-disturbance generation rescheduling capability in the context of economic dispatch is similar to the requirement of incorporating the effect of generator outage. In the former, generation rescheduling is treated as a calculated response to alleviate possible overloading in the remaining lines should a line outage occur. In the latter case, generation shifts are actuated to compensate the loss of a generator. In both cases, generation rescheduling is involved although the need of such actions are originated from quite different considerations. In Chapter 7, the inclusion of generator outage contingencies in a LP based dispatch algorithm has been discussed in detail. A similar approach can also be adopted for incorporating post contingency rescheduling capability. Without loss of generality, the security constrained dispatch considering corrective generation rescheduling capabilities may be formulated as follows:

$$\text{Minimize} \quad C = \sum_{g=1}^{N_g} F_g(P_g^0) \tag{8.9}$$

Subject to :

For the intact system:

(a) Power balance constraints:

$$\sum_{j=1}^{N_n} D_j = \sum_{g=1}^{N_g} P_g^{0} \tag{8.10}$$

(b) Generator output limits:

$$[P_g]^{min} \leq [P_g^{0}] \leq [P_g]^{max} \tag{8.11}$$

(c) Transmission line power flow limits:

$$-[F^0]^{max} \leq [S^0][G][P^0]+[C^0] \leq [F^0]^{max} \tag{8.12}$$

For each line/generator contingency k and response time allowance t,

(d) Power balance constraint

$$\sum_{j=1}^{N_n} D_j = \sum_{g=1}^{N_g} P_g^{k} \tag{8.13}$$

(e) Generator output limits:

$$[P_g]^{min} \leq [P_g^{k}] \leq [P_g]^{max} \tag{8.14}$$

and

$$-[R_g^{down}]\, t \leq [P_g^{k} - P_g^{0}] \leq [R_g^{up}]\, t \tag{8.15}$$

(f) Transmission line power flow limits:

$$-[F^k]^{max} \leq [S^k][G][P_g^{k}]+[C^k] \leq [F^k]^{max} \tag{8.16}$$

where

$F_g(P_g)$ = operating cost functions of the generator g

$D_j$ = nodal system load demand including any transmission losses

$[P_g^{0}],[P_g^{k}]$ = generator outputs for the intact and post-contingency system

$[P_g]^{min}$, $[P_g]^{max}$ = generator stable lower and upper output limits

$[G]$ = $N_n x N_g$ incidence matrix linking generators to buses which are connected directly

$[F^0]^{max}$, $[F^k]^{max}$ = circuit rating for normal and emergency system operation

$[R_g^{down}]$, $[R_g^{up}]$ = ramping down and ramping up rate of the generator

$[S^0]$, $[S^k]$ = sensitivity matrix for intact and emergency system state which relates the line current to bus injections

$[C^0]$, $[C^k]$ = line flow for intact and contingency cases due to nodal load demand

t = time allowance for the generators to react to the line/generator outage condition to bring the system to a tolerable state as defined by Eq.(8.16)

$N_g$ = number of on-line generators

$N_n$ = number of nodes.

Eq.(8.9) states that the objective of the dispatch is to minimize the total generation production cost of the <u>intact</u> system. This is subject to the power balance, unit capacity limits, line flow limits and unit generation shift limits for the intact and contingency conditions. Further constraints such as tie line flow agreements and generator group output restrictions may also be added to the constraint set. It is possible to see in Eq.(8.15) that if we make t=0, i.e. no corrective action allowed, the problem is identical to the conventionally security constrained dispatch. Conversely, if we make $t=\infty$, i.e. full range of rescheduling actions allowed in the period, the post-disturbance operating points $P^k$ become independent of the operating point $P^0$ and the problem becomes identical to the "pure" economic dispatch. For generator outage consideration, Eqs.(8.13) to (8.16) define the regulating margin requirement since they ensure that the load will be pick-up by the remaining units upon failure of a unit within the specified time t. For any given contingency, a series of Eqs.(8.13) to (8.16) constraints corresponding to different regulating time margins may also be utilised. These, coupled with the associated generation shifts and temporary line flow limits, may then be used to reflect the dynamic limitations of the system. The results of such a multistage dispatch would define a time sequence of controls to revert a disturbed system to the normal state.

It is also possible to see that the operating costs of the conventional security constrained dispatch and of the pure economic dispatch represent the upper and lower bounds for the solution of Eq.(8.9). From this observation, it is clear that the operating cost of a security constrained dispatch with rescheduling depends on the allowable time t for the generator to react and the achievable response rate of the generators.

### 8.2.2 Constraint Relaxation

It is apparent from the above that for a relative large system, the number of variables and constraints in the LP formulation can be very large indeed. For example, the test system provided by the CEGB has 115 generating units and 275 transmission lines. Assuming that all single line failures are considered and that all generators will participate in the correction process, for a single stage problem, there would be over 200,000 variables (including constraints) in the LP formulation. A LP solution is generally efficient only when the number of variables is reasonable, i.e. under a few thousands. Since the CPU time of LP execution increases quadratically with the number of variables[101], the CPU time requirement for a large scale problem with hundreds of thousands of variables would be impractical from both the execution time and computer storage requirements point of view. To overcome the dimensionality problem, an Iterative Constraint Selection (ICS)[203] process is implemented with details described as follows.

The success of the iterative constraint selection process is based on the exploitation of the special feature of the economic dispatch problem in which, although the potential number of constraints is large, the number of active constraints are normally small. By relaxing the economic dispatch problem (EDP) to include a small set of known active constraints initially, resulting in a much smaller LP problem, the EDP may be solved very quickly. When such an initial EDP solution is obtained, the full set of constraints is checked for violations. Any violated constraint detected is then added to the original set and a second LP iteration is performed. The final EDP solution is obtained when there is no constraint violation detected in the checking phase. This constraint relaxation was also employed in the proposed *(N-1)* security constrained dispatch algorithm described in the last chapter. In the present implementation, constraint relaxation is exploited in two areas: line flow constraints and generator limits, based on the following observations. For a well designed power system, the number of line outage contingencies which may lead to insecurity is relatively small, say 0.5%. For each such potentially insecured contingency, there may be 0.5% lines in the remaining transmission network near or over their rated limits, and the generators, say 10%, may need to reschedule their ramping or capacity limits to rectify the abnormalities. For the 115-unit and 275-line example system, the number of active line constraints is roughly about 196 (=14x14), the number of active generator constraints is about 140 (=14x10) and the number of generator variables is about 1610 (=115x14). Therefore the number of variables in the final relaxed LP iteration is in the region of two thousands instead of hundreds of thousands as originally estimated. Although based on this approach, a number of iterations will be required before reaching this final iteration, the dimensionality problem is now under control. If this method of building up the constraint necessary to be monitored is still too large, the number of constraints can be further reduced by omitting those constraints which become inactive in subsequent iterations. Ultimately the number of constraints in any iteration can be restricted to a pre-set total number of most violated constraints for the contingency cases. In the test studies carried out, the largest number of variables encountered is under three thousand and is within the capability of the LP algorithm employed. These measures which can be used to further cut down the number of constraints therefore are not implemented.

### 8.2.3 Post Contingency Power Flow Simulation

Another major stumbling block to overcome for the successful solution of post-contingency rescheduling is the effective simulation of the post-contingency system states. This is of paramount important since, as indicated above, for large systems there may be hundreds of thousands of constraints which need to be checked for violation. The number of operational constraints even after relaxation is also in the order of thousands. An AC load flow technique for power flow calculations is unlikely to be sufficiently computationally economical. Non-linear system characteristics included in an A.C. load flow also preclude the availability of a simple linear expression linking power flow to generation schedules. A D.C. load flow type sensitivity approach is therefore adopted in the LP formulation as described by Eq.(8.12) and Eq.(8.16). There are various techniques[87,183,228] reported in the literature for the calculation of the sensitivity matrices $[S^0]$ and $[S^k]$ and these have been reviewed in Chapter 7. These existing techniques, however, are relatively complex and involve much matrix manipulation and/or off-line pre-processing. A different approach, called current injection method (CIM), is therefore introduced in Chapter 7 and was utilized to calculate the post-contingency power flow as a function of the pre-fault system condition. In the following paragraphs, the CIM concept is extended to derive a simple expression for the post-contingency sensitivity matrices in terms of the pre-contingency sensitivity matrix.

Consider a system whose line flows and generator outputs for the intact system configuration are related by the simple linear function in Eq.(8.12) which is rewritten as:

$$[F] = [S][P] \tag{8.17}$$

where [P] is the nodal injection. Since Eq.(8.17) is a linear relationship, the superposition theorem applies. Let us assume that line k whose sending and receiving ends are m and n respectively is tripped due to fault or faulty operation, the new load flow in the remaining lines may be calculated by:

$$[F]' = [S]'[P] \tag{8.18}$$

where

$[F]'$ = post-contingency load flow;

$[S]'$ = post-contingency sensitivity matrix for line k outage.

Applying the superposition theorem, [F]' may also be calculated using:

$$[F]' = [F] + [\Delta F] = [F] + [S]'[\Delta P]' \tag{8.19}$$

where

$[\Delta F]$ = Incremental change in the remaining lines due to line k failure

$[\Delta P]'$= Thevenin equivalent nodal injection to simulate line outages with all elements equal to zero except:

= + $F_k$ pre-outage current of line k for node m
= - $F_k$ pre-outage current of line k for node n

Based on CIM concepts, it has been proved in Chapter 7 that the post-contingency sensitivity matrix [S]′ can be eliminated by modifying [△P]′in Eq.(8.19) such that

$$[\triangle F] = [S]'[\triangle P]' = [S][\triangle P^*] \qquad (8.20)$$

where

$$[\triangle P^*] = +F_k/(1-(S(k,m)-S(k,n))) \qquad \text{for the } m^{th} \text{ element}$$

$$= -F_k/(1-(S(k,m)-S(k,n))) \qquad \text{for the } n^{th} \text{ element}$$

$$= 0.0 \qquad \text{for other elements}$$

Substituting from Eq.(8.17) :

$$F_k = [S(k,1) \quad S(k,2) \quad S(k,3) \quad .... \quad S(k,N_n)] \; [P]$$

Eq.(8.20) becomes:

$$[S][\triangle P^*] =$$

$$\frac{1}{1-S(k,m)-S(k,n)} [S] \begin{array}{r} \\ \\ m\text{-->} \\ \\ \\ \\ n\text{-->} \\ \\ \end{array} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ . & . & \cdots & . \\ S(k,1) & S(k,2) & \cdots & S(k,N_n) \\ 0 & 0 & \cdots & 0 \\ . & . & \cdots & . \\ 0 & 0 & \cdots & 0 \\ -S(k,1) & -S(k,2) & \cdots & -S(k,N_n) \\ 0 & 0 & \cdots & 0 \end{bmatrix} [P]$$

$$\Rightarrow [S][\triangle P*] = [\triangle S][P] \qquad (8.21)$$

where

$$\triangle S(h,j) = \frac{1}{1-(S(k,m)-S(k,n))} S(k,j).(S(h,m)-S(h,n))$$

$h$ = index for remaining lines = $1,2,...N_L \neq k$

$j$ = node number

Substituting Eq.(8.17) and Eq.(8.21) into Eq.(8.19), we have

$$[F]' = [S][P] + [\triangle S][P]$$

$$\Rightarrow [S]'[P] = [S][P] + [\Delta S][P]$$

$$\Rightarrow [S]' = [S] + [\Delta S] \qquad (8.22)$$

$[S]'$ is the sensitivity matrix for the outage condition. Substituting the element of $[S]$ and $[\Delta S]$ in Eq.(8.22),

$$S'(h,j) = S(h,j) + \frac{1}{1-(S(k,m)-S(k,n))}(S(k,j)(S(h,m)-S(h,n)) \qquad \text{for } h \neq k$$

$$S'(h,j) = 0 \qquad \text{for } h = k \qquad (8.23)$$

The relative simple expression of Eq.(8.23) allows rapid calculation of post-contingency sensitivity coefficients as and when they are needed. The sensitivity matrix of the intact case can be stored in sparse factorized form avoiding a very large storage overhead. There is no requirement for any pre-processing as normally needed in the approaches using distribution factors such as Schnyner's solution scheme. In the case of any line failure causing system split, the denominator **$1-((S(k,m)-S(k,n))$** will be equal to zero and hence provide a convenient way of identifying a split system condition. In the present implementation, system split is regarded as an insecurity which may not be rectified by any generation adjustment. Such contingency is therefore not treated further in the optimization procedures once such condition is detected.

## 8.3 Computational Examples

The feasibility of the proposed LP approach on large practical system is investigated. The test system used has 115 units, 275 lines and 145 nodes and is based on a data set provided by the Central Electricity Research Laboratory of the former Central Electricity Generating Board in the United Kingdom. Two issues which are of primary importance are analysed when including post-contingency generation rescheduling capability in a dispatch: effect on system operating cost and on computer CPU time requirement.

### 8.3.1 Effect on System Operating Cost

The effect of including generation rescheduling capability on the optimum operating cost of a power system can be studied by comparing its solution with those given by dispatches which do not consider rescheduling capabilities. In Table 8.1 follows, optimum operating costs from three levels of dispatch sophistication are shown:

(1) A 'pure' economic dispatch in which line constraints for the intact system are considered;

(2) Conventional *(N-1)* secure constrained dispatch in which transmission lines are allowed to load to their emergency rating after the occurrence of a line outage;

(3) *(N-1)* secure constrained dispatch as in (2) but 8 minutes are allowed for the generators to shift their outputs to bring the transmission lines to or below their emergency rating after line outage occurrence.

Results for four loading conditions of the test network are included. For comparison purposes, the solutions of the 'pure' economic dispatch are used as the reference. For the test system and the given load patterns, the operating costs when considering rescheduling capability are the same as those of the 'pure' economic dispatch. Comparing with the conventional security constrained dispatch, the economic savings achieved by taking into account the effect of the generation shift capability is apparent. For the summer trough load, the saving can be as much as a quarter of the total expected fuel bill. The actual benefit realizable in practice may be less than the figures postulated in the table because of other limitations such as practical generator response rate achievable and maximum short time rating of the transmission line immediately after a line outage; but the potential is evident.

**Table 8.1** Comparison of Dispatch Results With Different Security Requirements

(1) Pure Economic Dispatch
(2) (N-1) Security Constrained
(3) (N-1) Security with Post-contingency Corrective Rescheduling

| Load Condition | Pure E. Dispatch | (N-1) Security | Security with Rescheduling |
|---|---|---|---|
| Winter Plateau | £914292 | £916255 (+0.2 %) | £914279 (+0.0 %) |
| Winter Trough | £479269 | £495842 (+3.5 %) | £479244 (+0.0 %) |
| Summer Plateau | £471334 | £491187 (+4.2 %) | £471317 (+0.0 %) |
| Summer Trough | £124240 | £153764 (+23.8 %) | £124230 (+0.0 %) |

It is also obvious from Eq.(8.15) that the amount of savings by considering rescheduling capability is affected by the time allowance for the generators to react to the outage conditions since the solution space for the dispatch is a function of response time. The response time allowance will depend on the dynamic capacity ratings of the transmission lines. If the transmission lines can be safely overloaded for a longer time, then the potential amount of generation shift permissible will be greater, leading to possible greater savings. The short time rating of a transmission line will depend on the operating environment such as wind speed, ambient temperature as well as the history of operation. This implies that continuous monitoring, recording and modeling of the critical transmission lines may be necessary in order to take full advantage of this new dispatch technique. The response rate of the generator is also part of the formula for calculating the solution space. For a given generation configuration and unit response characteristic, the relationship between the economic saving and the time allowance of a system may be studied. Table 8.2 shows the various optimal operating costs with respect to response time allowance for the four load cases of the CEGB test system. Table 8.3 shows the percentage of the maximum saving achievable against the response time allowance. The maximum saving is the case in which the dispatch results are identical a 'pure' economic dispatch solution.

The results of Table 8.3 is also depicted in Fig.8.3. In the tables, the blanks represent those cases where the optimal solution is not obtainable due to high dimensionality (exceeding 3000 variables) causing instability of the LP algorithm. The missing data, however, will not affect the

**Table 8.2** Variation of Operating Costs with Response Time Allowance

| Response Time(min.) | Winter Plateau | Winter Trough | Summer Plateau | Summer Trough |
|---|---|---|---|---|
| 10. | £914279 | £479244 | £471317 | £124230 |
| 8. | £914279 | £479244 | £471317 | £124230 |
| 7. | £914279 | £479244 | £471317 | £126249 |
| 6. | £914279 | £479244 | £471317 | £128979 |
| 5. | £914282 | £479250 | £471378 | £131857 |
| 4. | £914288 | £479280 | - | £135327 |
| 3. | £914591 | £479320 | - | £138667 |
| 2. | £914977 | £481556 | - | £143893 |
| 1. | - | - | - | £148549 |

**Table 8.3** Variation of Operating Cost Savings with Response Time Allowance

| Response Time(min.) | Winter Plateau | Winter Trough | Summer Plateau | Summer Trough |
|---|---|---|---|---|
| 10. | 100 % | 100 % | 100 % | 100 % |
| 8. | 100 % | 100 % | 100 % | 100 % |
| 7. | 100 % | 100 % | 100 % | 93 % |
| 6. | 100 % | 100 % | 100 % | 84 % |
| 5. | 100 % | 100 % | 100 % | 74 % |
| 4. | 100 % | 100 % | - | 62 % |
| 3. | 84 % | 100 % | - | 51 % |
| 2. | 64 % | 86 % | - | 33 % |
| 1. | - | - | - | 18 % |

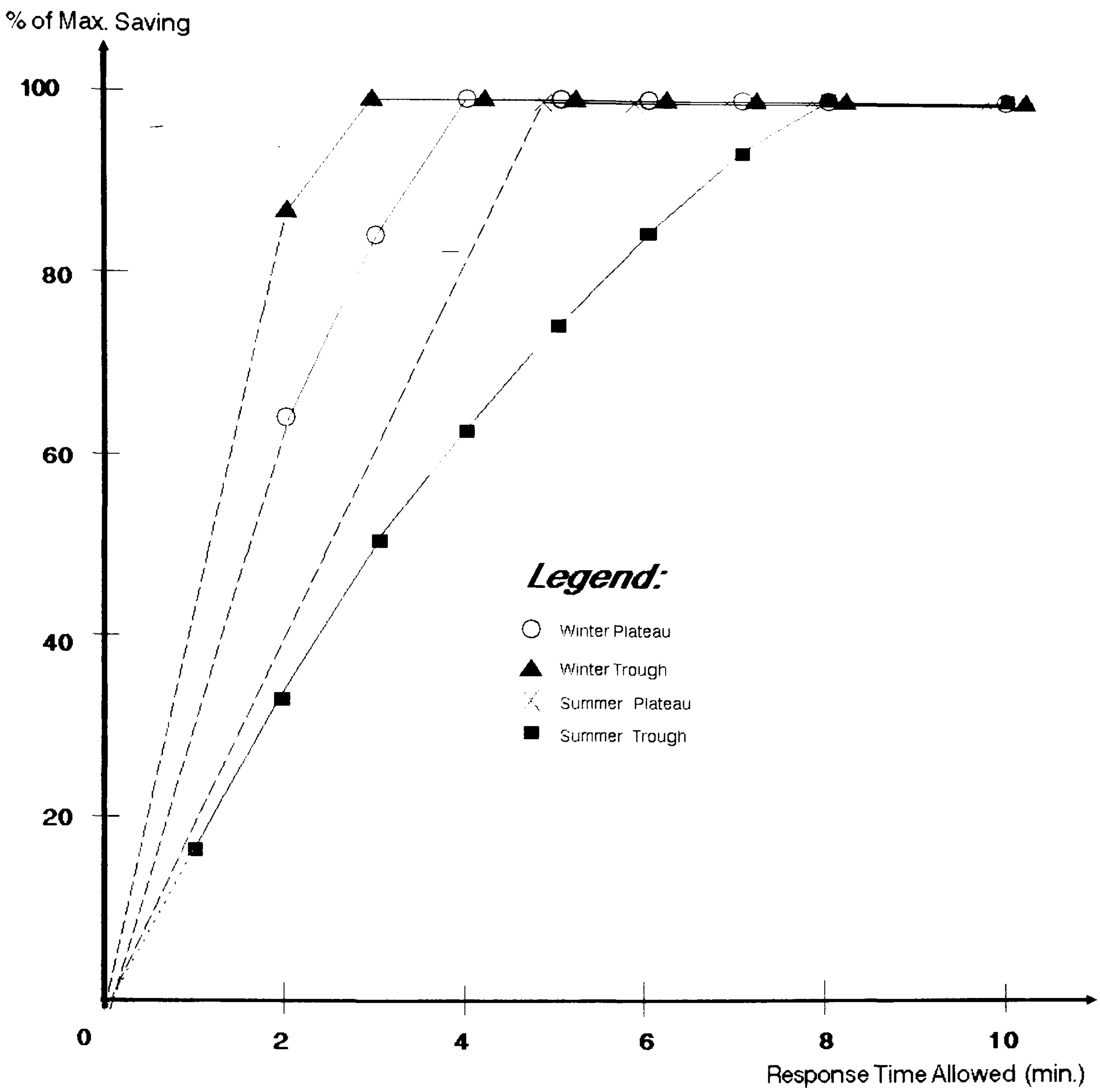



Fig.8.3 Operating Cost Savings V.S. Response Time Requirements for Four Different Load Conditions

conclusions which can be extracted from the study results. From the figure, it is clear that most economic savings when considering post-contingency rescheduling is realized in the first few minutes; in fact over 50% in the first three minutes. In the study example, a fixed small percent of the generator maximum output are used as their respective ramping rate. For gas turbine and hydro electric generators whose response rates are much faster, a greater generation shift will be achievable in the same time span. The important role of fast start up gas turbine and hydroelectric generators in terms of alleviating line overloading and establishing security of the system is therefore obvious. By siting these plants at strategic points of the system, they can also contribute to the economic operation of the system by allowing the more economical plant to generate at a higher output which may not be advisable otherwise. Because of the significant saving that can be achieved through fast, accurate and concerted reactions of the generators to the contingencies, it seems beneficial to store the calculated response requirements of the generators of some critical contingencies in the central control centre ready for instructing the automatic generation control units should any of these contingencies occur. Furthermore the locations of the generators also affect the effectiveness of its corrective injection to alleviate overloading. With these considerations, it is possible to see that the proposed approach can be used to assess the impact of the size, location, type of generator and type of generation control of a power station on the economic operation of a system for any given short time transmission capability of the network.

### 8.3.2 Effect on Computer Execution Time

Table 8.4 below depicts the CPU time requirements for the four load conditions assuming the response time for the generating units is 8 minutes. It also provides further information regarding number of active contingencies, number of line overloads, number of active generator ramping limits and number of variables. The data also plotted graphically in Fig.8.4.

**Table 8.4** CPU time for Security Constrained Dispatch with Post-contingency Corrective Rescheduling

| Load Condition | No. of Variables | **No. of Active Constraints** Cont'ncy | Line | Gen | CPU Min:Sec |
|---|---|---|---|---|---|
| Winter Plateau | 783 | 5 | 5 | 82 | 0:31 |
| Winter Trough | 1467 | 10 | 17 | 174 | 1:29 |
| Summer Plateau | 2973 | 20 | 63 | 474 | 6:33 |
| Summer Trough | 1441 | 10 | 14 | 151 | 1:27 |

For each of the four load cases, the pure economic dispatch and security constrained dispatch require only 5 and 8 CPU seconds respectively. The execution time for the corrective rescheduling dispatches is considerably longer but is still tolerable for real time applications. By inspection of Fig.8.4, the CPU time requirement bares a close

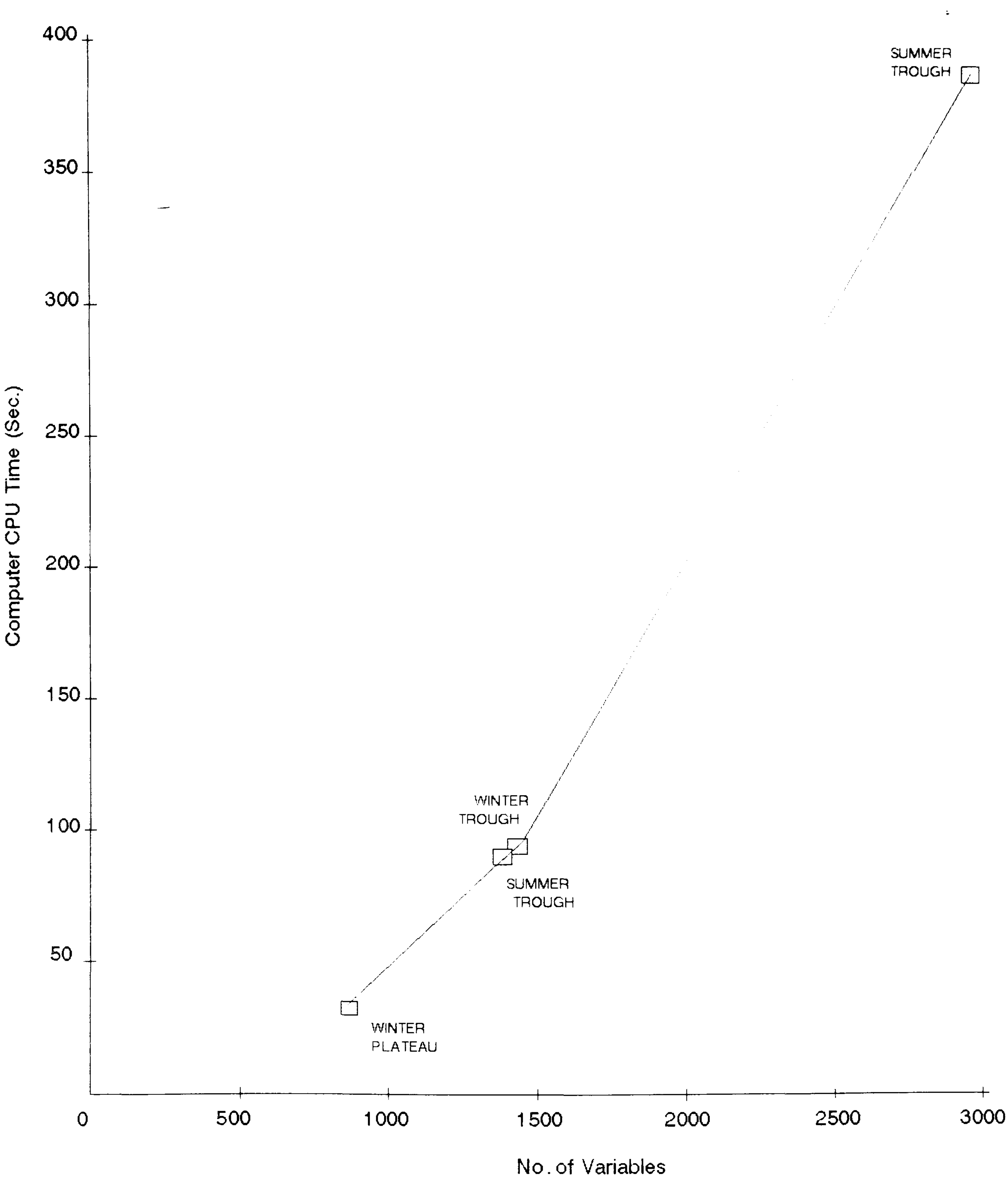



Fig.8.4

Variation of CPU time Requirements against the No. of Variables for the proposed (N-1) Security Constrained Dispatch considering post-contingency Rescheduling Capability relationship to the number of variables of the LP problem. This is a well known LP solution characteristic. Comparing Tables 8.1 and 8.4, there seems, however, no simple relationship between the computational time and the corresponding economic benefits. This is logical since the economic saving depends on the design and operation of the system and on the nominal and overload capacity ratings of the equipment. When the economic operation of the system is seriously affected by the limitations of a small number of lines, as indicated by a large increase in operational cost for a *(N-1)* security constrained dispatch in comparison to a pure economic dispatch, there will be a good chance that dispatch with corrective rescheduling consideration has a significant impact on the operational costs. Given that a system exhibits the characteristic of having great potential economic saving from considering corrective capability, it is likely that an experienced system operator will have already instigated an *ad hoc* scheme similar to the corrective dispatch. This is another factor which will affect the theoretical maximum economic saving achievable by implementing the rigourous approach. The present methodology, however, offers a basic framework for further progress in analysing and maximizing the security and economic potentials inherent in the dynamic capacity ratings of the plants.

## 8.4 Summary

This chapter has presented a new LP algorithms for the solution of security constrained dispatch considering corrective rescheduling capability of the generating plants.

The major draw back of the existing approaches, namely, suboptimality of Schnyder and Glavitsch's Lagrangian solution scheme, and non-convergence of decomposition method offered by Monticelli el al are eliminated. The strength of the new formulation stem from the inherent simplicity and robustness of LP approaches. It has been shown, using a large test system, that computationally the proposed method is practical for on-line application. The efficiency of the method, however, has not been exploited exhaustively. For example, the number of generators in the system capable to perform the corrective rescheduling duty may be limited. The number of line power flow constraints may also be minimized by including only the most violated constraints in the constraints sets. However, the main objective of the thesis to demonstrate the applicability of LP formulation in this interesting and challenging problem is largely achieved. Using the test system, the chapter also indicates clearly the potential significant economic savings by considering post-contingency rescheduling capacity and that further work in pursuing the concept will prove to be worthwhile.

# CHAPTER 9

# CONCLUSIONS

An electricity supply system is a large integrated process which requires continuous monitoring and control to ensure a secure and economic operation. The significant increase in the use of computer aided on-line control in recent years reflects the world-wide recognition of the necessity, as well as economic benefit, of employing such advanced technology. There are estimates that, for some systems, the pay back period for full SCADA and EMS control hardware/software can be as short as one or two years. On-line computer assisted control reduces the overall system cost in several main areas. Firstly, there is the possible saving in staffing cost due to automation. More significantly, it is the capability to exploit the economic generation sources, the power transfer capacity of existing transmission lines, and to minimize losses, supply interruption and spinning reserve, together with operational flexibility and management planning information collection, which give the ultimate advantage of a modern power system computer control facility. This thesis is concerned with the subjects of unit commitment and economic dispatch. These two control functions are at the heart of a power system EMS computer control software suite and represent a time decomposed hierarchical approach to achieve the complex economic operation optimization objective. Unit commitment deals with a longer time span problem, typically of 24 hours or one week period. Economic savings are achieved by controlling the on/off schedule of the appropriate units.

Economic dispatch deals with problem with a much shorter time span, typically of 5 to 30 minutes ahead. Cost savings are accomplished by maximizing the output of economic generation sources.

Traditionally, the two problems, because of their different emphasis and nature, are solved by different optimization techniques. The problem structure and the operational constraints which need to be considered for the two control functions, however, are closely related in many respects. One of the common threads running through a major part of the thesis is the application of dynamic programming methods to solve these two optimization problems. The operational characteristics of system components directly affect the production cost of a system. In Chapter 2, the modelling of the system requirements and limitations of its major components, such as generators and the transmission network, are described. The important aspect of system security is also outlined. Depending on the capability of the optimization techniques employed, detailed or otherwise simplified models may be used.

The work on unit commitment solution reported in the literature is surveyed in Chapter 3. The prominent algorithms are grouped under five categories: merit order, mixed integer-linear programming, branch-and-bound, Lagrangian relaxation and dynamic programming. These technique have achieved various degrees of success in terms of model accuracy, flexibility and computability. It is found that even with the rapid advance in performance/cost of computer hardware, the merit order method which is the simplest of all algorithms is still perceived as the only feasible approach for some large system. Mixed integer-linear programming and branch-and-bound are probably sufficiently efficient for small systems only. The Lagrangian relaxation method is the most mathematically rigourous of all approaches. It is reported that it has been applied successfully to a 250 unit system; but the technique has limited capability to include many practical operational constraints. The dynamic programming (DP) technique is inherently computationally intensive and requires enormous computer memory; but its flexibility and capability of dealing with non-linear constraints have attracted considerable interest in its applications ranging from power system planning to operational control. In the context of unit commitment, DP has been applied in two principally different classes of implementation: one may be generally classified as the time variant DP approach and the other the time static DP approach. By limiting the number of possible generator combinations in each sub-interval, time variant DP approaches have been able to control the execution time and storage requirement. To date, time variant DP approaches have become an accepted alternative to merit order methods in the industry. On the other hand, time static DP approaches employ a DP derived optimal generator combination table and assign the optimal generation combination of each sub-interval according to this table. The methods have a characteristic simplicity similar to the merit order techniques; but the CPU time taken to derive the optimal generator combination table has restricted the efficiency and flexibility of the approach.

In Chapter 4, the deficiency of the time static DP approach is overcome by a novel recursive DP formula. Using the proposed formula, CPU time required to build the optimal generation table is dramatically reduced. More significantly, the CPU time requirement is now independent of the number of units in a system but can be controlled by selecting an appropriate step size for the discrete representation of the generator cost functions. As a result of the computational efficiency improvement, it is practical to build a revised optimal generation table for each sub-interval of the study period. This new capability leads flexibility in considering many practical constraints, such as derated capacity, pre-assigned on/off generator schedules, which may not be possible in the conventional time static DP methods. In order to take into consideration start up/shut down costs, a composite cost function is proposed which combines production dependent fuel costs with start up/shut down costs of a generating unit. This generation cost model allows the application of the proposed DP recursive formula to decide efficiently the on/off schedule of the generating units satisfying many physical and operational constraints. Another new algorithm described in the chapter is the evaluation of the total load carrying capability of any unit combination to cover the lose of any loaded unit. The method is responsive to margin time allowance and ramping capability of individual units. It is believed that this is a major improvement from the conventional simple criterion of satisfying a fixed amount of spinning reserve disregarding the expected loading and response rate of the synchronized units. Tests carried out show that the proposed method is potentially compatible for on-line large scale applications.

Active power economic dispatch algorithms are reviewed in Chapter 5. The dominant techniques for the solution of this crucial operational cost minimisation function are grouped into five categories: equal incremental cost, gradient methods, linear programming (LP), quadratic programming (QP) and dynamic programming. The equal incremental cost concept is the simplest and most widely applied technique and is manifest in the applications of merit order loading in many long term and operational planning studies. It also has the advantage of easy incorporation of the consideration of losses by the use of penalty factors. The main disadvantage of the method is the inability to include operational constraints such as transmission line power transfer limitations in the optimization process. Gradient techniques have a similar inadequacy. LP and QP are by far the most intensively researched methods. They are flexible, robust and, in general, can deal with any constraints which can be modelled by linear functions. QP has the additional capability of dealing with a quadratic cost function which is particularly relevant for loss reduction. LP is generally regarded as the most computationally efficient and versatile of all methods. There are only a limited number of trial implementations of the DP approach. The inherently long CPU time and enormous storage represent the major obstacles for the DP methods in practice. Furthermore, their inability to incorporate transmission constraints restricts the methods to special cases where their unique capability to consider a non-linear cost function is overwhelmingly significant.

The shortcomings of DP methods, however, can be eliminated by applying the novel DP recursive formula proposed in Chapter 4. An iterative procedure developed in Chapter 6 optimizes the generator output taking into consideration individual line flow constraints and losses. By using a successive estimation technique, not only are the inherent CPU time and memory obstacles overcome, the solution can also be controlled to any desired accuracy. Tests included indicate that this novel approach is both highly efficient and accurate; comparable to LP and DP techniques. A new loss formula is also described. This loss formula eliminates the requirement of a 'base case' as used in the conventional B coefficients methods. It also has the advantage of computational simplicity and is responsive to the rapid changes in system topology, load distribution and generation pattern.

It has been recognized for some years that consideration of operational constraints for the intact system will not ensure the security of the system under contingency conditions. A new LP based (N-1) security constrained dispatch algorithm is described in Chapter 7. A new simulation technique for post line failure real power flow estimation, called Current Injection Method (CIM), is derived based on the well known superposition theorem. Using this algorithm, the need to modify the system impedance matrix or its inverse, as in many existing methods, is no longer required. The security constrained dispatch algorithm implemented utilizes the CIM technique to generate a list of critical line failures within the solution process. The resulting optimal generator dispatch ensures that any unscheduled single line outage will not cause overloading in any of the remaining lines in the system. Tests using a CEGB test network demonstrate that the proposed algorithm takes only marginally more computation time than a dispatch without contingency consideration. The economic cost for the added security however can be significant.

In *(N-1)* security constrained dispatch, the possible response such generation rescheduling, line switching and load shedding have not been included in the analysis. *(N-1)* security constrained dispatch therefore represent a pessimistic assessment of the operational capability of the system. The new security constrained algorithm is therefore extended in Chapter 8 to include the post-contingency corrective generation rescheduling capability. There are two major technical problems in dealing with such a formulation: problem size and functional relationship of post-contingency power to rescheduled generation outputs. These are solved by employing a constraint selection scheme and by deriving a simple formula for the determination of the post-contingency power flow sensitivity coefficients. By allowing transmission lines to overload to their short time rating during the short period in which the generator outputs are being adjusted by the operator in response to a line failure, significant economic saving is demonstrated. It is shown in the thesis that this new algorithm although significantly complex is potentially capable of dealing with a large realistic network for real time application.

Many possibilities exist for further work in the area of DP applications in unit commitment and economic dispatch. For example, in the thesis, the new recursive formula is applied separately to the unit commitment and dispatch problem. It is conceivable that its capability to consider line flow constraints as implemented in the generation dispatch can be included in the unit commitment phase. This may lead to an efficient technique which integrates the unit on/off scheduling technique with inherent consideration of line flow constraints. The unique capability of DP for non-linear cost and constraint representation is the major incentive for such a development. On the hand, the new *(N-1)* security constrained dispatch with post-contingency generation rescheduling algorithm induces many issues which need to be examined rigourously. Firstly, by considering the possible actions of the system operator and the inherent response rates of the synchronized units, the spinning reserve requirements may be defined analytically with consideration to the dynamic response capability of the generators and of the transmission network. The economic benefit of rapid response/short time rating of the generators and of the transmission lines may also be quantified. The dynamic rating data requirements of *the (N-1)* contingency constrained dispatch also raises the question of what level of increased plant monitoring and performance calculations will be adequate to capture the economic benefit of considering corrective actions. The proposed algorithm represents a first attempt to include post-contingency generation rescheduling capability in a LP based dispatch technique. It would be vital to further develop the algorithm and research into new techniques for improved efficiency or capability.

In summary, the unit commitment and economic dispatch functions in large scale power system operational control have been considered. DP and LP techniques have been applied to these problem, and computer requirements have been compared with existing methods. It is found that the proposed techniques are highly efficient and have contributed to the subjects by expanding the capability of DP and LP based techniques to deal with these two vital control functions in a modern EMS software package.

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APPENDIX A

O.C.E.P.S. ENERGY MANAGEMENT SOFTWARE

The suite of power system application software produced by the OCEPS research group has been in development for over 17 years. Parallel to the tremendous progress of digital computer technology, the operational control algorithms of the OCEPS group are also evolving continually in step with the availability of affordable powerful minicomputers. The following description serves to give a brief outline of the major control elements of this suite of programme. The data flow diagram for the major control functions is depicted in Fig.1.1 (Chapter 1).

The OCEPS software package is divided into two parts: system simulation , and analysis and control functions. The simulation programs are installed in two *Perkin Elmer 3230* computer, one of which equipped with array processor FPS 5000. The analysis and control functions are installed in a *VAX 8600*. The communication between the simulation and control function is provided by a GEC SCADA computer reflecting the real world situation in which the control computer communicate with the remote terminal units via some form of communication links and data pre-processor.

**A.1 System Simulation**

The major aim of the simulation package is to provide a 'test bed' for the development, testing, verification and evaluation of the control algorithms. In order to represent realistic response to load frequency control action and other control inputs, power plants are simulated individually with detail dynamic models on the turbo-generators, automatic voltage regulators, governors, turbines and boilers. Over-speed and under frequency protection are simulated. The network models include transmission line, transformers, static compensators which can be capacitive or inductive, and consumer loads. Complex transformer tap and line overloading protection are simulated. Random gross analog and digital measurement errors of the system are also introduced before data transmitted to the control function to reflect the operating environment of a real system. Time-varying network connectivity information is sent to the control function to enable the islanding re-synchronization simulation.

- **Load variation and disturbances:**
  These are the load data and disturbances, such as load shedding, which impress on a physical system to create the effect of load distribution variation in a system.

- **Exact topology determination:**
  This is the determination of system connectivity and islanding considering any switching operation as a result of line tripping initiated by the control action or by the protective relays.

## A.2 Analysis and Control Functions

The dynamic simulator creates telemetry data which are communicated to the global data area within the analysis and control computer. The received date are then undergone various processes to achieve the monitoring and control objectives.

- **Data validation and state estimation:**
  In here, the raw data received are systematically filtered to eliminate any bad data, gross error due to measurement noise, miscalibration and to provide necessary limit checking, consistency checking and exponential smoothing capabilities. Estimates for unmeasured quantities and a consistent data set are produced.

- **Security analysis and fault studies:**
  These programs allow operator initiated and automatic "what if" analysis, to determine the viability of power system under various hypothetical contingencies. The impact of generator or line outages on the power flow and bus voltages are ranked according to severity of constraint violations and presented to the operator. Detailed assessment results of any outage may be displayed graphical and in tabular forms on operator demand.

- **Emergency rescheduling and load shedding:**
  During emergency conditions in which insufficient generation is available to meet the demand or where one or more unexpected generation plant outage have occurred, there is a need for rapid redeployment of generation schedule and to initiate optimal load shedding. Under emergency conditions, economic operation of the system has a lower priority than the minimisation of load shedding. Artificial costs are assigned to the load supply points for the determination of the optimal degree of load shedding subject to the power flow and generation pick up limitations.

- **Load prediction:**
  Estimation of future load is required in order that prior warning of output requirements may be given to power stations, enabling limitations on boiler fuel feed rates, and generator rate of change of output constraints, to be observed. Furthermore, the economic start-up and shut-down of generating units is dependent on the predicted load so that expensive spinning reserve may be minimized. The optimal sharing of load among the synchronized generators also depends on the forecast load so that the consumer demand may be satisfied at minimum fuel cost. The predictor employs a weather-corrected ARIMA (Auto Regressive Integrated Moving Average) model where history of network loads, network configurations and corresponding meteorological data are utilized.

- **Unit commitment:**
  The ordering of unit start-up and shut-down is performed in this module. Based on the forecast load provided by the load forecaster, an optimal unit on/off schedule is devised by this control module so that minimum operation cost will incur satisfying the load balance and spinning reserve requirements. Dynamic programming technique is the principle algorithm employed. Heuristic merit order approach is also available within the package.

- **Economic dispatch:**
  The optimal sharing of generation output among the scheduled 'ON' units are provided by economic dispatch. The principle objective of this module is to minimize fuel cost to meet the predicted demand subject to system security and operational constraints. Various algorithm based on linear programming, quadratic programming and dynamic programming techniques are implemented.

- **Load frequency control:**
  This control module calculates the generator set points in order to maintain a stable frequency and pre-scheduled tie line power flow. The continue variation of load necessitate a smoothed continual modification of the set points and avoiding any step changes which may introduce transient disturbances. Depending on the system characteristics, frequency biased or tie line biased automatic frequency control may be employed. In common with the other analysis and control functions in the package, the load frequency controller is able to operate in circumstances in which the power system has split into two or more electrically independent islands and drives each island to the nominal system frequency.

## APPENDIX B

## BRANCH-AND-BOUND TECHNIQUE

Branch and bound is a powerful and flexible optimization technique. It found the widest application in those problems with a mixture of continuous and discrete variables. The method essentially subdivides the possible solution space into mutually exclusive groups by assigning fixed values to some of the discrete valuables with the remaining discrete variables relaxed to be treated as continuous. This process is called branching. The objective function for each group is then evaluated to give an estimation of the best possible optimal solution obtainable from each respective group. This is called bounding. The method concentrates initially on those groups which gives the best possibility of finding the global optimal. The branching and bounding process are repeated many times until all the discrete variables have been assigned a value. The global optimum is the solution with the best objective cost function and all the discrete variable have taken some fixed values. The efficiency of the approach is achieved by discarding those groups leading to infeasibility and also those non-promising groups without actually evaluating them. The following simple example serves to clarify the optimization steps.

Consider the problem:

Minimize: $C = 4\ X_1 + 10\ X_2$

Subject to:

$$2\ X_1 - X_2 \leq 1$$
$$- X_1 - X_2 \leq -2$$
$$X_1,\ X_2 = 0,\ 1 \text{ or } 2$$

The solution method of the branch and bound technique can be organised as a sequential decision problem represented by a tree structure in which a branch represents the branching process and a node represents the subproblem after the branching. For this example, the branching represents the integer variables committed to 0-1-2 as shown in Fig.B.1. The essence of the method is to terminate the search from as many nodes and as early in the calculation as possible. The steps of the branch-and-bound solution method is summarized as follows:

1. Generate upper bound, $F_u^*$.

2. Select a free variable say $X_1$, and set $X_1=0$, $X_1=1$, $X_1=2$. This is the branching process to create three subproblem for each fixed value of X1. These subproblems are represented by nodes S1, S2 and S3.

3. Evaluate the objective function for nodes S1, S2 and S3 with the unassigned discrete variables (X2) relaxed and treated as continuous variables. This is to make the subproblem easier to solve by making use of an efficient computational technique such as a linear programming algorithm. The efficiency of branch and-bound scheme relies on how efficiently the subproblems can be solved. The solutions of these problems are partial solution to the original problem as they have not taken all constraints into consideration. These partial solutions represent the lower bounds of the original problem and designated as $F_L$.

4. For each recently created node, fathom if:
   (a) No feasible solution.
   (b) $F_U^* \leq F_L$.
   (c) If there is no further free variable, and $F_L \leq F_U^*$ then a new solution for the original problem is found. Set $F_U^* = F_L$. All partial solution shall be checked against this revised $F_U^*$.

5. Select the node with the best $F_L$ and repeat steps 2-3-4. The optimal solution of the original problem is found when no better $F_U^*$ can be found.

Fig.B.1 shows the possible branch-and-bound steps to find the optimal solution F=14 for the example problem. As can be seen, many possible solutions for the problem do not need to be evaluated because of the fathoming criterion.

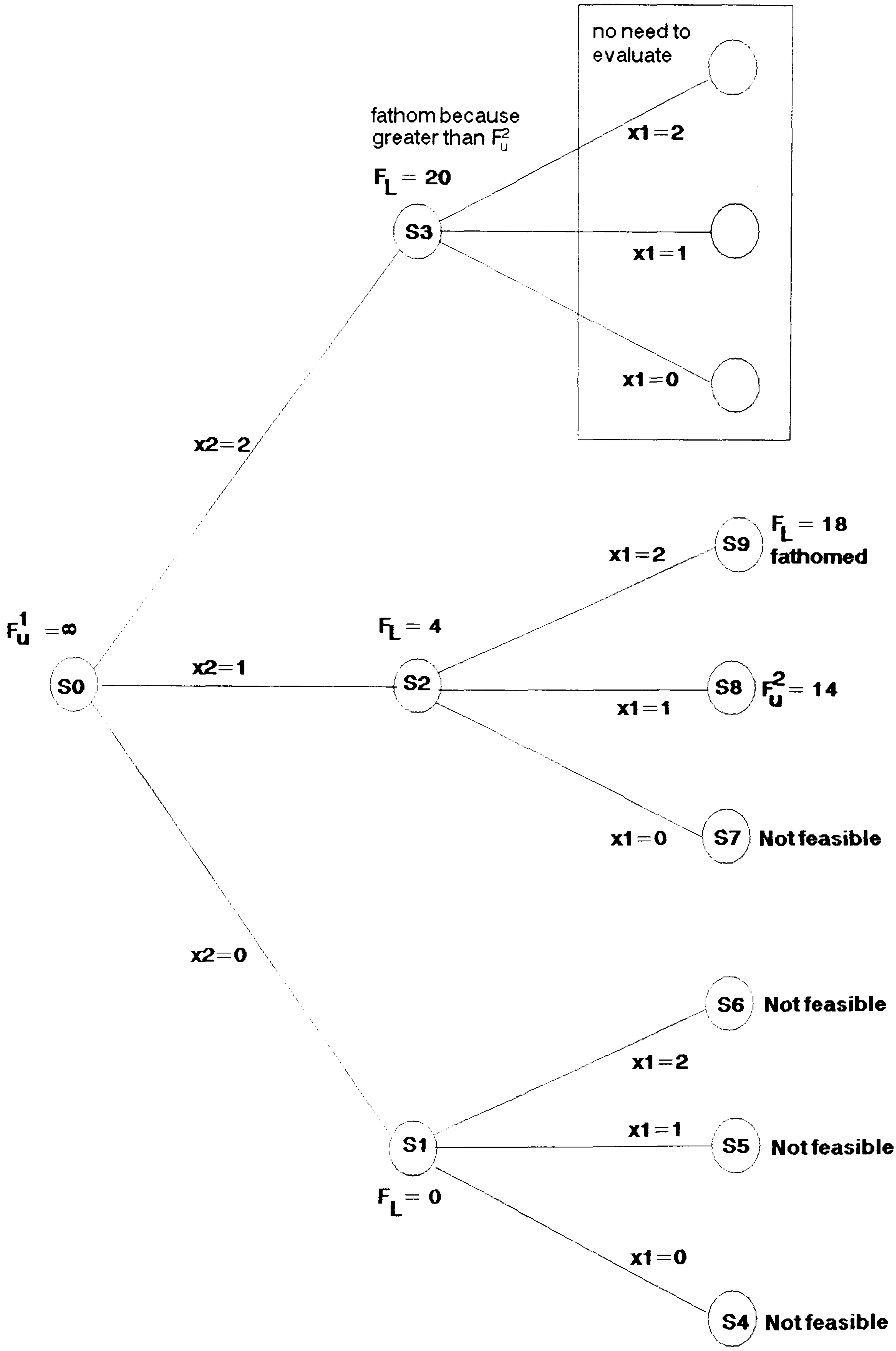


Fig. B.1 Simple Example for Branch-and Bound Tree

## APPENDIX C

## DYNAMIC PROGRAMMING TECHNIQUE

Dynamic programming (DP) is an optimization technique, which breaks a problem into a series of simpler sub-problems (stages) by applying the principle of optimality. In most combinatorial problems, it is theoretically possible to evaluate all solutions and select the best. However, for problem with many variables, the number of possible combinations can be very large and the amount of computational effort required will be excessive. DP offers a systematic approach to reduce the number of combinations which need to be considered and has been successfully solving a variety of control and process problems. Introduced by Dr. Richard Bellman in 1957, **"The Principle of Optimality"** is:

> *A policy is optimal if, at a stated stage, whatever preceding decisions may have been, the decisions still to be taken constitute an optimal policy when the result of the previous decisions is included.*

In other words, an optimal policy must contain only optimal subpolicies. The following simple example serves to give an introduction of the concept of this powerful method.

Consider a salesman who wishes to travel from city A to city N. The network diagram of Fig.C.1 represents the possible routes he may follow and the values on the arcs represent the associated cost, from one intermediate city to the other. The problem is to find the minimum cost path from A to N. The problem can be divided into 5 stages as shown in the figure. The stages are numbered so that the number of a stage represents the number cities he visited from city A. When the salesman is at a particular stage, he will be in a particular state defined to be the particular city of that stage at which he is located. The cost incurred travelling from one state to another is the transition cost. The total minimum cost incur to arriving to a state is called a return. A single decision of how to travel from one stage to the next is called a policy choice. A set of policy choices from the initial stage to some state in an intermediate stage is a subpolicy. A complete set of decisions from the starting point to the final destination representing a solution to the problem is a policy. The policy with the minimum return is the optimal solution to the problem. The DP approaches normally proceed in two steps: forward calculation and then back tracking.

<u>Forward calculation:</u>

**Stage 0:** The return is zero since no cost is incurred so far. The return is shown above the state designation.

**Stage 1:** Three possible states: B, C, D. There is only one possible policy choice to any of these state. Let f{X} represents return of state X and t(x1,x2) represent the transition cost from state x1 to x2, then

f{B} = 5; f{C} = 2; f{D} = 3

**Stage 2:** Three possible states: E, F and G each having two possible ways to reach from the last stage.

```
f{E}    = min{ f{B}+t(A,B), f{C}+t(C,E) }
        = min{ 5+11, 2+8 } = 10

f{F}    = min{ f{C}+t(C,F), f{D}+t(D,F) }
        = min{ 2+4, 3+6 } = 6

f{G}    = min{ f{C}+t(C,G), f{D}+t(D,G) }
        = min{ 2+9, 3+6} = 9
```

Note that to calculate the return of the a stage, the return of the incident state of the previous stage is utilized. One of the key characteristic of DP is that the return of the destined state is independent of the way in which the return of the incident state arrived. In general, the following recursive formula applied to this salesman travelling problem:

f(x1) = min{ f{x2}, t(x2,x1)}

**Stage (3) to (5):** By applying this recursive formula, the return of each state in the network can be calculated and is show in the node circles in the figure.

<u>Back tracking:</u>

The minimum total cost to city N is 19. The actual path to travel to attain this minimum cost can be found by unravelling the information contained in the state returns. To begin with it was the return of state L (15) plus the transition cost from the state L to state N which produced the return of 19 at state N. Hence state L and the arc (L,N) is on the optimal path. By the same token, it was the return at state I (12) plus the transition cost from the state I to state L that produced the return of 15 at state L. Hence state I and the arc (I,L) is on the optimal path. Applying this back tracking technique, the optimal path is then <A,C,E,I,L,N>.

There are 19 possible routes from A to N. It is possible to evaluate them all and chosen the least cost. The above approach involves less calculation and the reduction of effort becomes even more significant when the network size increases.

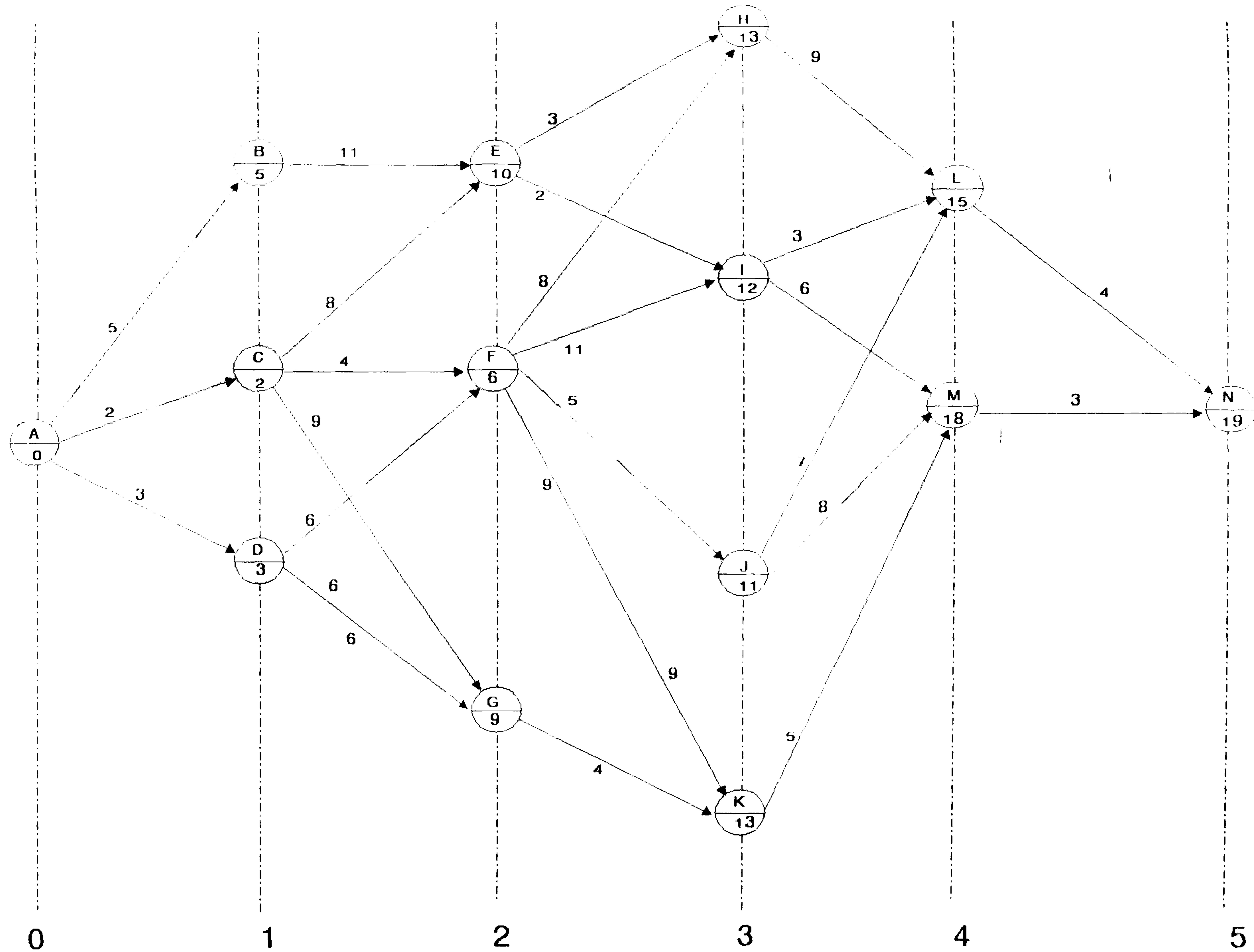


Fig. C.1 Dynamic Programming Example Showing Minimum Cost at each State

## APPENDIX - D

## 22-UNIT TEST SYSTEM DATA

The 22-unit test network data is extracted from the book "Power System Control" by M.J.H. Sterling[196], 1978.

### D.1 System Data

No.of active nodes = 10
No. of active generators = 22
No. of active nodes = 14
No. of generator groups = 7

### D.2 Generator Data

Note: Generator operating cost function is modelled as quadratic, i.e. $F(P) = a + bP + cP^2$

| Gen | Node | Statn | Upper Limit | Lower Limit | Ramp Up | Ramp Down | Cost *a* | Cost *b* | Cost *c* |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0.60 | 0.10 | 0.02 | 0.03 | 0.00 | 190. | 100.0 |
| 2 | 1 | 1 | 0.60 | 0.10 | 0.02 | 0.03 | 0.00 | 190. | 100.0 |
| 3 | 1 | 1 | 0.60 | 0.10 | 0.02 | 0.03 | 0.00 | 200. | 100.0 |
| 4 | 1 | 1 | 0.60 | 0.10 | 0.02 | 0.03 | 0.00 | 200. | 100.0 |
| 5 | 1 | 1 | 0.60 | 0.10 | 0.02 | 0.03 | 0.00 | 190. | 150.0 |
| 6 | 1 | 1 | 0.60 | 0.10 | 0.02 | 0.03 | 0.00 | 190. | 150.0 |
| 7 | 2 | 2 | 1.00 | 0.20 | 0.01 | 0.01 | 0.00 | 200. | 0.0 |
| 8 | 2 | 2 | 1.00 | 0.20 | 0.01 | 0.01 | 0.00 | 200. | 0.0 |
| 9 | 2 | 2 | 1.00 | 0.20 | 0.01 | 0.01 | 0.00 | 200. | 0.0 |
| 10 | 2 | 2 | 1.00 | 0.20 | 0.01 | 0.01 | 0.00 | 200. | 0.0 |
| 11 | 2 | 2 | 1.00 | 0.20 | 0.01 | 0.01 | 0.00 | 200. | 0.0 |
| 12 | 2 | 2 | 1.00 | 0.20 | 0.01 | 0.01 | 0.00 | 200. | 0.0 |
| 13 | 3 | 3 | 0.60 | 0.10 | 0.02 | 0.03 | 0.00 | 200. | 100.0 |
| 14 | 3 | 3 | 0.60 | 0.10 | 0.02 | 0.03 | 0.00 | 200. | 100.0 |
| 15 | 4 | 4 | 0.50 | 0.20 | 0.03 | 0.03 | 0.00 | 210. | 0.0 |
| 16 | 6 | 5 | 0.60 | 0.10 | 0.01 | 0.02 | 0.00 | 220. | 0.0 |
| 17 | 6 | 5 | 0.60 | 0.10 | 0.01 | 0.02 | 0.00 | 220. | 0.0 |
| 18 | 7 | 6 | 0.30 | 0.05 | 0.02 | 0.02 | 0.00 | 195. | 200.0 |
| 19 | 7 | 6 | 0.30 | 0.05 | 0.02 | 0.02 | 0.00 | 195. | 200.0 |
| 20 | 7 | 6 | 0.30 | 0.05 | 0.02 | 0.02 | 0.00 | 195. | 200.0 |
| 21 | 10 | 7 | 0.60 | 0.10 | 0.01 | 0.02 | 0.00 | 200. | 50.0 |
| 22 | 10 | 7 | 0.60 | 0.10 | 0.01 | 0.02 | 0.00 | 200. | 50.0 |

## D.3 Generator Group or Station Constraints

| GROUP | Upper Limit | Lower Limit | Ramp Up Rate | Ramp Down Rate |
|---|---|---|---|---|
| 1 | 3.500 | 0.500 | 0.120 | 0.180 |
| 2 | 5.000 | 0.500 | 0.060 | 0.060 |
| 3 | 1.200 | 0.500 | 0.040 | 0.060 |
| 4 | 0.500 | 0.200 | 0.030 | 0.030 |
| 5 | 1.200 | 0.100 | 0.020 | 0.040 |
| 6 | 0.900 | 0.100 | 0.060 | 0.060 |
| 7 | 1.200 | 0.050 | 0.060 | 0.060 |

## D.4 Line Data

| Line | Node | R(p.u.) | X(p.u.) | SUS(p.u.) | P-limit |
|---|---|---|---|---|---|
| 1 | 1- 2 | 0.0030 | 0.0280 | 0.0000 | 6.200 |
| 2 | 1- 3 | 0.0590 | 0.1510 | 0.0000 | 1.000 |
| 3 | 1- 5 | 0.1430 | 0.3640 | 0.0000 | 1.000 |
| 4 | 1- 9 | 0.0440 | 0.1120 | 0.0000 | 1.000 |
| 5 | 1-10 | 0.0290 | 0.0730 | 0.0000 | 1.000 |
| 6 | 2- 3 | 0.0010 | 0.0100 | 0.0000 | 6.200 |
| 7 | 3- 4 | 0.0040 | 0.0320 | 0.0000 | 6.200 |
| 8 | 4- 5 | 0.0050 | 0.0420 | 0.0000 | 6.200 |
| 9 | 5- 6 | 0.0550 | 0.1400 | 0.0000 | 1.000 |
| 10 | 5- 7 | 0.0730 | 0.1850 | 0.0000 | 1.000 |
| 11 | 6- 7 | 0.1320 | 0.3360 | 0.0000 | 1.000 |
| 12 | 7- 8 | 0.0290 | 0.0730 | 0.0000 | 1.000 |
| 13 | 8- 9 | 0.0330 | 0.0840 | 0.0000 | 1.000 |
| 14 | 9-10 | 0.0330 | 0.0840 | 0.0000 | 1.000 |

## D.5 Estimated Nodal Loadings at Target Time (Not Including Losses)

| Node | P (p.u) | Q (p.u) |
|---|---|---|
| 1 | 2.0000 | 1.0000 |
| 2 | 1.0000 | 0.5000 |
| 3 | 1.5000 | 0.7500 |
| 4 | 1.0000 | 0.5000 |
| 5 | 0.5000 | 0.2500 |
| 6 | 0.5000 | 0.2500 |
| 7 | 1.0000 | 0.5000 |
| 8 | 0.5000 | 0.2500 |
| 9 | 1.0000 | 0.5000 |
| 10 | 1.0000 | 0.5000 |

Total Load Demand = 10.000 p.u.

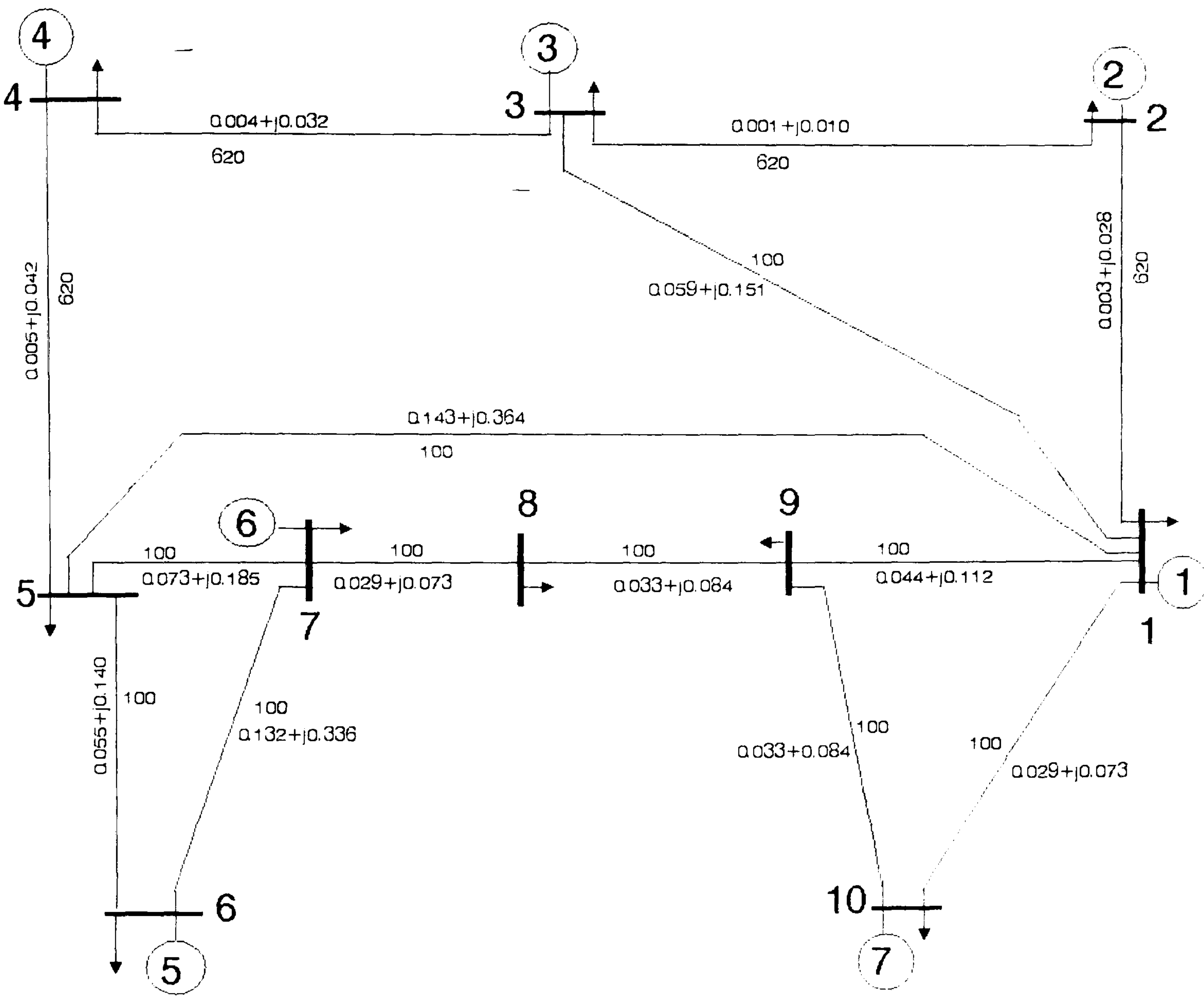


Fig. D.1 22 Unit Test Network
(extract from "Power System Control" by Sterling, 1978)

APPENDIX E

CEGB TEST NETWORK DATA

## E.1 Data for Thermal Plants

| Name | Connt'n | Bus Volt *(kv.)* | No. of *Units* | Type | OP Cost *\$/MwHr* | MVA | MW. | Unavailability *Break -down* | *Maint* |
|---|---|---|---|---|---|---|---|---|---|
| DUNB | DUNG4 | 400 | 2 | N | 4.14 | 1129 | 960 | 0.15 | 0.46 |
| DUNC | DUNG4 | 400 | 1 | N | 3.87 | 1358 | 1155 | 0.15 | 0.40 |
| DUND | DUNG4 | 400 | 1 | N | 3.87 | 1358 | 1155 | 0.15 | 0.40 |
| HTPL | HATL2 | 275 | 2 | N | 4.14 | 1176 | 1000 | 0.15 | 0.40 |
| SIZB | SIZE4 | 400 | 1 | N | 3.87 | 1358 | 1155 | 0.19 | 0.48 |
| SIZC | SIZE4 | 400 | 1 | N | 3.87 | 1358 | 1155 | 0.15 | 0.40 |
| HINB | HINP4 | 400 | 2 | N | 4.14 | 1352 | 1150 | 0.15 | 0.46 |
| HINC | HINP4 | 400 | 1 | N | 3.87 | 1358 | 1155 | 0.15 | 0.40 |
| WIN1 | WINF4 | 400 | 1 | N | 3.87 | 1358 | 1155 | 0.15 | 0.40 |
| HEYA | HEYS4 | 400 | 2 | N | 4.14 | 1176 | 1000 | 0.15 | 0.46 |
| HEYB | HEYS4 | 400 | 2 | N | 4.14 | 1447 | 1230 | 0.15 | 0.46 |
| HUNT | ELVA2Q | 275 | 2 | N | 4.14 | 1223 | 1040 | 0.15 | 0.46 |
| WYLF | PENT4 | 400 | 4 | N | 9.00 | 988 | 840 | 0.12 | 0.33 |
| LONG | TORN4 | 400 | 2 | C | 26.32 | 1472 | 1252 | 0.12 | 0.28 |
| LONG | ELVA2R | 275 | 2 | C | 26.32 | 1472 | 1252 | 0.12 | 0.28 |
| KINC | ECCL4S | 400 | 3 | C | 26.19 | 441 | 375 | 0.09 | 0.18 |
| COCK | TORN4 | 400 | 4 | C | 27.00 | 1355 | 1152 | 0.21 | 0.18 |
| WTHA | WTHU2 | 275 | 2 | C | 30.96 | 442 | 376 | 0.09 | 0.18 |
| WTHB | WTHU2 | 275 | 3 | C | 29.29 | 1016 | 864 | 0.21 | 0.18 |
| ABTB | ABTH2 | 275 | 1 | C | 28.68 | 647 | 550 | 0.12 | 0.28 |
| ABTB | ABTH2 | 275 | 2 | C | 28.68 | 1011 | 860 | 0.12 | 0.28 |
| ABTA | ABTH2 | 275 | 4 | C | 30.54 | 442 | 376 | 0.09 | 0.18 |
| DCOT | DIDC4 | 400 | 4 | C | 28.66 | 2141 | 1820 | 0.12 | 0.28 |
| USKM | WHSO4 | 336 | 6 | C | 30.59 | 395 | 336 | 0.09 | 0.18 |
| CDON | WILL2 | 275 | 3 | C | 30.48 | 263 | 224 | 0.09 | 0.18 |
| CDON | WILL2 | 275 | 3 | C | 30.48 | 352 | 300 | 0.09 | 0.18 |
| COTT | COTT4 | 400 | 4 | C | 26.34 | 2164 | 1840 | 0.12 | 0.28 |
| DRB1 | DRAK2 | 275 | 1 | C | 30.09 | 131 | 112 | 0.09 | 0.18 |
| DRB2 | DRAK2 | 275 | 2 | C | 30.09 | 395 | 336 | 0.09 | 0.18 |
| DRC1 | DRAK2 | 275 | 1 | C | 29.68 | 364 | 310 | 0.55 | 0.44 |
| DRC1 | DRAK2 | 275 | 1 | C | 29.68 | 411 | 350 | 0.55 | 0.44 |
| DRC3 | DRAK2 | 275 | 2 | C | 29.68 | 647 | 550 | 0.55 | 0.44 |
| HMH1 | NECH2 | 275 | 4 | C | 32.43 | 287 | 244 | 0.09 | 0.18 |
| HMH2 | NECH2 | 275 | 2 | C | 32.43 | 215 | 183 | 0.09 | 0.18 |
| HIGM | HIGM2 | 275 | 5 | C | 28.70 | 1094 | 930 | 0.21 | 0.18 |
| IRON | IRON4 | 400 | 2 | C | 27.65 | 1081 | 919 | 0.12 | 0.28 |
| MEAB | CELL4 | 400 | 4 | C | 29.25 | 263 | 224 | 0.09 | 0.18 |

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|---|---|---|---|---|---|---|---|---|---|
| RATC | RATS4 | 400 | 4 | C | 26.31 | 2272 | 1932 | 0.12 | 0.28 |
| RUGA | BUSH2 | 275 | 3 | C | 28.90 | 395 | 336 | 0.09 | 0.18 |
| RUGA | CELL4 | 400 | 2 | C | 28.90 | 263 | 224 | 0.09 | 0.18 |
| RUGB | RUGE4 | 400 | 2 | C | 25.85 | 1082 | 920 | 0.12 | 0.28 |
| STYB | STAY4 | 400 | 1 | C | 28.31 | 131 | 112 | 0.09 | 0.18 |
| STYB | CHTE2 | 275 | 2 | C | 28.31 | 263 | 224 | 0.09 | 0.18 |
| WBUR | WBUR4 | 400 | 4 | C | 26.94 | 2164 | 1840 | 0.12 | 0.28 |
| WILL | WILL2 | 275 | 4 | C | 29.30 | 461 | 392 | 0.09 | 0.18 |
| WILL | WILL2 | 275 | 2 | C | 28.51 | 442 | 376 | 0.21 | 0.18 |
| AGEC | KEAR4 | 400 | 2 | C | 28.04 | 192 | 164 | 0.09 | 0.18 |
| AGEC | KEAR4 | 400 | 1 | C | 28.04 | 79 | 68 | 0.09 | 0.18 |
| BOLD | FIDF2 | 275 | 2 | C | 32.29 | 192 | 164 | 0.09 | 0.18 |
| CARN | DAIN4 | 400 | 1 | C | 32.71 | 282 | 240 | 0.21 | 0.18 |
| FIDF | FIDF2 | 275 | 4 | C | 26.52 | 2211 | 1880 | 0.12 | 0.28 |
| PADI | PADI4 | 400 | 1 | C | 27.87 | 263 | 224 | 0.21 | 0.18 |
| BLYA | BLYT2 | 275 | 4 | C | 28.40 | 527 | 448 | 0.09 | 0.18 |
| BLYB | BLYT2 | 275 | 2 | C | 28.59 | 729 | 620 | 0.21 | 0.18 |
| BLYB | BLYT2 | 275 | 2 | C | 30.54 | 565 | 480 | 0.21 | 0.18 |
| DRAX | DRAX4J | 400 | 3 | C | 25.55 | 2205 | 1875 | 0.12 | 0.28 |
| DRAX | DRAX4K | 400 | 3 | C | 25.55 | 2205 | 1875 | 0.12 | 0.28 |
| EGGB | EGGB4J | 400 | 2 | C | 27.09 | 1011 | 860 | 0.12 | 0.28 |
| EGGB | EGGB4K | 400 | 2 | C | 27.09 | 1011 | 860 | 0.12 | 0.28 |
| SKGR | SKLG2 | 275 | 4 | C | 29.55 | 527 | 448 | 0.09 | 0.18 |
| ELLA | ELLA2 | 275 | 3 | C | 32.71 | 197 | 168 | 0.09 | 0.18 |
| FERR | FERR4Q | 400 | 3 | C | 29.02 | 331 | 282 | 0.09 | 0.18 |
| FERR | FERR4Q | 400 | 4 | C | 26.14 | 2272 | 1932 | 0.12 | 0.28 |
| STLN | STEW4 | 400 | 4 | C | 32.71 | 263 | 224 | 0.09 | 0.18 |
| STLS | STEW4 | 400 | 5 | C | 32.71 | 352 | 300 | 0.09 | 0.18 |
| THOM | THOM2 | 275 | 2 | C | 27.80 | 1108 | 942 | 0.12 | 0.28 |
| TILB | TILB2 | 275 | 4 | C | 30.21 | 1581 | 1344 | 0.21 | 0.18 |
| KINO | KINO4 | 400 | 4 | C | 29.97 | 2258 | 1920 | 0.12 | 0.28 |
| GRAI | GRAI4 | 400 | 4 | F | 53.65 | 2969 | 2524 | 0.10 | 0.23 |
| FAWL | FAWL4 | 400 | 4 | F | 53.86 | 2272 | 1932 | 0.10 | 0.23 |
| PEMB | PEMB4 | 400 | 4 | F | 52.93 | 2235 | 1900 | 0.10 | 0.23 |
| INCB | DEES4 | 400 | 2 | F | 53.65 | 1117 | 950 | 0.10 | 0.23 |
| LITT | LITT4 | 400 | 3 | F | 54.20 | 2205 | 1875 | 0.10 | 0.23 |
| RBGH | CANT4 | 400 | 1 | F | 54.45 | 134 | 114 | 0.12 | 0.18 |
| NFLW | NFLW4Q | 400 | 1 | F | 54.45 | 201 | 171 | 0.10 | 0.23 |
| NFLW | NFLW4Q | 400 | 1 | F | 54.45 | 201 | 171 | 0.10 | 0.23 |
| NFLW | CANT4 | 400 | 2 | F | 54.45 | 268 | 228 | 0.10 | 0.23 |
| NFLW | NINF4 | 400 | 1 | F | 54.45 | 134 | 114 | 0.10 | 0.23 |
| IKIP | ELVA2Q | 275 | 2 | F | 54.45 | 755 | 642 | 0.10 | 0.23 |
| PEHD | TORN4 | 400 | 2 | F | 54.45 | 755 | 642 | 0.10 | 0.23 |
| RYEH | RYEH4Q | 400 | 2 | TG | 107.51 | 164 | 140 | 0.12 | 0.18 |
| GRAI | GRAI4 | 400 | 5 | TG | 107.51 | 170 | 145 | 0.12 | 0.18 |
| KINO | KINO4 | 400 | 4 | TG | 107.51 | 79 | 68 | 0.12 | 0.18 |
| LETC | WYMO4 | 400 | 2 | TG | 107.51 | 164 | 140 | 0.12 | 0.18 |
| LITT | LITT4 | 400 | 3 | TG | 107.51 | 123 | 105 | 0.12 | 0.18 |
| NORW | NORW4 | 400 | 2 | TG | 107.51 | 129 | 110 | 0.12 | 0.18 |
| TAYL | WISD2 | 275 | 2 | TG | 107.51 | 164 | 140 | 0.12 | 0.18 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| TILB | TILB2 | 275 | 4 | TG | 107.51 | 79 | 68 | 0.12 | 0.18 |
| WATF | ELST1J | 132 | 1 | TG | 107.51 | 82 | 70 | 0.12 | 0.18 |
| WATF | ELST1L | 132 | 2 | TG | 107.51 | 82 | 70 | 0.12 | 0.18 |
| ABTH | ABTH2 | 275 | 3 | TG | 107.51 | 59 | 51 | 0.12 | 0.18 |
| BULB | IVER2J | 275 | 4 | TG | 107.51 | 329 | 280 | 0.12 | 0.18 |
| COWS | FAWL4 | 400 | 2 | TG | 107.51 | 164 | 140 | 0.12 | 0.18 |
| DIDC | DIDC4 | 400 | 4 | TG | 107.51 | 117 | 100 | 0.12 | 0.18 |
| FAWL | FAWL4 | 400 | 4 | TG | 107.51 | 79 | 68 | 0.12 | 0.18 |
| HINP | HINP4 | 400 | 2 | TG | 107.51 | 82 | 70 | 0.12 | 0.18 |
| PEMB | PEMB4 | 400 | 4 | TG | 107.51 | 117 | 100 | 0.12 | 0.18 |
| COTT | COTT4 | 400 | 4 | TG | 107.51 | 117 | 100 | 0.12 | 0.18 |
| IRON | IRON4 | 400 | 2 | TG | 107.51 | 39 | 34 | 0.12 | 0.18 |
| LEIC | WILL2 | 275 | 2 | TG | 107.51 | 119 | 102 | 0.12 | 0.18 |
| OCKH | BUSH2 | 275 | 2 | TG | 107.51 | 164 | 140 | 0.12 | 0.18 |
| OCKH | PENN2 | 275 | 2 | TG | 107.51 | 164 | 140 | 0.12 | 0.18 |
| RATC | RATS4 | 400 | 4 | TG | 107.51 | 79 | 68 | 0.12 | 0.18 |
| RUGB | RUGE4 | 400 | 2 | TG | 107.51 | 58 | 50 | 0.12 | 0.18 |
| WBUR | WBUR4 | 400 | 4 | TG | 107.51 | 79 | 68 | 0.12 | 0.18 |
| FIDF | FIDF2 | 275 | 4 | TG | 107.51 | 79 | 68 | 0.12 | 0.18 |
| HEYS | HEYS4 | 400 | 2 | TG | 107.51 | 82 | 70 | 0.12 | 0.18 |
| INCE | DEES4 | 400 | 2 | TG | 107.51 | 58 | 50 | 0.12 | 0.18 |
| LIDR | FIDF2 | 275 | 2 | TG | 107.51 | 164 | 140 | 0.12 | 0.18 |
| DRAX | DRAX4J | 400 | 3 | TG | 107.51 | 123 | 105 | 0.12 | 0.18 |
| DRAX | DRAX4K | 400 | 3 | TG | 107.51 | 123 | 105 | 0.12 | 0.18 |
| EGGB | EGGB4J | 400 | 2 | TG | 107.51 | 39 | 34 | 0.12 | 0.18 |
| EGGB | EGGB4K | 400 | 2 | TG | 107.51 | 39 | 34 | 0.12 | 0.18 |
| FERR | FERR4Q | 400 | 4 | TG | 107.51 | 79 | 68 | 0.12 | 0.18 |
| THOM | THOM2 | 275 | 2 | TG | 107.51 | 65 | 56 | 0.12 | 0.18 |

## E.2 NETWORK DATA

| 1st End Bus | | 2nd End Bus | | R | X | U | Max. Trans. Limits | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Name | Volt | Name | Volt | In | Ohms | Rate | Normal | | Emergency | |
| | *(Kv)* | | *(Kv)* | | | % | *Sum* | *Win* | *Sum* | *Win* |
| ABHA4Q | 400 | EXET4 | 400 | 1.88 | 15.64 | 1 | 748 | 935 | 880 | 1100 |
| ABHA4Q | 400 | LAND4 | 400 | 1.77 | 14.39 | 1 | 748 | 935 | 880 | 1100 |
| ABHA4R | 400 | EXET4 | 400 | 1.89 | 15.73 | 1 | 748 | 935 | 880 | 1100 |
| ABHA4R | 400 | INDQ4 | 400 | 3.65 | 29.30 | 1 | 748 | 935 | 880 | 1100 |
| ABTH2 | 275 | CILF4 | 400 | 1.41 | 23.61 | 0 | 3400 | 4250 | 4000 | 5000 |
| ABTH2 | 275 | SWAN2 | 275 | 5.75 | 38.27 | 0 | 3400 | 4250 | 4000 | 5000 |
| ABTH2 | 275 | WHSO2 | 275 | 3.08 | 19.66 | 0 | 3400 | 4250 | 4000 | 5000 |
| ALVE4Q | 400 | INDQ4 | 400 | 3.77 | 31.41 | 1 | 1040 | 1300 | 1224 | 1530 |
| ALVE4Q | 400 | TAUN4Q | 400 | 2.78 | 23.18 | 1 | 1115 | 1394 | 1312 | 1640 |
| AMEM4Q | 400 | ECLA4 | 400 | 1.34 | 10.64 | 1 | 921 | 1151 | 1084 | 1355 |
| AMEM4R | 400 | ECLA4 | 400 | 1.34 | 10.64 | 1 | 921 | 1151 | 1084 | 1355 |
| AXMI4 | 400 | WINF4 | 400 | 2.26 | 16.34 | 1 | 1496 | 1870 | 1760 | 2200 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| BEDD2 | 275 | NFLW4S | 400 | 2.35 | 60.38 | 0 | 3400 | 4250 | 4000 | 5000 |
| BEDD2 | 275 | WWEY2 | 275 | 1.33 | 10.82 | 0 | 3400 | 4250 | 4000 | 5000 |
| BEDD4K | 400 | BEDD2 | 275 | 0.37 | 42.72 | 1 | 510 | 637 | 600 | 750 |
| BEDD4K | 400 | NFLW4T | 400 | 1.19 | 10.05 | 1 | 960 | 1200 | 1129 | 1412 |
| BLYT2 | 275 | HARK2 | 275 | 17.08 | 70.12 | 1 | 353 | 442 | 416 | 520 |
| BLYT2 | 275 | HARK2 | 275 | 17.09 | 70.35 | 1 | 353 | 442 | 416 | 520 |
| BLYT2 | 275 | NORT4 | 400 | 3.98 | 83.85 | 0 | 3400 | 4250 | 4000 | 5000 |
| BOLN4 | 400 | LOVE4 | 400 | 1.25 | 16.62 | 1 | 1496 | 1870 | 1760 | 2200 |
| BOLN4 | 400 | LOVE4 | 400 | 1.25 | 16.62 | 1 | 1496 | 1870 | 1760 | 2200 |
| BOLN4 | 400 | NINF4 | 400 | 0.99 | 13.24 | 1 | 1842 | 2303 | 2168 | 2710 |
| BOLN4 | 400 | NINF4 | 400 | 0.99 | 13.24 | 1 | 1842 | 2303 | 2168 | 2710 |
| BRAI4Q | 400 | PELH4 | 400 | 1.28 | 17.13 | 1 | 1842 | 2303 | 2168 | 2710 |
| BRAI4Q | 400 | RAYL4 | 400 | 0.61 | 8.17 | 1 | 1842 | 2303 | 2168 | 2710 |
| BRAI4R | 400 | BRFO4 | 400 | 0.95 | 12.65 | 1 | 1842 | 2303 | 2168 | 2710 |
| BRAI4R | 400 | RAYL4 | 400 | 0.61 | 8.17 | 1 | 1842 | 2303 | 2168 | 2710 |
| BRAW2 | 275 | ELLA2 | 275 | 1.16 | 8.95 | 0 | 3400 | 4250 | 4000 | 5000 |
| BRAW2 | 275 | FERR4Q | 400 | 0.52 | 93.67 | 0 | 3400 | 4250 | 4000 | 5000 |
| BRAW2 | 275 | OSBA2 | 275 | 15.61 | 69.15 | 0 | 3400 | 4250 | 4000 | 5000 |
| BRAW2 | 275 | PADI4 | 400 | 1.51 | 41.24 | 1 | 510 | 637 | 600 | 750 |
| BRAW2 | 275 | SKLG2 | 275 | 1.84 | 43.91 | 0 | 3400 | 4250 | 4000 | 5000 |
| BRFO4 | 400 | NORW4 | 400 | 1.18 | 15.75 | 1 | 924 | 1156 | 1088 | 1360 |
| BRLE4 | 400 | DIDC4 | 400 | 0.82 | 11.00 | 1 | 1496 | 1870 | 1760 | 2200 |
| BRLE4 | 400 | DIDC4 | 400 | 0.82 | 11.00 | 1 | 1496 | 1870 | 1760 | 2200 |
| BRLE4 | 400 | FLEE4 | 400 | 0.34 | 4.50 | 1 | 1496 | 1870 | 1760 | 2200 |
| BRLE4 | 400 | FLEE4 | 400 | 0.34 | 4.50 | 1 | 1496 | 1870 | 1760 | 2200 |
| BRLE4 | 400 | MELK4 | 400 | 2.62 | 24.87 | 1 | 1258 | 1572 | 1480 | 1850 |
| BRLE4 | 400 | MELK4 | 400 | 2.62 | 24.87 | 1 | 1258 | 1572 | 1480 | 1850 |
| BRWA4Q | 400 | DUM24 | 400 | 0.58 | 5.55 | 1 | 1264 | 1581 | 1488 | 1860 |
| BRWA4R | 400 | DUM24 | 400 | 0.58 | 5.55 | 1 | 1264 | 1581 | 1488 | 1860 |
| HINP4 | 400 | DUM24 | 400 | 0.01 | 0.01 | 0 | 3400 | 4250 | 4000 | 5000 |
| BURW4 | 400 | PELH4 | 400 | 0.77 | 10.26 | 1 | 1842 | 2303 | 2168 | 2710 |
| BURW4 | 400 | WALP4 | 400 | 0.99 | 13.17 | 1 | 1842 | 2303 | 2168 | 2710 |
| BUSH2 | 275 | DRAK2 | 275 | 2.34 | 20.31 | 0 | 3400 | 4250 | 4000 | 5000 |
| BUSH2 | 275 | FECK2 | 275 | 6.82 | 76.16 | 0 | 3400 | 4250 | 4000 | 5000 |
| BUSH2 | 275 | NECH2 | 275 | 4.35 | 59.20 | 0 | 3400 | 4250 | 4000 | 5000 |
| BUSH2 | 275 | PENN2 | 275 | 1.51 | 12.46 | 0 | 3400 | 4250 | 4000 | 5000 |
| CANT4 | 400 | KEMS4J | 400 | 0.91 | 8.66 | 1 | 1258 | 1572 | 1480 | 1850 |
| CANT4 | 400 | KEMS4K | 400 | 0.91 | 8.66 | 1 | 1258 | 1572 | 1480 | 1850 |
| CANT4 | 400 | SELL4 | 400 | 0.87 | 8.22 | 1 | 1258 | 1572 | 1480 | 1850 |
| CANT4 | 400 | SELL4 | 400 | 0.87 | 8.22 | 1 | 1258 | 1572 | 1480 | 1850 |
| CELL4 | 400 | DRAK4 | 400 | 1.73 | 13.67 | 1 | 921 | 1151 | 1084 | 1355 |
| CELL4 | 400 | STSB4 | 400 | 3.13 | 27.64 | 1 | 748 | 935 | 880 | 1100 |
| CELL4 | 400 | WILL4 | 400 | 2.39 | 18.90 | 1 | 748 | 935 | 880 | 1100 |
| CHIC4 | 400 | DUM34 | 400 | 1.46 | 19.41 | 1 | 1496 | 1870 | 1760 | 2200 |
| CHIC4 | 400 | WINF4 | 400 | 0.47 | 6.27 | 1 | 1496 | 1870 | 1760 | 2200 |
| CHTE2 | 275 | HIGM2 | 275 | 0.84 | 10.80 | 0 | 3400 | 4250 | 4000 | 5000 |
| CHTE2 | 275 | NEEP2 | 275 | 1.59 | 18.20 | 0 | 3400 | 4250 | 4000 | 5000 |
| CHTE2 | 275 | THOM2 | 275 | 2.40 | 39.87 | 0 | 3400 | 4250 | 4000 | 5000 |
| CHTE2 | 275 | THOM4 | 400 | -1.59 | 71.19 | 0 | 3400 | 4250 | 4000 | 5000 |
| CILF4 | 400 | PEMB4 | 400 | 2.80 | 37.31 | 1 | 1884 | 2355 | 2216 | 2771 |

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|---|---|---|---|---|---|---|---|---|---|---|
| CILF4 | 400 | PEMB4 | 400 | 2.76 | 36.81 | 1 | 1884 | 2355 | 2216 | 2771 |
| CILF4 | 400 | WALH4 | 400 | 1.94 | 26.01 | 1 | 754 | 943 | 888 | 1110 |
| CILF4 | 400 | WHSO4Q | 400 | 0.77 | 10.24 | 1 | 1884 | 2355 | 2216 | 2771 |
| CITR4 | 400 | NFLW4Q | 400 | 1.34 | 11.10 | 1 | 980 | 1225 | 1153 | 1442 |
| CITR4 | 400 | NFLW4R | 400 | 1.34 | 11.10 | 1 | 980 | 1225 | 1153 | 1442 |
| CITR4 | 400 | SJOW2 | 275 | 0.28 | 22.23 | 0 | 3400 | 4250 | 4000 | 5000 |
| COTT4 | 400 | GREN4 | 400 | 4.02 | 38.95 | 1 | 1292 | 1615 | 1520 | 1900 |
| COTT4 | 400 | STAY4 | 400 | 2.38 | 7.81 | 1 | 897 | 1122 | 1056 | 1320 |
| COTT4 | 400 | THOM4 | 400 | 1.31 | 16.43 | 1 | 1638 | 2048 | 1928 | 2410 |
| COWL4 | 400 | DIDC4 | 400 | 0.22 | 2.92 | 1 | 1496 | 1870 | 1760 | 2200 |
| COWL4 | 400 | DIDC4 | 400 | 0.22 | 2.92 | 1 | 1496 | 1870 | 1760 | 2200 |
| COWL4 | 400 | ECLA4 | 400 | 0.70 | 9.32 | 1 | 1496 | 1870 | 1760 | 2200 |
| COWL4 | 400 | MITY4 | 400 | 0.38 | 29.06 | 1 | 754 | 943 | 888 | 1110 |
| COWL4 | 400 | SUND4 | 400 | 1.27 | 16.88 | 1 | 1496 | 1870 | 1760 | 2200 |
| COWL4 | 400 | WALH4 | 400 | 1.99 | 24.31 | 1 | 748 | 935 | 880 | 1100 |
| CREB4 | 400 | KEAD4 | 400 | 1.43 | 12.02 | 1 | 921 | 1151 | 1084 | 1355 |
| CREB4 | 400 | KEAD4 | 400 | 1.43 | 12.02 | 1 | 921 | 1151 | 1084 | 1355 |
| CREB4 | 400 | NORT4 | 400 | 2.35 | 31.36 | 1 | 1842 | 2303 | 2168 | 2710 |
| DAIN4 | 400 | CELL4 | 400 | 2.11 | 16.70 | 1 | 1040 | 1300 | 1224 | 1530 |
| DAIN4 | 400 | DEES4 | 400 | 2.32 | 18.33 | 1 | 1115 | 1394 | 1312 | 1640 |
| DAIN4 | 400 | DEES4 | 400 | 2.32 | 18.33 | 1 | 1115 | 1394 | 1312 | 1640 |
| DAIN4 | 400 | KEAR4 | 400 | 0.50 | 4.78 | 1 | 1264 | 1581 | 1488 | 1860 |
| DEES4 | 400 | FIDF2 | 275 | 0.38 | 17.60 | 0 | 3400 | 4250 | 4000 | 5000 |
| DEES4 | 400 | LEGA4 | 400 | 1.02 | 8.24 | 1 | 1040 | 1300 | 1224 | 1530 |
| DEES4 | 400 | PENT4 | 400 | 1.52 | 20.20 | 1 | 1842 | 2303 | 2168 | 2710 |
| DEES4 | 400 | PENT4 | 400 | 1.52 | 20.20 | 1 | 1842 | 2303 | 2168 | 2710 |
| DEES4 | 400 | TRAW4 | 400 | 3.02 | 23.89 | 1 | 1115 | 1394 | 1312 | 1640 |
| DRAK2 | 275 | NECH2 | 275 | 0.82 | 11.83 | 0 | 3400 | 4250 | 4000 | 5000 |
| DRAK4 | 400 | DRAK2 | 275 | 0.34 | 25.64 | 1 | 510 | 637 | 600 | 750 |
| DRAK4 | 400 | DRAK2 | 275 | 0.35 | 25.80 | 1 | 510 | 637 | 600 | 750 |
| DRAK4 | 400 | HAMH4 | 400 | 0.95 | 8.99 | 1 | 1258 | 1572 | 1480 | 1850 |
| DRAK4 | 400 | RATS4 | 400 | 1.27 | 10.06 | 1 | 921 | 1151 | 1084 | 1355 |
| DRAK4 | 400 | RUGE4 | 400 | 0.80 | 6.34 | 1 | 1040 | 1300 | 1224 | 1530 |
| DRAX4J | 400 | CREB4 | 400 | 0.99 | 13.25 | 1 | 1842 | 2303 | 2168 | 2710 |
| DRAX4J | 400 | DRAX4K | 400 | 0.32 | 32.00 | 1 | 1360 | 1700 | 1600 | 2000 |
| DRAX4J | 400 | EGGB4J | 400 | 0.21 | 2.83 | 1 | 1496 | 1870 | 1760 | 2200 |
| DRAX4J | 400 | KEAD4 | 400 | 0.67 | 6.70 | 1 | 1842 | 2303 | 2168 | 2710 |
| DRAX4K | 400 | EGGB4K | 400 | 0.20 | 2.68 | 1 | 1842 | 2303 | 2168 | 2710 |
| DRAX4K | 400 | OSBA4Q | 400 | 0.75 | 10.03 | 1 | 1842 | 2303 | 2168 | 2710 |
| DRAX4K | 400 | THOM4 | 400 | 0.40 | 4.64 | 1 | 1795 | 2244 | 2112 | 2640 |
| DUNG4 | 400 | NINF4 | 400 | 0.85 | 11.35 | 1 | 1842 | 2303 | 2168 | 2710 |
| DUNG4 | 400 | NINF4 | 400 | 0.85 | 11.35 | 1 | 1842 | 2303 | 2168 | 2710 |
| DUNG4 | 400 | SELL4 | 400 | 0.88 | 7.56 | 1 | 921 | 1151 | 1084 | 1355 |
| DUNG4 | 400 | SELL4 | 400 | 0.88 | 7.56 | 1 | 921 | 1151 | 1084 | 1355 |
| ECLA4 | 400 | ENDE4 | 400 | 2.70 | 26.04 | 1 | 1258 | 1572 | 1480 | 1850 |
| ECLA4 | 400 | RATS4 | 400 | 3.40 | 35.39 | 1 | 1258 | 1572 | 1480 | 1850 |
| ECLA4 | 400 | SUND4 | 400 | 0.56 | 7.41 | 1 | 1496 | 1870 | 1760 | 2200 |
| EGGB4J | 400 | ROCH2 | 275 | 2.67 | 50.72 | 1 | 647 | 809 | 761 | 952 |
| EGGB4J | 400 | STSB4 | 400 | 0.86 | 11.46 | 1 | 1842 | 2303 | 2168 | 2710 |
| EGGB4K | 400 | FERR4Q | 400 | 0.26 | 3.45 | 1 | 1842 | 2303 | 2168 | 2710 |

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|---|---|---|---|---|---|---|---|---|---|---|
| EGGB4K | 400 | PADI4 | 400 | 1.89 | 25.24 | 1 | 1842 | 2303 | 2168 | 2710 |
| EGGB4K | 400 | THOM4 | 400 | 0.54 | 7.20 | 1 | 1496 | 1870 | 1760 | 2200 |
| ELLA2 | 275 | FERR4Q | 400 | -4.68 | 158.57 | 0 | 3400 | 4250 | 4000 | 5000 |
| ELLA2 | 275 | SKLG2 | 275 | 3.95 | 27.16 | 0 | 3400 | 4250 | 4000 | 5000 |
| ELLA2 | 275 | STAL2 | 275 | 2.41 | 25.78 | 1 | 761 | 952 | 896 | 1120 |
| ELST2 | 275 | IVER2J | 275 | 0.80 | 6.46 | 0 | 3400 | 4250 | 4000 | 5000 |
| ELST2 | 275 | SUND4 | 400 | 1.67 | 36.15 | 1 | 510 | 637 | 600 | 750 |
| ELST2 | 275 | SUND4 | 400 | 1.67 | 36.15 | 1 | 510 | 637 | 600 | 750 |
| ELST2 | 275 | TILB2 | 275 | 2.26 | 23.75 | 0 | 3400 | 4250 | 4000 | 5000 |
| ELVA2Q | 275 | HARK2 | 275 | 14.21 | 49.79 | 0 | 3400 | 4250 | 4000 | 5000 |
| ELVA2R | 275 | HARK2 | 275 | 8.54 | 65.79 | 0 | 3400 | 4250 | 4000 | 5000 |
| DUM34 | 400 | AXMI4 | 400 | 0.70 | 9.28 | 1 | 1496 | 1870 | 1760 | 2200 |
| EXET4 | 400 | DUM34 | 400 | 0.01 | 0.01 | 0 | 3400 | 4250 | 4000 | 5000 |
| EXET4 | 400 | BRWA4Q | 400 | 1.78 | 18.83 | 1 | 1638 | 2048 | 1928 | 2410 |
| EXET4 | 400 | BRWA4R | 400 | 1.78 | 18.83 | 1 | 1638 | 2048 | 1928 | 2410 |
| MANN4 | 400 | DUM54 | 400 | 0.01 | 0.01 | 0 | 3400 | 4250 | 4000 | 5000 |
| FAWL4 | 400 | DUM54 | 400 | 1.20 | 16.03 | 1 | 1496 | 1870 | 1760 | 2200 |
| FAWL4 | 400 | NURS4 | 400 | 0.42 | 5.64 | 1 | 1496 | 1870 | 1760 | 2200 |
| FECK2 | 275 | NECH2 | 275 | 4.33 | 56.70 | 0 | 3400 | 4250 | 4000 | 5000 |
| FECK2 | 275 | PENN2 | 275 | 5.42 | 48.68 | 0 | 3400 | 4250 | 4000 | 5000 |
| FECK2 | 275 | WILL2 | 275 | 9.61 | 88.19 | 0 | 3400 | 4250 | 4000 | 5000 |
| FECK4 | 400 | FECK2 | 275 | 0.29 | 25.77 | 1 | 680 | 850 | 800 | 1000 |
| FECK4 | 400 | HAMH4 | 400 | 1.40 | 13.25 | 1 | 1264 | 1581 | 1488 | 1860 |
| FECK4 | 400 | IRON4 | 400 | 2.50 | 19.77 | 1 | 1040 | 1300 | 1224 | 1530 |
| FECK4 | 400 | MELK4 | 400 | 3.43 | 32.58 | 1 | 1258 | 1572 | 1480 | 1850 |
| FECK4 | 400 | WALH4 | 400 | 2.18 | 20.71 | 1 | 1258 | 1572 | 1480 | 1850 |
| FIDF2 | 275 | PEWO4 | 400 | 0.93 | 31.08 | 0 | 3400 | 4250 | 4000 | 5000 |
| FLEE4 | 400 | LOVE4 | 400 | 1.25 | 11.85 | 1 | 1264 | 1581 | 1488 | 1860 |
| FLEE4 | 400 | LOVE4 | 400 | 1.25 | 11.85 | 1 | 1264 | 1581 | 1488 | 1860 |
| GRAI4 | 400 | KEMS4J | 400 | 0.17 | 2.21 | 1 | 1496 | 1870 | 1760 | 2200 |
| GRAI4 | 400 | KEMS4K | 400 | 0.17 | 2.21 | 1 | 1496 | 1870 | 1760 | 2200 |
| GRAI4 | 400 | KINO4 | 400 | 0.22 | 3.01 | 1 | 1496 | 1870 | 1760 | 2200 |
| GRAI4 | 400 | TILB4K | 400 | 0.54 | 7.21 | 1 | 1496 | 1870 | 1760 | 2200 |
| GREN4 | 400 | SUND4 | 400 | 1.26 | 12.01 | 1 | 1264 | 1581 | 1488 | 1860 |
| GREN4 | 400 | SUND4 | 400 | 1.26 | 12.01 | 1 | 1264 | 1581 | 1488 | 1860 |
| GREN4 | 400 | WBUR4 | 400 | 4.23 | 40.94 | 1 | 1292 | 1615 | 1520 | 1900 |
| HAMH4 | 400 | NECH2 | 275 | 0.72 | 31.90 | 1 | 647 | 809 | 761 | 952 |
| HARK1L | 132 | HARK2 | 275 | 3.01 | 133.33 | 1 | 122 | 153 | 144 | 180 |
| HATL2 | 275 | BLYT2 | 275 | 4.40 | 33.60 | 0 | 3400 | 4250 | 4000 | 5000 |
| HATL2 | 275 | NORT4 | 400 | 0.71 | 14.12 | 0 | 3400 | 4250 | 4000 | 5000 |
| HEYS4 | 400 | HARK2 | 275 | 2.52 | 35.21 | 0 | 3400 | 4250 | 4000 | 5000 |
| HEYS4 | 400 | PEWO4 | 400 | 0.43 | 3.50 | 0 | 3400 | 4250 | 4000 | 5000 |
| HIGM2 | 275 | THOM2 | 275 | 20.78 | 104.86 | 0 | 3400 | 4250 | 4000 | 5000 |
| HIGM4 | 400 | HIGM2 | 275 | 0.27 | 25.60 | 1 | 647 | 809 | 761 | 952 |
| HIGM4 | 400 | HIGM2 | 275 | 0.69 | 54.53 | 1 | 340 | 425 | 400 | 500 |
| HIGM4 | 400 | RATS4 | 400 | 2.48 | 19.64 | 1 | 921 | 1151 | 1084 | 1355 |
| HIGM4 | 400 | WBUR4 | 400 | 4.37 | 4.13 | 1 | 921 | 1151 | 1084 | 1355 |
| HINP4 | 400 | DUM14 | 400 | 0.01 | 0.01 | 0 | 3400 | 4250 | 4000 | 5000 |
| DUM14 | 400 | MELK4 | 400 | 2.77 | 26.31 | 1 | 1264 | 1581 | 1488 | 1860 |
| DUM14 | 400 | MELK4 | 400 | 2.77 | 26.31 | 1 | 1264 | 1581 | 1488 | 1860 |

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|---|---|---|---|---|---|---|---|---|---|---|
| HINP4 | 400 | TAUN4Q | 400 | 0.87 | 8.22 | 1 | 1264 | 1581 | 1488 | 1860 |
| HINP4 | 400 | TAUN4R | 400 | 0.87 | 8.22 | 1 | 1264 | 1581 | 1488 | 1860 |
| INDQ4 | 400 | TAUN4R | 400 | 6.55 | 54.60 | 1 | 1040 | 1300 | 1224 | 1530 |
| IROA2 | 275 | IROA4Q | 400 | 0.27 | 25.60 | 1 | 680 | 850 | 800 | 1000 |
| IROA2 | 275 | MELK4 | 400 | 3.09 | 23.91 | 0 | 3400 | 4250 | 4000 | 5000 |
| IROA2 | 275 | WHSO2 | 275 | 3.28 | 15.10 | 0 | 3400 | 4250 | 4000 | 5000 |
| IROA4Q | 400 | CILF4 | 400 | 1.45 | 19.53 | 1 | 1700 | 2125 | 2000 | 2500 |
| IROA4Q | 400 | MELK4 | 400 | 0.58 | 7.68 | 1 | 1788 | 2235 | 2104 | 2630 |
| IRON4 | 400 | PENN2 | 275 | 1.34 | 34.08 | 1 | 647 | 809 | 761 | 952 |
| IRON4 | 400 | PENN2 | 275 | 1.34 | 34.08 | 1 | 647 | 809 | 761 | 952 |
| IRON4 | 400 | RUGE4 | 400 | 2.25 | 17.81 | 1 | 921 | 1151 | 1084 | 1355 |
| IVER2J | 200 | AMEM4Q | 400 | 1.76 | 49.20 | 0 | 3400 | 4250 | 4000 | 5000 |
| IVER2J | 200 | AMEM4R | 400 | 2.05 | 50.55 | 0 | 3400 | 4250 | 4000 | 5000 |
| IVER2K | 200 | AMEM4R | 400 | 2.05 | 50.55 | 0 | 3400 | 4250 | 4000 | 5000 |
| IVER2K | 200 | WWEY2 | 275 | 2.07 | 16.29 | 1 | 516 | 646 | 608 | 760 |
| KEAD4 | 400 | WBUR4 | 400 | 0.52 | 4.77 | 1 | 1842 | 2303 | 2168 | 2710 |
| KEAD4 | 400 | WBUR4 | 400 | 0.52 | 6.98 | 1 | 1496 | 1870 | 1760 | 2200 |
| KEAR4 | 400 | PADI4 | 400 | 1.02 | 10.79 | 1 | 1264 | 1581 | 1488 | 1860 |
| KEMS4J | 400 | NFLW4S | 400 | 1.14 | 11.22 | 1 | 1258 | 1572 | 1480 | 1850 |
| KEMS4K | 400 | NFLW4T | 400 | 1.14 | 11.28 | 1 | 1258 | 1572 | 1480 | 1850 |
| KINO4 | 400 | NFLW4Q | 400 | 0.57 | 6.56 | 1 | 1115 | 1394 | 1312 | 1640 |
| KINO4 | 400 | NFLW4R | 400 | 0.57 | 6.56 | 1 | 1115 | 1394 | 1312 | 1640 |
| KINO4 | 400 | TILB4J | 400 | 0.32 | 4.26 | 1 | 1496 | 1870 | 1760 | 2200 |
| LALE2 | 275 | AMEM4Q | 400 | 3.87 | 63.36 | 1 | 340 | 425 | 400 | 500 |
| LALE2 | 275 | BRLE4 | 400 | 5.67 | 64.69 | 0 | 3400 | 4250 | 4000 | 5000 |
| LALE2 | 275 | WWEY2 | 275 | -2.75 | 109.29 | 0 | 3400 | 4250 | 4000 | 5000 |
| LAND4 | 400 | INDQ4 | 400 | 1.89 | 14.97 | 1 | 748 | 935 | 880 | 1100 |
| LEGA4 | 400 | IRON4 | 400 | 2.39 | 18.92 | 1 | 1040 | 1300 | 1224 | 1530 |
| LEGA4 | 400 | IRON4 | 400 | 2.39 | 18.92 | 1 | 1040 | 1300 | 1224 | 1530 |
| LEGA4 | 400 | TRAW4 | 400 | 2.93 | 23.43 | 1 | 1040 | 1300 | 1224 | 1530 |
| LITT4 | 400 | NFLW4S | 400 | 0.30 | 2.40 | 1 | 1040 | 1300 | 1224 | 1530 |
| LITT4 | 400 | NFLW4T | 400 | 0.30 | 2.40 | 1 | 1040 | 1300 | 1224 | 1530 |
| LOVE4 | 400 | FAWL4 | 400 | 0.49 | 6.16 | 1 | 754 | 943 | 888 | 1110 |
| LOVE4 | 400 | FAWL4 | 400 | 0.49 | 6.16 | 1 | 754 | 943 | 888 | 1110 |
| LOVE4 | 400 | DUM54 | 400 | 1.53 | 20.46 | 1 | 1509 | 1887 | 1776 | 2220 |
| LOVE4 | 400 | NURS4 | 400 | 0.77 | 10.30 | 1 | 1496 | 1870 | 1760 | 2200 |
| DUM44 | 400 | MANN4 | 400 | 0.74 | 9.93 | 1 | 1496 | 1870 | 1760 | 2200 |
| DUM44 | 400 | MANN4 | 400 | 0.74 | 9.93 | 1 | 1496 | 1870 | 760 | 2200 |
| WINF4 | 400 | DUM44 | 400 | 0.01 | 0.01 | 0 | 3400 | 4250 | 4000 | 5000 |
| MELK4 | 400 | MITY4 | 400 | 0.90 | 8.53 | 1 | 1258 | 1572 | 1480 | 1850 |
| MELK4 | 400 | WHSO4Q | 400 | 1.26 | 16.96 | 1 | 1700 | 2125 | 2000 | 2500 |
| NECH2 | 275 | WILL2 | 275 | 3.26 | 53.52 | 0 | 3400 | 4250 | 4000 | 5000 |
| NEEP2 | 275 | STSB4 | 400 | 0.83 | 29.70 | 1 | 510 | 637 | 600 | 750 |
| NEEP2 | 275 | THOM2 | 275 | 2.14 | 19.60 | 0 | 3400 | 4250 | 4000 | 5000 |
| NFLW4Q | 400 | NFLW4R | 400 | -0.49 | 85.42 | 0 | 3400 | 4250 | 4000 | 5000 |
| NORT4 | 400 | OSBA4Q | 400 | 2.11 | 28.14 | 1 | 1842 | 2303 | 2168 | 2710 |
| NORW4 | 400 | WALP4 | 400 | 3.07 | 25.60 | 1 | 921 | 1151 | 1084 | 1355 |
| NORW4 | 400 | WALP4 | 400 | 3.07 | 25.60 | 1 | 921 | 1151 | 1084 | 1355 |
| OSBA2 | 275 | FERR4Q | 400 | 11.05 | 64.38 | 0 | 3400 | 4250 | 4000 | 5000 |
| OSBA4Q | 400 | OSBA2 | 275 | 0.34 | 25.60 | 1 | 510 | 637 | 600 | 750 |

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|---|---|---|---|---|---|---|---|---|---|---|
| PADI4 | 400 | PEWO4 | 400 | 0.97 | 10.36 | 1 | 1264 | 1581 | 1488 | 1860 |
| PELH4 | 400 | RYEH4Q | 400 | 0.48 | 6.47 | 1 | 1842 | 2303 | 2168 | 2710 |
| PELH4 | 400 | RYEH4R | 400 | 0.48 | 6.47 | 1 | 1842 | 2303 | 2168 | 2710 |
| PELH4 | 400 | SUND4 | 400 | 0.86 | 11.49 | 1 | 1842 | 2303 | 2168 | 2710 |
| PELH4 | 400 | WALP4 | 400 | 1.76 | 23.42 | 1 | 1842 | 2303 | 2168 | 2710 |
| PEMB4 | 400 | SWAN4 | 400 | 1.61 | 21.42 | 1 | 1496 | 1870 | 1760 | 2200 |
| PEMB4 | 400 | WALH4 | 400 | 4.22 | 56.42 | 1 | 748 | 935 | 880 | 1100 |
| PENT4 | 400 | TRAW4 | 400 | 0.95 | 13.03 | 1 | 748 | 935 | 880 | 1100 |
| PEWO4 | 400 | DAIN4 | 400 | 1.88 | 17.84 | 1 | 1264 | 1581 | 1488 | 1860 |
| PEWO4 | 400 | HARK2 | 275 | 9.40 | 114.87 | 0 | 3400 | 4250 | 4000 | 5000 |
| RATS4 | 400 | ENDE4 | 400 | 0.71 | 9.46 | 1 | 1496 | 1870 | 1760 | 2200 |
| RATS4 | 400 | STAY4 | 400 | 0.17 | 13.68 | 1 | 897 | 1122 | 1056 | 1320 |
| RATS4 | 400 | WILL4 | 400 | 0.84 | 6.65 | 1 | 921 | 1151 | 1084 | 1355 |
| RAYL4 | 400 | TILB4J | 400 | 0.79 | 6.22 | 1 | 921 | 1151 | 1084 | 1355 |
| RAYL4 | 400 | TILB4K | 400 | 0.80 | 6.35 | 1 | 921 | 1151 | 1084 | 1355 |
| ROCH2 | 275 | KEAR4 | 400 | 0.89 | 21.39 | 0 | 3400 | 4250 | 4000 | 5000 |
| ROCH2 | 275 | STAL2 | 275 | 1.91 | 20.41 | 1 | 870 | 1088 | 1024 | 1280 |
| RYEH4Q | 400 | WALX2 | 275 | 0.34 | 25.60 | 0 | 3400 | 4250 | 4000 | 5000 |
| RYEH4Q | 400 | WALX2 | 275 | 0.34 | 25.60 | 0 | 3400 | 4250 | 4000 | 5000 |
| RYEH4R | 400 | WALX2 | 275 | 0.34 | 25.60 | 0 | 3400 | 4250 | 4000 | 5000 |
| RYEH4R | 400 | WALX2 | 275 | 0.34 | 25.60 | 0 | 3400 | 4250 | 4000 | 5000 |
| SIZE4 | 400 | BRFO4 | 400 | 0.82 | 10.93 | 1 | 2080 | 2601 | 2448 | 3060 |
| SIZE4 | 400 | BRFO4 | 400 | 0.82 | 10.93 | 1 | 2080 | 2601 | 2448 | 3060 |
| SIZE4 | 400 | NORW4 | 400 | 2.00 | 26.66 | 1 | 2080 | 2601 | 2448 | 3060 |
| SIZE4 | 400 | PELH4 | 400 | 2.15 | 28.66 | 1 | 1842 | 2303 | 2168 | 2710 |
| SKLG2 | 275 | FERR4Q | 400 | 3.72 | 37.87 | 0 | 3400 | 4250 | 4000 | 5000 |
| STAL2 | 275 | DAIN4 | 400 | 1.71 | 27.00 | 0 | 3400 | 4250 | 4000 | 5000 |
| STAL2 | 275 | THOM4 | 400 | 1.67 | 43.68 | 1 | 647 | 809 | 761 | 952 |
| STEW4 | 400 | BLYT2 | 275 | 1.05 | 21.82 | 0 | 3400 | 4250 | 4000 | 5000 |
| STEW4 | 400 | ECCL4S | 400 | 3.95 | 32.96 | 1 | 921 | 1151 | 1084 | 1355 |
| STEW4 | 400 | NORT4 | 400 | 1.25 | 16.94 | 1 | 1638 | 2048 | 1928 | 2410 |
| STEW4 | 400 | NORT4 | 400 | 2.30 | 33.16 | 0 | 3400 | 4250 | 4000 | 5000 |
| STEW4 | 400 | TORN4 | 400 | 4.69 | 43.47 | 1 | 921 | 1151 | 1084 | 1355 |
| SUND4 | 400 | WYMO4 | 400 | 0.35 | 4.65 | 1 | 1842 | 2303 | 2168 | 2710 |
| SWAN4 | 400 | CILF4 | 400 | 1.17 | 15.66 | 1 | 1496 | 1870 | 1760 | 2200 |
| SWAN4 | 400 | SWAN2 | 275 | 0.34 | 25.60 | 1 | 510 | 637 | 600 | 750 |
| SWAN4 | 400 | SWAN2 | 275 | 0.34 | 25.60 | 1 | 510 | 637 | 600 | 750 |
| THOM2 | 275 | THOM4 | 400 | 0.34 | 25.65 | 1 | 510 | 637 | 600 | 750 |
| THOM2 | 275 | THOM4 | 400 | 0.35 | 25.71 | 1 | 510 | 637 | 600 | 750 |
| TILB4J | 400 | TILB2 | 275 | 0.34 | 25.60 | 1 | 510 | 637 | 600 | 750 |
| TILB4K | 400 | TILB2 | 275 | 0.34 | 25.60 | 1 | 510 | 637 | 600 | 750 |
| TORN4 | 400 | ECCL4S | 400 | 2.18 | 82.11 | 0 | 3400 | 4250 | 4000 | 5000 |
| TORN4 | 400 | ELVA2Q | 275 | 14.76 | 112.19 | 0 | 3400 | 4250 | 4000 | 5000 |
| TORN4 | 400 | ELVA2R | 275 | 2.09 | 167.14 | 0 | 3400 | 4250 | 4000 | 5000 |
| WALP4 | 400 | WBUR4 | 400 | 1.99 | 26.55 | 1 | 1496 | 1870 | 1760 | 2200 |
| WALP4 | 400 | WBUR4 | 400 | 1.99 | 26.55 | 1 | 1496 | 1870 | 1760 | 2200 |
| WALX2 | 275 | SJOW2 | 275 | 0.76 | 7.26 | 0 | 3400 | 4250 | 4000 | 5000 |
| WALX2 | 275 | WTHU2 | 275 | 1.60 | 15.32 | 0 | 3400 | 4250 | 4000 | 5000 |
| WHSO4Q | 400 | WHSO2 | 275 | 0.34 | 25.60 | 1 | 510 | 637 | 600 | 750 |
| WIDO2 | 275 | BEDD4K | 400 | 0.65 | 18.84 | 0 | 3400 | 4250 | 4000 | 5000 |

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|---|---|---|---|---|---|---|---|---|---|---|
| WIDO2 | 275 | LITT4 | 400 | 0.84 | 36.15 | 0 | 3400 | 4250 | 4000 | 5000 |
| WIDO2 | 275 | NFLW4S | 400 | 3.39 | 65.69 | 0 | 3400 | 4250 | 4000 | 5000 |
| WIDO2 | 275 | WISD2 | 275 | 0.83 | 19.28 | 1 | 510 | 637 | 600 | 750 |
| WILL4 | 400 | WILL2 | 275 | 0.53 | 38.62 | 1 | 340 | 425 | 400 | 500 |
| WILL4 | 400 | WILL2 | 275 | 0.53 | 38.65 | 1 | 340 | 425 | 400 | 500 |
| WISD2 | 275 | CITR4 | 400 | 0.40 | 23.97 | 0 | 3400 | 4250 | 4000 | 5000 |
| WISD2 | 275 | LALE2 | 275 | 0.62 | 5.46 | 0 | 3400 | 4250 | 4000 | 5000 |
| WTHU2 | 275 | LITT4 | 400 | 0.68 | 45.38 | 1 | 510 | 637 | 600 | 750 |
| WTHU2 | 275 | LITT4 | 400 | 0.69 | 45.54 | 1 | 510 | 637 | 600 | 750 |
| WWEY2 | 275 | BRLE4 | 400 | 1.31 | 18.27 | 0 | 3400 | 4250 | 4000 | 5000 |
| WYMO4 | 400 | COTT4 | 400 | 3.24 | 43.19 | 1 | 1496 | 1870 | 1760 | 2200 |
| WYMO4 | 400 | COTT4 | 400 | 3.24 | 43.19 | 1 | 1496 | 1870 | 1760 | 2200 |
| WYMO4 | 400 | PELH4 | 400 | 0.51 | 6.84 | 1 | 1842 | 2303 | 2168 | 2710 |

## E.3 DEMAND DATA

| NODE | VOLTAGE (Kv.) | DEMAND MW. *Winter Plateau* | *Winter Trough* | *Summer Plateau* | *Summer Trough* |
|---|---|---|---|---|---|
| WALX2 | 275 | 635.422 | 408.098 | 411.214 | 185.901 |
| WISD2 | 275 | 476.894 | 306.283 | 308.623 | 139.522 |
| WIDO2 | 275 | 983.724 | 631.793 | 636.619 | 287.801 |
| ELST2 | 275 | 863.860 | 554.811 | 559.048 | 252.734 |
| IVER2J | 275 | 360.573 | 231.577 | 233.345 | 105.490 |
| CITR4 | 400 | 443.459 | 284.810 | 286.985 | 129.740 |
| SJOW2 | 275 | 1074.373 | 690.012 | 695.282 | 314.322 |
| BEDD2 | 275 | 645.516 | 414.580 | 417.747 | 188.854 |
| BEDD4K | 400 | 55.965 | 35.943 | 36.218 | 16.373 |
| IVER2K | 275 | 152.123 | 97.701 | 98.447 | 44.506 |
| LALE2 | 275 | 245.099 | 157.414 | 158.616 | 71.707 |
| WWEY2 | 275 | 383.487 | 246.293 | 248.174 | 112.194 |
| BOLN4 | 400 | 632.357 | 433.423 | 390.038 | 216.076 |
| FAWL4 | 400 | 228.542 | 156.645 | 140.965 | 78.093 |
| FLEE4 | 400 | 885.230 | 606.744 | 546.010 | 302.483 |
| LOVE4 | 400 | 590.973 | 405.057 | 364.512 | 201.936 |
| NINF4 | 400 | 319.713 | 219.134 | 197.199 | 109.246 |
| NURS4 | 400 | 330.981 | 226.857 | 204.149 | 113.096 |
| BRAI4Q | 400 | 84.461 | 35.580 | 34.513 | 16.215 |
| BRAI4R | 400 | 84.461 | 35.580 | 34.513 | 16.215 |
| BRFO4 | 400 | 612.178 | 257.888 | 250.153 | 117.531 |
| RAYL4 | 400 | 359.459 | 151.427 | 146.885 | 69.012 |
| SIZE4 | 400 | 0. | 0. | 0. | 0. |
| TILB4J | 400 | 0. | 0. | 0. | 0. |
| TILB4K | 400 | 0. | 0. | 0. | 0. |
| CANT4 | 400 | 210.001 | 88.466 | 85.812 | 40.318 |
| KEMS4J | 400 | 0. | 0. | 0. | 0. |

| | | | | | |
|---|---|---|---|---|---|
| KEMS4K | 400 | 223.523 | 94.162 | 91.338 | 42.914 |
| KINO4 | 400 | 185.518 | 78.152 | 75.808 | 35.617 |
| NFLW4Q | 400 | 125.585 | 52.904 | 51.318 | 24.111 |
| NFLW4R | 400 | 125.585 | 52.904 | 51.318 | 24.111 |
| NFLW4S | 400 | 0. | 0. | 0. | 0. |
| NFLW4T | 400 | 0. | 0. | 0. | 0. |
| TILB2 | 275 | 350.091 | 147.481 | 143.057 | 67.213 |
| WTHU2 | 275 | 457.615 | 192.776 | 186.995 | 87.856 |
| DUNG4 | 400 | 0. | 0. | 0. | 0. |
| GRAI4 | 400 | 0. | 0. | 0. | 0. |
| LITT4 | 400 | 303.812 | 127.985 | 124.146 | 58.328 |
| SELL4 | 400 | 122.825 | 51.742 | 50.190 | 23.581 |
| BRLE4 | 400 | 206.806 | 130.316 | 127.483 | 63.033 |
| AMEM4Q | 400 | 50.809 | 32.017 | 31.321 | 15.486 |
| AMEM4R | 400 | 50.809 | 32.017 | 31.321 | 15.486 |
| COWL4 | 400 | 462.718 | 291.576 | 285.237 | 141.034 |
| DIDC4 | 400 | 0. | 0. | 0. | 0. |
| ECLA4 | 400 | 237.966 | 149.951 | 146.691 | 72.531 |
| PELH4 | 400 | 192.791 | 121.485 | 118.844 | 58.762 |
| RYEH4Q | 400 | 133.274 | 83.981 | 82.155 | 40.621 |
| RYEH4R | 400 | 91.653 | 57.754 | 56.498 | 27.935 |
| SUND4 | 400 | 462.719 | 291.576 | 285.238 | 141.034 |
| WYMO4 | 400 | 154.273 | 97.213 | 95.100 | 47.022 |
| AXMI4 | 400 | 161.752 | 110.436 | 102.827 | 55.413 |
| CHIC4 | 400 | 88.097 | 60.148 | 56.004 | 30.180 |
| MANN4 | 400 | 578.884 | 395.233 | 368.000 | 198.315 |
| WINF4 | 400 | 0. | 0. | 0. | 0. |
| ABHA4Q | 400 | 122.026 | 85.933 | 77.584 | 43.261 |
| ABHA4R | 400 | 122.026 | 85.933 | 77.584 | 43.261 |
| ALVE4Q | 400 | 134.481 | 94.704 | 85.503 | 47.677 |
| BRWA4Q | 400 | 87.746 | 61.591 | 55.607 | 31.007 |
| BRWA4R | 400 | 87.746 | 61.591 | 55.607 | 31.007 |
| EXET4 | 400 | 215.310 | 151.625 | 136.894 | 76.333 |
| INDQ4R | 400 | 310.616 | 218.741 | 197.489 | 110.122 |
| LAND4 | 400 | 213.560 | 150.392 | 135.781 | 75.713 |
| TAUN4Q | 400 | 76.977 | 54.208 | 48.942 | 27.290 |
| TAUN4R | 400 | 76.977 | 54.208 | 48.942 | 27.290 |
| IROA2 | 275 | 755.302 | 545.081 | 488.467 | 287.462 |
| IROA4Q | 400 | 0. | 0. | 0. | 0. |
| MELK4 | 400 | 310.885 | 224.357 | 201.055 | 118.321 |
| MITY4 | 400 | 214.405 | 224.357 | 201.055 | 118.321 |
| WALH4 | 400 | 440.080 | 317.594 | 284.607 | 167.491 |
| ABTH2 | 275 | 530.087 | 387.025 | 364.540 | 236.375 |
| SWAN2 | 275 | 549.722 | 401.361 | 378.043 | 245.131 |
| WHSO2 | 275 | 615.013 | 449.031 | 422.944 | 274.245 |
| CILF4 | 400 | 76.250 | 55.671 | 52.437 | 34.001 |
| PEMB4 | 400 | 114.732 | 83.768 | 78.901 | 51.161 |
| SWAN4 | 400 | 0. | 0. | 0. | 0. |
| WHSO4Q | 400 | 0. | 0. | 0. | 0. |
| BURW4 | 400 | 217.889 | 147.861 | 139.286 | 76.294 |

| | | | | | |
|---|---|---|---|---|---|
| ENDE4 | 400 | 232.538 | 157.802 | 148.651 | 81.423 |
| GREN4 | 400 | 659.402 | 447.475 | 421.525 | 230.891 |
| NORW4 | 400 | 409.143 | 277.647 | 261.546 | 143.262 |
| WALP4 | 400 | 463.245 | 314.361 | 296.131 | 162.206 |
| COTT4 | 400 | 0. | 0. | 0. | 0. |
| HIGM4 | 400 | 0. | 0. | 0. | 0. |
| RATS4 | 400 | 349.113 | 170.441 | 234.840 | 91.515 |
| STAY4 | 400 | 177.551 | 86.682 | 119.434 | 46.542 |
| WBUR4 | 400 | 194.020 | 94.723 | 130.513 | 50.860 |
| BUSH2 | 275 | 830.792 | 529.390 | 561.255 | 286.790 |
| DRAK2 | 275 | 330.374 | 210.518 | 223.190 | 114.045 |
| FECK2 | 275 | 607.938 | 387.385 | 410.703 | 209.860 |
| NECH2 | 275 | 1027.858 | 654.962 | 694.387 | 354.817 |
| PENN2 | 275 | 443.156 | 282.384 | 299.381 | 152.978 |
| WILL2 | 275 | 753.062 | 479.859 | 508.743 | 259.957 |
| CELL4 | 400 | 535.962 | 341.521 | 362.078 | 185.014 |
| DRAK4 | 400 | 0. | 0. | 0. | 0. |
| FECK4 | 400 | 0. | 0. | 0. | 0. |
| HAMH4 | 400 | 0. | 0. | 0. | 0. |
| IRON4 | 400 | 189.410 | 120.694 | 127.959 | 65.384 |
| RUGE4 | 400 | 0. | 0. | 0. | 0. |
| WILL4 | 400 | 0. | 0. | 0. | 0. |
| DEES4 | 400 | 513.339 | 271.270 | 274.655 | 203.135 |
| LEGA4 | 400 | 281.714 | 148.869 | 150.727 | 111.478 |
| PENT4 | 400 | 365.300 | 193.040 | 195.449 | 144.554 |
| TRAW4 | 400 | 52.650 | 27.822 | 28.170 | 20.834 |
| FIDF2 | 275 | 1603.703 | 1116.420 | 1160.067 | 694.608 |
| ROCH2 | 275 | 751.076 | 522.862 | 543.304 | 325.312 |
| STAL2 | 275 | 659.353 | 459.009 | 476.955 | 285.584 |
| DAIN4 | 400 | 410.965 | 286.094 | 297.279 | 178.000 |
| HEYS4 | 400 | 306.807 | 213.584 | 221.934 | 132.887 |
| KEAR4 | 400 | 273.717 | 190.548 | 197.998 | 118.554 |
| PADI4 | 400 | 190.544 | 132.647 | 137.833 | 82.530 |
| PEWO4 | 400 | 506.505 | 352.604 | 366.389 | 219.381 |
| BRAW2 | 275 | 505.809 | 364.125 | 365.863 | 222.748 |
| ELLA2 | 275 | 334.783 | 241.006 | 242.156 | 147.432 |
| SKLG2 | 275 | 472.642 | 340.249 | 341.873 | 208.142 |
| OSBA2 | 275 | 360.926 | 259.826 | 261.066 | 158.945 |
| CHTE2 | 275 | 843.583 | 607.284 | 610.182 | 371.497 |
| HIGM2 | 275 | 95.975 | 69.091 | 69.421 | 42.265 |
| NEEP2 | 275 | 500.276 | 360.142 | 361.861 | 220.311 |
| THOM2 | 275 | 618.786 | 445.456 | 447.582 | 272.501 |
| DRAX4J | 400 | 47.839 | 34.439 | 34.603 | 21.067 |
| DRAX4K | 400 | 0. | 0. | 0. | 0. |
| EGGB4J | 400 | 0. | 0. | 0. | 0. |
| EGGB4K | 400 | 0. | 0. | 0. | 0. |
| FERR4Q | 400 | 158.989 | 114.454 | 115.000 | 70.016 |
| OSBA4Q | 400 | 0. | 0. | 0. | 0. |
| CREB4 | 400 | 418.875 | 301.542 | 302.981 | 184.464 |
| KEAD4 | 400 | 453.910 | 326.764 | 328.323 | 199.893 |

| | | | | | |
|---|---|---|---|---|---|
| STSB4 | 400 | 66.484 | 47.861 | 48.089 | 29.278 |
| THOM4 | 400 | 69.553 | 50.070 | 50.309 | 30.630 |
| HATL2 | 275 | 773.310 | 598.121 | 559.024 | 361.655 |
| STEW4 | 400 | 294.075 | 227.454 | 212.586 | 137.530 |
| BLYT2 | 275 | 710.539 | 549.571 | 513.646 | 332.299 |
| NORT4 | 400 | 279.059 | 215.840 | 201.731 | 130.508 |
| HARK2 | 275 | 295.941 | 277.944 | 300.940 | 166.967 |
| ECCL4S | 400 | 795.100 | 557.746 | 555.541 | 344.494 |
| ELVA2Q | 275 | 1261.000 | 884.564 | 881.068 | 546.355 |
| ELVA2R | 275 | 996.000 | 698.673 | 695.911 | 431.539 |
| HARK1L | 132 | 269.000 | 188.698 | 187.952 | 116.550 |
| TORN4 | 400 | 2089.000 | 1465.389 | 1459.597 | 905.104 |
| HINP4 | 400 | 0. | 0. | 0. | 0. |
| *TOTALS* | | 46528. | 30879. | 30575. | 17126. |

Fig.E.1 Supergrid System: 400/275kV circuits
(extract from National Grid Seven Year Statement, 1990)

*APPENDIX F*

# DYNAMIC PROGRAMMING LOSS MINIMISATION (DPLM) TECHNQIUE

## Dispatch Result - CEGB Winter Plateau Load

*UNIVERSITY OF DURHAM*
*OPERATIONAL CONTROL OF ELECTRIC POWER SYSTEM*
*(O.C.E.P.S.) PROJECT*
*PROGRAM: ECONOMIC DISPATCH (DPLM) V3.1.0*

## F.1 SYSTEM DATA

NO. OF ACTIVE ISLANDS = 1
NO. OF ACTIVE NODES = 145
NO. OF ACTIVE GENERATORS = 115
NO. OF ACTIVE LINES = 275
NO. OF ACTIVE SHUNTS = 0
NO. OF GENERATOR GROUP = 0

DISPATCH TIME IN ADVANCE = 30.00 MINUTES

## F.2 GENERATOR DATA

| GEN | NODE | GRP | G_now | G_high | G_low | RAMP_I | RAMP_D | COST-A | COST-B | COST-C |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 0 | 5.000 | 5.500 | 0.000 | 0.500 | 0.500 | 0.00 | 2868. | 0.00 |
| 2 | 6 | 0 | 5.000 | 8.600 | 0.000 | 0.500 | 0.500 | 0.00 | 2868. | 0.00 |
| 3 | 6 | 0 | 0.200 | 0.510 | 0.000 | 0.500 | 0.500 | 0.00 | 2868. | 0.00 |
| 4 | 6 | 0 | 2.000 | 3.760 | 0.000 | 0.500 | 0.500 | 0.00 | 2868. | 0.00 |
| 5 | 72 | 0 | 2.000 | 3.360 | 0.000 | 0.500 | 0.500 | 0.00 | 3059. | 0.00 |
| 6 | 16 | 0 | 5.000 | 11.55 | 5.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 7 | 22 | 0 | 2.000 | 4.480 | 0.000 | 0.500 | 0.500 | 0.00 | 2840. | 0.00 |
| 8 | 22 | 0 | 5.000 | 6.200 | 0.000 | 0.500 | 0.500 | 0.00 | 2840. | 0.00 |
| 9 | 22 | 0 | 2.000 | 4.800 | 0.000 | 0.500 | 0.500 | 0.00 | 2840. | 0.00 |
| 10 | 27 | 0 | 1.000 | 1.140 | 0.000 | 0.500 | 0.500 | 0.00 | 5445. | 0.00 |
| 11 | 34 | 0 | 1.000 | 1.680 | 0.000 | 0.500 | 0.500 | 0.00 | 3271. | 0.00 |
| 12 | 35 | 0 | 1.000 | 2.820 | 0.000 | 0.500 | 0.500 | 0.00 | 2614. | 0.00 |
| 13 | 35 | 0 | 5.000 | 19.32 | 0.000 | 0.500 | 0.500 | 0.00 | 2614. | 0.00 |
| 14 | 35 | 0 | 0.500 | 0.680 | 0.000 | 0.500 | 0.500 | 0.00 | 2614. | 0.00 |
| 15 | 37 | 0 | 1.000 | 2.240 | 0.000 | 0.500 | 0.500 | 0.00 | 2787. | 0.00 |
| 16 | 38 | 0 | 2.000 | 4.480 | 0.000 | 0.500 | 0.500 | 0.00 | 2955. | 0.00 |
| 17 | 39 | 0 | 0.500 | 1.100 | 0.000 | 0.500 | 0.500 | 0.00 | 10751. | 0.00 |
| 18 | 41 | 0 | 5.000 | 18.20 | 0.000 | 0.500 | 0.500 | 0.00 | 2866. | 0.00 |
| 19 | 41 | 0 | 0.500 | 1.000 | 0.000 | 0.500 | 0.500 | 0.00 | 2866. | 0.00 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 47 | 0 | 5.000 | 11.50 | 0.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 21 | 47 | 0 | 5.000 | 11.55 | 0.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 22 | 47 | 0 | 0.300 | 0.700 | 0.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 23 | 50 | 0 | 2.000 | 3.360 | 0.000 | 0.500 | 0.500 | 0.00 | 2890. | 0.00 |
| 24 | 50 | 0 | 0.500 | 1.400 | 0.000 | 0.500 | 0.500 | 0.00 | 2890. | 0.00 |
| 25 | 51 | 0 | 1.000 | 1.120 | 0.000 | 0.500 | 0.500 | 0.00 | 2968. | 0.00 |
| 26 | 51 | 0 | 2.000 | 3.360 | 0.000 | 0.500 | 0.500 | 0.00 | 2968. | 0.00 |
| 27 | 51 | 0 | 2.000 | 3.100 | 0.000 | 0.500 | 0.500 | 0.00 | 2968. | 0.00 |
| 28 | 51 | 0 | 2.000 | 3.500 | 0.000 | 0.500 | 0.500 | 0.00 | 2968. | 0.00 |
| 29 | 51 | 0 | 5.000 | 5.500 | 0.000 | 0.500 | 0.500 | 0.00 | 2968. | 0.00 |
| 30 | 53 | 0 | 2.000 | 2.440 | 0.000 | 0.500 | 0.500 | 0.00 | 3243. | 0.00 |
| 31 | 53 | 0 | 1.000 | 1.830 | 0.000 | 0.500 | 0.500 | 0.00 | 3243. | 0.00 |
| 32 | 54 | 0 | 0.500 | 1.400 | 0.000 | 0.500 | 0.500 | 0.00 | 10751. | 0.00 |
| 33 | 55 | 0 | 1.000 | 1.140 | 0.000 | 0.500 | 0.500 | 0.00 | 5445. | 0.00 |
| 34 | 55 | 0 | 1.000 | 2.280 | 0.000 | 0.500 | 0.500 | 0.00 | 5445. | 0.00 |
| 35 | 59 | 0 | 1.000 | 2.240 | 0.000 | 0.500 | 0.500 | 0.00 | 2890. | 0.00 |
| 36 | 59 | 0 | 1.000 | 2.240 | 0.000 | 0.500 | 0.500 | 0.00 | 2890. | 0.00 |
| 37 | 65 | 0 | 1.000 | 2.240 | 0.000 | 0.500 | 0.500 | 0.00 | 2831. | 0.00 |
| 38 | 66 | 0 | 5.000 | 9.300 | 0.000 | 0.500 | 0.500 | 0.00 | 2870. | 0.00 |
| 39 | 68 | 0 | 5.000 | 9.420 | 0.000 | 0.500 | 0.500 | 0.00 | 2780. | 0.00 |
| 40 | 68 | 0 | 0.500 | 0.560 | 0.000 | 0.500 | 0.500 | 0.00 | 2780. | 0.00 |
| 41 | 70 | 0 | 0.500 | 1.000 | 0.000 | 0.500 | 0.500 | 0.00 | 5293. | 0.00 |
| 42 | 70 | 0 | 5.000 | 19.00 | 0.000 | 0.500 | 0.500 | 0.00 | 5293. | 0.00 |
| 43 | 74 | 0 | 1.000 | 1.710 | 0.000 | 0.500 | 0.500 | 0.00 | 5445. | 0.00 |
| 44 | 74 | 0 | 1.000 | 1.710 | 0.000 | 0.500 | 0.500 | 0.00 | 5445. | 0.00 |
| 45 | 77 | 0 | 0.500 | 1.000 | 0.000 | 0.500 | 0.500 | 0.00 | 2634. | 0.00 |
| 46 | 77 | 0 | 5.000 | 18.40 | 0.000 | 0.500 | 0.500 | 0.00 | 2634. | 0.00 |
| 47 | 79 | 0 | 1.000 | 1.120 | 0.000 | 0.500 | 0.500 | 0.00 | 2831. | 0.00 |
| 48 | 85 | 0 | 1.000 | 2.400 | 0.000 | 0.500 | 0.500 | 0.00 | 271. | 0.00 |
| 49 | 86 | 0 | 5.000 | 9.500 | 0.000 | 0.500 | 0.500 | 0.00 | 5365. | 0.00 |
| 50 | 86 | 0 | 0.500 | 0.500 | 0.000 | 0.500 | 0.500 | 0.00 | 5365. | 0.00 |
| 51 | 87 | 0 | 1.000 | 1.640 | 0.000 | 0.500 | 0.500 | 0.00 | 2804. | 0.00 |
| 52 | 87 | 0 | 0.200 | 0.680 | 0.000 | 0.500 | 0.500 | 0.00 | 2804. | 0.00 |
| 53 | 88 | 0 | 1.000 | 1.640 | 0.000 | 0.500 | 0.500 | 0.00 | 2652. | 0.00 |
| 54 | 88 | 0 | 5.000 | 18.80 | 0.000 | 0.500 | 0.500 | 0.00 | 2652. | 0.00 |
| 55 | 88 | 0 | 0.500 | 0.680 | 0.000 | 0.500 | 0.500 | 0.00 | 2652. | 0.00 |
| 56 | 88 | 0 | 0.500 | 1.400 | 0.000 | 0.500 | 0.500 | 0.00 | 2652. | 0.00 |
| 57 | 90 | 0 | 5.000 | 8.400 | 0.000 | 0.500 | 0.500 | 0.00 | 900. | 0.00 |
| 58 | 93 | 0 | 0.500 | 0.680 | 0.000 | 0.500 | 0.500 | 0.00 | 2631. | 0.00 |
| 59 | 93 | 0 | 5.000 | 19.32 | 0.000 | 0.500 | 0.500 | 0.00 | 2631. | 0.00 |
| 60 | 94 | 0 | 0.500 | 0.500 | 0.000 | 0.500 | 0.500 | 0.00 | 2585. | 0.00 |
| 61 | 94 | 0 | 5.000 | 9.200 | 0.000 | 0.500 | 0.500 | 0.00 | 2585. | 0.00 |
| 62 | 95 | 0 | 5.000 | 18.75 | 0.000 | 0.500 | 0.500 | 0.00 | 2555. | 0.00 |
| 63 | 95 | 0 | 0.500 | 1.050 | 0.000 | 0.500 | 0.500 | 0.00 | 2555. | 0.00 |
| 64 | 96 | 0 | 0.500 | 1.050 | 0.000 | 0.500 | 0.500 | 0.00 | 2555. | 0.00 |
| 65 | 96 | 0 | 5.000 | 18.75 | 0.000 | 0.500 | 0.500 | 0.00 | 2555. | 0.00 |
| 66 | 97 | 0 | 5.000 | 8.600 | 0.000 | 0.500 | 0.500 | 0.00 | 2709. | 0.00 |
| 67 | 97 | 0 | 0.100 | 0.340 | 0.000 | 0.500 | 0.500 | 0.00 | 2709. | 0.00 |
| 68 | 98 | 0 | 0.100 | 0.340 | 0.000 | 0.500 | 0.500 | 0.00 | 2709. | 0.00 |
| 69 | 98 | 0 | 5.000 | 8.600 | 0.000 | 0.500 | 0.500 | 0.00 | 2709. | 0.00 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 70 | 100 | 0 | 5.000 | 9.600 | 0.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 71 | 100 | 0 | 5.000 | 11.55 | 0.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 72 | 100 | 0 | 5.000 | 11.55 | 0.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 73 | 104 | 0 | 0.500 | 0.700 | 0.000 | 0.500 | 0.500 | 0.00 | 10751. | 0.00 |
| 74 | 104 | 0 | 0.500 | 0.700 | 0.000 | 0.500 | 0.500 | 0.00 | 10751. | 0.00 |
| 75 | 105 | 0 | 2.000 | 2.800 | 0.000 | 0.500 | 0.500 | 0.00 | 10751. | 0.00 |
| 76 | 106 | 0 | 0.500 | 0.680 | 0.000 | 0.500 | 0.500 | 0.00 | 3021. | 0.00 |
| 77 | 106 | 0 | 5.000 | 13.44 | 0.000 | 0.500 | 0.500 | 0.00 | 3021. | 0.00 |
| 78 | 107 | 0 | 5.000 | 6.420 | 0.000 | 0.500 | 0.500 | 0.00 | 414. | 0.00 |
| 79 | 107 | 0 | 5.000 | 10.40 | 0.000 | 0.500 | 0.500 | 0.00 | 414. | 0.00 |
| 80 | 108 | 0 | 5.000 | 12.52 | 0.000 | 0.500 | 0.500 | 0.00 | 2632. | 0.00 |
| 81 | 111 | 0 | 5.000 | 19.32 | 0.000 | 0.500 | 0.500 | 0.00 | 5386. | 0.00 |
| 82 | 111 | 0 | 1.000 | 1.400 | 0.000 | 0.500 | 0.500 | 0.00 | 5386. | 0.00 |
| 83 | 111 | 0 | 0.200 | 0.680 | 0.000 | 0.500 | 0.500 | 0.00 | 5386. | 0.00 |
| 84 | 113 | 0 | 0.500 | 1.020 | 0.000 | 0.500 | 0.500 | 0.00 | 2851. | 0.00 |
| 85 | 113 | 0 | 2.000 | 2.240 | 0.000 | 0.500 | 0.500 | 0.00 | 2851. | 0.00 |
| 86 | 113 | 0 | 2.000 | 3.000 | 0.000 | 0.500 | 0.500 | 0.00 | 2851. | 0.00 |
| 87 | 113 | 0 | 2.000 | 3.920 | 0.000 | 0.500 | 0.500 | 0.00 | 2851. | 0.00 |
| 88 | 113 | 0 | 2.000 | 3.760 | 0.000 | 0.500 | 0.500 | 0.00 | 2851. | 0.00 |
| 89 | 115 | 0 | 0.100 | 0.340 | 0.000 | 0.500 | 0.500 | 0.00 | 2765. | 0.00 |
| 90 | 115 | 0 | 5.000 | 9.190 | 0.000 | 0.500 | 0.500 | 0.00 | 2765. | 0.00 |
| 91 | 117 | 0 | 5.000 | 25.24 | 0.000 | 1.500 | 1.500 | 0.00 | 5365. | 0.00 |
| 92 | 117 | 0 | 1.000 | 1.450 | 0.000 | 0.500 | 0.500 | 0.00 | 5365. | 0.00 |
| 93 | 118 | 0 | 0.500 | 0.680 | 0.000 | 0.500 | 0.500 | 0.00 | 2997. | 0.00 |
| 94 | 118 | 0 | 5.000 | 19.20 | 0.000 | 0.500 | 0.500 | 0.00 | 2997. | 0.00 |
| 95 | 120 | 0 | 5.000 | 18.40 | 0.000 | 0.500 | 0.500 | 0.00 | 2694. | 0.00 |
| 96 | 120 | 0 | 0.500 | 0.680 | 0.000 | 0.500 | 0.500 | 0.00 | 2694. | 0.00 |
| 97 | 122 | 0 | 5.000 | 10.00 | 0.000 | 0.500 | 0.500 | 0.00 | 414. | 0.00 |
| 98 | 123 | 0 | 0.500 | 0.700 | 0.000 | 0.500 | 0.500 | 0.00 | 414. | 0.00 |
| 99 | 123 | 0 | 5.000 | 10.00 | 0.000 | 0.500 | 0.500 | 0.00 | 414. | 0.00 |
| 100 | 123 | 0 | 5.000 | 12.30 | 0.000 | 0.500 | 0.500 | 0.00 | 414. | 0.00 |
| 101 | 132 | 0 | 5.000 | 18.75 | 0.000 | 0.500 | 0.500 | 0.00 | 5420. | 0.00 |
| 102 | 132 | 0 | 0.500 | 1.050 | 0.000 | 0.500 | 0.500 | 0.00 | 5420. | 0.00 |
| 103 | 134 | 0 | 1.000 | 1.400 | 0.000 | 0.500 | 0.500 | 0.00 | 10751. | 0.00 |
| 104 | 138 | 0 | 5.000 | 11.55 | 0.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 105 | 138 | 0 | 5.000 | 11.55 | 0.000 | 0.500 | 0.500 | 0.00 | 387. | 0.00 |
| 106 | 139 | 0 | 1.000 | 2.240 | 0.000 | 0.500 | 0.500 | 0.00 | 3271. | 0.00 |
| 107 | 139 | 0 | 1.000 | 3.000 | 0.000 | 0.500 | 0.500 | 0.00 | 3271. | 0.00 |
| 108 | 140 | 0 | 2.000 | 3.750 | 0.000 | 0.500 | 0.500 | 0.00 | 2619. | 0.00 |
| 109 | 141 | 0 | 5.000 | 6.420 | 0.000 | 0.500 | 0.500 | 0.00 | 2632. | 0.00 |
| 110 | 141 | 0 | 5.000 | 12.52 | 0.000 | 0.500 | 0.500 | 0.00 | 2632. | 0.00 |
| 111 | 141 | 0 | 5.000 | 11.52 | 0.000 | 0.500 | 0.500 | 0.00 | 2632. | 0.00 |
| 112 | 142 | 0 | 1.000 | 1.400 | 0.000 | 0.500 | 0.500 | 0.00 | 10751. | 0.00 |
| 113 | 143 | 0 | 2.000 | 3.760 | 0.000 | 0.500 | 0.500 | 0.00 | 2929. | 0.00 |
| 114 | 143 | 0 | 5.000 | 8.640 | 0.000 | 0.500 | 0.500 | 0.00 | 2929. | 0.00 |
| 115 | 145 | 0 | 0.500 | 1.400 | 0.000 | 0.500 | 0.500 | 0.00 | 10751. | 0.00 |

## F.3 LINE DATA

| LINE | NODE | R(p.u.) | X(p.u.) | SUS(p.u.) | P-LIMIT |
|---|---|---|---|---|---|
| 1 | 1- 2 | 0.0012 | 0.0098 | 0.0000 | 9.3500 |
| 2 | 1- 3 | 0.0011 | 0.0090 | 0.0000 | 9.3500 |
| 3 | 4- 2 | 0.0012 | 0.0098 | 0.0000 | 9.3500 |
| 4 | 4- 5 | 0.0023 | 0.0183 | 0.0000 | 9.3500 |
| 5 | 6- 7 | 0.0009 | 0.0148 | 0.0000 | 42.5000 |
| 6 | 6- 8 | 0.0036 | 0.0239 | 0.0000 | 42.5000 |
| 7 | 6- 9 | 0.0019 | 0.0123 | 0.0000 | 42.5000 |
| 8 | 10- 5 | 0.0024 | 0.0196 | 0.0000 | 13.0000 |
| 9 | 10- 11 | 0.0017 | 0.0145 | 0.0000 | 13.9400 |
| 10 | 12- 13 | 0.0008 | 0.0066 | 0.0001 | 11.5100 |
| 11 | 14- 13 | 0.0008 | 0.0066 | 0.0001 | 11.5100 |
| 12 | 15- 16 | 0.0014 | 0.0102 | 0.0000 | 18.7000 |
| 13 | 17- 18 | 0.0015 | 0.0377 | 0.0000 | 42.5000 |
| 14 | 17- 19 | 0.0008 | 0.0068 | 0.0001 | 42.5000 |
| 15 | 20- 17 | 0.0002 | 0.0267 | 0.0000 | 6.3700 |
| 16 | 20- 21 | 0.0007 | 0.0063 | 0.0001 | 12.0000 |
| 17 | 22- 23 | 0.0107 | 0.0438 | 0.0000 | 4.4200 |
| 18 | 22- 23 | 0.0107 | 0.0440 | 0.0000 | 4.4200 |
| 19 | 22- 24 | 0.0025 | 0.0524 | 0.0000 | 42.5000 |
| 20 | 25- 26 | 0.0008 | 0.0104 | 0.0000 | 18.7000 |
| 21 | 25- 26 | 0.0008 | 0.0104 | 0.0000 | 18.7000 |
| 22 | 25- 27 | 0.0006 | 0.0083 | 0.0000 | 23.0300 |
| 23 | 25- 27 | 0.0006 | 0.0083 | 0.0000 | 23.0300 |
| 24 | 28- 29 | 0.0008 | 0.0107 | 0.0000 | 23.0300 |
| 25 | 28- 30 | 0.0004 | 0.0051 | 0.0001 | 23.0300 |
| 26 | 31- 32 | 0.0006 | 0.0079 | 0.0000 | 23.0300 |
| 27 | 31- 30 | 0.0004 | 0.0051 | 0.0001 | 23.0300 |
| 28 | 33- 34 | 0.0007 | 0.0056 | 0.0001 | 42.5000 |
| 29 | 33- 35 | 0.0003 | 0.0585 | 0.0000 | 42.5000 |
| 30 | 33- 36 | 0.0098 | 0.0432 | 0.0000 | 42.5000 |
| 31 | 33- 37 | 0.0009 | 0.0258 | 0.0000 | 63.7000 |
| 32 | 33- 38 | 0.0012 | 0.0274 | 0.0000 | 42.5000 |
| 33 | 32- 39 | 0.0007 | 0.0098 | 0.0000 | 11.5600 |
| 34 | 40- 41 | 0.0005 | 0.0069 | 0.0001 | 18.7000 |
| 35 | 40- 41 | 0.0005 | 0.0069 | 0.0001 | 18.7000 |
| 36 | 40- 42 | 0.0002 | 0.0028 | 0.0001 | 18.7000 |
| 37 | 40- 42 | 0.0002 | 0.0028 | 0.0001 | 18.7000 |
| 38 | 40- 43 | 0.0016 | 0.0155 | 0.0000 | 15.7200 |
| 39 | 40- 43 | 0.0016 | 0.0155 | 0.0000 | 15.7200 |
| 40 | 44- 45 | 0.0004 | 0.0035 | 0.0001 | 15.8100 |
| 41 | 46- 45 | 0.0004 | 0.0035 | 0.0001 | 15.8100 |
| 42 | 47- 45 | 0.0000 | 0.0000 | 0.0313 | 42.5000 |
| 43 | 48- 29 | 0.0005 | 0.0064 | 0.0001 | 23.0300 |
| 44 | 48- 49 | 0.0006 | 0.0082 | 0.0000 | 23.0300 |
| 45 | 50- 51 | 0.0015 | 0.0127 | 0.0000 | 42.5000 |
| 46 | 50- 52 | 0.0043 | 0.0476 | 0.0000 | 42.5000 |

| | | | | | |
|---|---|---|---|---|---|
| 47 | 50- 53 | 0.0027 | 0.0370 | 0.0000 | 42.5000 |
| 48 | 50- 54 | 0.0009 | 0.0078 | 0.0000 | 42.5000 |
| 49 | 55- 56 | 0.0006 | 0.0054 | 0.0001 | 15.7200 |
| 50 | 55- 57 | 0.0006 | 0.0054 | 0.0001 | 15.7200 |
| 51 | 55- 58 | 0.0005 | 0.0051 | 0.0001 | 15.7200 |
| 52 | 55- 58 | 0.0005 | 0.0051 | 0.0001 | 15.7200 |
| 53 | 59- 60 | 0.0011 | 0.0085 | 0.0000 | 11.5100 |
| 54 | 59- 61 | 0.0020 | 0.0173 | 0.0000 | 9.3500 |
| 55 | 59- 62 | 0.0015 | 0.0118 | 0.0000 | 9.3500 |
| 56 | 63- 64 | 0.0009 | 0.0121 | 0.0000 | 18.7000 |
| 57 | 63- 16 | 0.0003 | 0.0039 | 0.0001 | 18.7000 |
| 58 | 65- 66 | 0.0005 | 0.0068 | 0.0001 | 42.5000 |
| 59 | 65- 67 | 0.0010 | 0.0114 | 0.0000 | 42.5000 |
| 60 | 65- 68 | 0.0015 | 0.0249 | 0.0000 | 42.5000 |
| 61 | 65- 69 | -0.0010 | 0.0445 | 0.0000 | 42.5000 |
| 62 | 7- 70 | 0.0018 | 0.0233 | 0.0000 | 23.5500 |
| 63 | 7- 70 | 0.0017 | 0.0230 | 0.0000 | 23.5500 |
| 64 | 7- 71 | 0.0012 | 0.0163 | 0.0000 | 9.4300 |
| 65 | 7- 72 | 0.0005 | 0.0064 | 0.0001 | 23.5500 |
| 66 | 73- 74 | 0.0008 | 0.0069 | 0.0001 | 12.2500 |
| 67 | 73- 75 | 0.0008 | 0.0069 | 0.0001 | 12.2500 |
| 68 | 73- 76 | 0.0002 | 0.0139 | 0.0000 | 42.5000 |
| 69 | 77- 78 | 0.0025 | 0.0243 | 0.0000 | 16.1500 |
| 70 | 77- 79 | 0.0015 | 0.0049 | 0.0001 | 11.2200 |
| 71 | 77- 69 | 0.0008 | 0.0103 | 0.0000 | 20.4800 |
| 72 | 80- 41 | 0.0001 | 0.0018 | 0.0002 | 18.7000 |
| 73 | 80- 41 | 0.0001 | 0.0018 | 0.0002 | 18.7000 |
| 74 | 80- 13 | 0.0004 | 0.0058 | 0.0001 | 18.7000 |
| 75 | 80- 81 | 0.0002 | 0.0182 | 0.0000 | 9.4300 |
| 76 | 80- 82 | 0.0008 | 0.0105 | 0.0000 | 18.7000 |
| 77 | 80- 71 | 0.0012 | 0.0152 | 0.0000 | 9.3500 |
| 78 | 83- 84 | 0.0009 | 0.0075 | 0.0001 | 11.5100 |
| 79 | 83- 84 | 0.0009 | 0.0075 | 0.0001 | 11.5100 |
| 80 | 83- 24 | 0.0015 | 0.0196 | 0.0000 | 23.0300 |
| 81 | 85- 59 | 0.0013 | 0.0104 | 0.0000 | 13.0000 |
| 82 | 85- 86 | 0.0014 | 0.0115 | 0.0000 | 13.9400 |
| 83 | 85- 86 | 0.0014 | 0.0115 | 0.0000 | 13.9400 |
| 84 | 85- 87 | 0.0003 | 0.0030 | 0.0001 | 15.8100 |
| 85 | 86- 88 | 0.0002 | 0.0110 | 0.0000 | 42.5000 |
| 86 | 86- 89 | 0.0006 | 0.0052 | 0.0001 | 13.0000 |
| 87 | 86- 90 | 0.0010 | 0.0126 | 0.0000 | 23.0300 |
| 88 | 86- 90 | 0.0010 | 0.0126 | 0.0000 | 23.0300 |
| 89 | 86- 91 | 0.0019 | 0.0149 | 0.0000 | 13.9400 |
| 90 | 51- 53 | 0.0005 | 0.0074 | 0.0001 | 42.5000 |
| 91 | 60- 51 | 0.0002 | 0.0160 | 0.0000 | 6.3700 |
| 92 | 60- 51 | 0.0002 | 0.0161 | 0.0000 | 6.3700 |
| 93 | 60- 92 | 0.0006 | 0.0056 | 0.0001 | 15.7200 |
| 94 | 60- 93 | 0.0008 | 0.0063 | 0.0001 | 11.5100 |
| 95 | 60- 94 | 0.0005 | 0.0040 | 0.0001 | 13.0000 |
| 96 | 95- 83 | 0.0006 | 0.0083 | 0.0000 | 23.0300 |

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|---|---|---|---|---|---|
| 97 | 95- 96 | 0.0002 | 0.0200 | 0.0000 | 17.0000 |
| 98 | 95- 97 | 0.0001 | 0.0018 | 0.0002 | 18.7000 |
| 99 | 95- 84 | 0.0004 | 0.0042 | 0.0001 | 23.0300 |
| 100 | 96- 98 | 0.0001 | 0.0017 | 0.0002 | 21.0680 |
| 101 | 96- 99 | 0.0005 | 0.0063 | 0.0001 | 23.0300 |
| 102 | 96- 69 | 0.0003 | 0.0029 | 0.0001 | 22.4400 |
| 103 | 100- 27 | 0.0005 | 0.0071 | 0.0001 | 23.0300 |
| 104 | 100- 27 | 0.0005 | 0.0071 | 0.0001 | 23.0300 |
| 105 | 100- 58 | 0.0005 | 0.0047 | 0.0001 | 11.5100 |
| 106 | 100- 58 | 0.0005 | 0.0047 | 0.0001 | 11.5100 |
| 107 | 13-101 | 0.0017 | 0.0163 | 0.0000 | 15.7200 |
| 108 | 13- 93 | 0.0021 | 0.0221 | 0.0000 | 15.7200 |
| 109 | 13- 82 | 0.0003 | 0.0046 | 0.0001 | 18.7000 |
| 110 | 97-102 | 0.0017 | 0.0317 | 0.0000 | 8.0900 |
| 111 | 97- 61 | 0.0005 | 0.0072 | 0.0001 | 23.0300 |
| 112 | 98- 35 | 0.0002 | 0.0022 | 0.0002 | 23.0300 |
| 113 | 98- 37 | 0.0012 | 0.0158 | 0.0000 | 23.0300 |
| 114 | 98- 69 | 0.0003 | 0.0045 | 0.0001 | 18.7000 |
| 115 | 34- 35 | -0.0029 | 0.0991 | 0.0000 | 42.5000 |
| 116 | 34- 38 | 0.0025 | 0.0170 | 0.0000 | 42.5000 |
| 117 | 34-103 | 0.0015 | 0.0161 | 0.0000 | 9.5200 |
| 118 | 104-105 | 0.0005 | 0.0040 | 0.0001 | 42.5000 |
| 119 | 104- 82 | 0.0010 | 0.0226 | 0.0000 | 6.3700 |
| 120 | 104- 82 | 0.0010 | 0.0226 | 0.0000 | 6.3700 |
| 121 | 104-106 | 0.0014 | 0.0148 | 0.0000 | 42.5000 |
| 122 | 107- 23 | 0.0089 | 0.0311 | 0.0000 | 42.5000 |
| 123 | 108- 23 | 0.0053 | 0.0411 | 0.0000 | 42.5000 |
| 124 | 64- 15 | 0.0004 | 0.0058 | 0.0001 | 18.7000 |
| 125 | 2- 64 | 0.0000 | 0.0000 | 0.0313 | 42.5000 |
| 126 | 2- 44 | 0.0011 | 0.0118 | 0.0000 | 20.4800 |
| 127 | 2- 46 | 0.0011 | 0.0118 | 0.0000 | 20.4800 |
| 128 | 109-110 | 0.0000 | 0.0000 | 0.0313 | 42.5000 |
| 129 | 111-110 | 0.0008 | 0.0100 | 0.0000 | 18.7000 |
| 130 | 111-112 | 0.0003 | 0.0035 | 0.0001 | 18.7000 |
| 131 | 52- 53 | 0.0027 | 0.0354 | 0.0000 | 42.5000 |
| 132 | 52- 54 | 0.0034 | 0.0304 | 0.0000 | 42.5000 |
| 133 | 52-113 | 0.0060 | 0.0551 | 0.0000 | 42.5000 |
| 134 | 114- 52 | 0.0002 | 0.0161 | 0.0000 | 8.5000 |
| 135 | 114- 92 | 0.0009 | 0.0083 | 0.0000 | 15.8100 |
| 136 | 114-115 | 0.0016 | 0.0124 | 0.0000 | 13.0000 |
| 137 | 114- 43 | 0.0021 | 0.0204 | 0.0000 | 15.7200 |
| 138 | 114- 71 | 0.0014 | 0.0129 | 0.0000 | 15.7200 |
| 139 | 88-116 | 0.0006 | 0.0194 | 0.0000 | 42.5000 |
| 140 | 42- 26 | 0.0008 | 0.0074 | 0.0001 | 15.8100 |
| 141 | 42- 26 | 0.0008 | 0.0074 | 0.0001 | 15.8100 |
| 142 | 117- 56 | 0.0001 | 0.0014 | 0.0003 | 18.7000 |
| 143 | 117- 57 | 0.0001 | 0.0014 | 0.0003 | 18.7000 |
| 144 | 117-118 | 0.0001 | 0.0019 | 0.0002 | 18.7000 |
| 145 | 117-119 | 0.0003 | 0.0045 | 0.0001 | 18.7000 |
| 146 | 78- 82 | 0.0008 | 0.0075 | 0.0001 | 15.8100 |

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|---|---|---|---|---|---|
| 147 | 78- 82 | 0.0008 | 0.0075 | 0.0001 | 15.8100 |
| 148 | 78-120 | 0.0026 | 0.0256 | 0.0000 | 16.1500 |
| 149 | 92- 53 | 0.0004 | 0.0199 | 0.0000 | 8.0900 |
| 150 | 121- 23 | 0.0019 | 0.0833 | 0.0000 | 4.0000 |
| 151 | 122- 22 | 0.0027 | 0.0210 | 0.0000 | 42.5000 |
| 152 | 122- 24 | 0.0004 | 0.0088 | 0.0000 | 42.5000 |
| 153 | 123- 23 | 0.0016 | 0.0220 | 0.0000 | 42.5000 |
| 154 | 123-116 | 0.0003 | 0.0022 | 0.0002 | 42.5000 |
| 155 | 66- 68 | 0.0130 | 0.0655 | 0.0000 | 42.5000 |
| 156 | 124- 66 | 0.0002 | 0.0160 | 0.0000 | 8.0900 |
| 157 | 124- 66 | 0.0004 | 0.0341 | 0.0000 | 4.2500 |
| 158 | 124- 93 | 0.0016 | 0.0123 | 0.0000 | 11.5100 |
| 159 | 124-120 | 0.0027 | 0.0026 | 0.0001 | 11.5100 |
| 160 | 47-125 | 0.0000 | 0.0000 | 0.0313 | 42.5000 |
| 161 | 125- 43 | 0.0017 | 0.0164 | 0.0000 | 15.8100 |
| 162 | 125- 43 | 0.0017 | 0.0164 | 0.0000 | 15.8100 |
| 163 | 47- 11 | 0.0005 | 0.0051 | 0.0001 | 15.8100 |
| 164 | 47-126 | 0.0005 | 0.0051 | 0.0001 | 15.8100 |
| 165 | 5-126 | 0.0041 | 0.0341 | 0.0000 | 13.0000 |
| 166 | 127-128 | 0.0002 | 0.0160 | 0.0000 | 8.5000 |
| 167 | 127- 43 | 0.0019 | 0.0149 | 0.0000 | 42.5000 |
| 168 | 127- 9 | 0.0021 | 0.0094 | 0.0000 | 42.5000 |
| 169 | 128- 7 | 0.0009 | 0.0122 | 0.0000 | 21.2500 |
| 170 | 128- 43 | 0.0004 | 0.0048 | 0.0001 | 22.3500 |
| 171 | 115- 54 | 0.0008 | 0.0213 | 0.0000 | 8.0900 |
| 172 | 115- 54 | 0.0008 | 0.0213 | 0.0000 | 8.0900 |
| 173 | 115- 94 | 0.0014 | 0.0111 | 0.0000 | 11.5100 |
| 174 | 105- 12 | 0.0011 | 0.0308 | 0.0000 | 42.5000 |
| 175 | 105- 14 | 0.0013 | 0.0316 | 0.0000 | 42.5000 |
| 176 | 129- 14 | 0.0013 | 0.0316 | 0.0000 | 42.5000 |
| 177 | 129- 19 | 0.0013 | 0.0102 | 0.0000 | 6.4600 |
| 178 | 84-120 | 0.0003 | 0.0030 | 0.0001 | 23.0300 |
| 179 | 84-120 | 0.0003 | 0.0044 | 0.0001 | 18.7000 |
| 180 | 87- 37 | 0.0006 | 0.0067 | 0.0001 | 15.8100 |
| 181 | 56- 18 | 0.0007 | 0.0070 | 0.0001 | 15.7200 |
| 182 | 57- 21 | 0.0007 | 0.0071 | 0.0001 | 15.7200 |
| 183 | 118- 74 | 0.0004 | 0.0041 | 0.0001 | 13.9400 |
| 184 | 118- 75 | 0.0004 | 0.0041 | 0.0001 | 13.9400 |
| 185 | 118-130 | 0.0002 | 0.0027 | 0.0001 | 18.7000 |
| 186 | 131- 12 | 0.0024 | 0.0396 | 0.0000 | 4.2500 |
| 187 | 131- 40 | 0.0035 | 0.0404 | 0.0000 | 42.5000 |
| 188 | 131- 19 | -0.0017 | 0.0683 | 0.0000 | 42.5000 |
| 189 | 3- 5 | 0.0012 | 0.0094 | 0.0000 | 9.3500 |
| 190 | 89-115 | 0.0015 | 0.0118 | 0.0000 | 13.0000 |
| 191 | 89-115 | 0.0015 | 0.0118 | 0.0000 | 13.0000 |
| 192 | 89- 91 | 0.0018 | 0.0146 | 0.0000 | 13.0000 |
| 193 | 132- 18 | 0.0002 | 0.0015 | 0.0003 | 13.0000 |
| 194 | 132- 21 | 0.0002 | 0.0015 | 0.0003 | 13.0000 |
| 195 | 26-111 | 0.0003 | 0.0038 | 0.0001 | 9.4300 |
| 196 | 26-111 | 0.0003 | 0.0038 | 0.0001 | 9.4300 |

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|---|---|---|---|---|---|
| 197 | 26-110 | 0.0010 | 0.0128 | 0.0000 | 18.8700 |
| 198 | 26-112 | 0.0005 | 0.0064 | 0.0001 | 18.7000 |
| 199 | 133-109 | 0.0005 | 0.0062 | 0.0001 | 18.7000 |
| 200 | 133-109 | 0.0005 | 0.0062 | 0.0001 | 18.7000 |
| 201 | 16-133 | 0.0000 | 0.0000 | 0.0313 | 42.5000 |
| 202 | 43- 81 | 0.0006 | 0.0053 | 0.0001 | 15.7200 |
| 203 | 43- 72 | 0.0008 | 0.0106 | 0.0000 | 21.2500 |
| 204 | 53-113 | 0.0020 | 0.0334 | 0.0000 | 42.5000 |
| 205 | 67- 61 | 0.0005 | 0.0186 | 0.0000 | 6.3700 |
| 206 | 67- 68 | 0.0013 | 0.0122 | 0.0000 | 42.5000 |
| 207 | 74- 75 | -0.0003 | 0.0534 | 0.0000 | 2.5000 |
| 208 | 24- 99 | 0.0013 | 0.0176 | 0.0000 | 23.0300 |
| 209 | 39- 49 | 0.0019 | 0.0160 | 0.0000 | 11.5100 |
| 210 | 39- 49 | 0.0019 | 0.0160 | 0.0000 | 11.5100 |
| 211 | 36- 35 | 0.0069 | 0.0402 | 0.0000 | 42.5000 |
| 212 | 99- 36 | 0.0002 | 0.0160 | 0.0000 | 6.3700 |
| 213 | 37-116 | 0.0006 | 0.0065 | 0.0001 | 15.8100 |
| 214 | 29-134 | 0.0003 | 0.0040 | 0.0001 | 23.0300 |
| 215 | 29-135 | 0.0003 | 0.0040 | 0.0001 | 23.0300 |
| 216 | 29- 82 | 0.0005 | 0.0072 | 0.0001 | 23.0300 |
| 217 | 29- 49 | 0.0011 | 0.0146 | 0.0000 | 23.0300 |
| 218 | 70-136 | 0.0010 | 0.0134 | 0.0000 | 18.7000 |
| 219 | 70- 71 | 0.0026 | 0.0353 | 0.0000 | 9.3500 |
| 220 | 90- 91 | 0.0006 | 0.0081 | 0.0000 | 9.3500 |
| 221 | 116- 85 | 0.0012 | 0.0111 | 0.0000 | 15.8100 |
| 222 | 116- 23 | 0.0059 | 0.0718 | 0.0000 | 42.5000 |
| 223 | 93-101 | 0.0004 | 0.0059 | 0.0001 | 18.7000 |
| 224 | 93- 79 | 0.0001 | 0.0086 | 0.0000 | 11.2200 |
| 225 | 93- 62 | 0.0005 | 0.0042 | 0.0001 | 11.5100 |
| 226 | 30-130 | 0.0005 | 0.0039 | 0.0001 | 11.5100 |
| 227 | 30-119 | 0.0005 | 0.0040 | 0.0001 | 11.5100 |
| 228 | 102- 87 | 0.0006 | 0.0134 | 0.0000 | 42.5000 |
| 229 | 102-103 | 0.0012 | 0.0128 | 0.0000 | 10.8800 |
| 230 | 134-137 | 0.0002 | 0.0160 | 0.0000 | 42.5000 |
| 231 | 134-137 | 0.0002 | 0.0160 | 0.0000 | 42.5000 |
| 232 | 135-137 | 0.0002 | 0.0160 | 0.0000 | 42.5000 |
| 233 | 135-137 | 0.0002 | 0.0160 | 0.0000 | 42.5000 |
| 234 | 138- 32 | 0.0005 | 0.0068 | 0.0001 | 26.0100 |
| 235 | 138- 32 | 0.0005 | 0.0068 | 0.0001 | 26.0100 |
| 236 | 138- 39 | 0.0012 | 0.0167 | 0.0000 | 26.0100 |
| 237 | 138- 29 | 0.0013 | 0.0179 | 0.0000 | 23.0300 |
| 238 | 38- 35 | 0.0023 | 0.0237 | 0.0000 | 42.5000 |
| 239 | 103- 85 | 0.0011 | 0.0169 | 0.0000 | 42.5000 |
| 240 | 103- 69 | 0.0010 | 0.0273 | 0.0000 | 8.0900 |
| 241 | 139- 22 | 0.0007 | 0.0136 | 0.0000 | 42.5000 |
| 242 | 139-140 | 0.0025 | 0.0206 | 0.0000 | 11.5100 |
| 243 | 139- 24 | 0.0008 | 0.0106 | 0.0000 | 20.4800 |
| 244 | 139- 24 | 0.0014 | 0.0207 | 0.0000 | 42.5000 |
| 245 | 139-141 | 0.0029 | 0.0272 | 0.0000 | 11.5100 |
| 246 | 82-142 | 0.0002 | 0.0029 | 0.0001 | 23.0300 |

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|---|---|---|---|---|---|
| 247 | 136- 7 | 0.0007 | 0.0098 | 0.0000 | 18.7000 |
| 248 | 136- 8 | 0.0002 | 0.0160 | 0.0000 | 6.3700 |
| 249 | 136- 8 | 0.0002 | 0.0160 | 0.0000 | 6.3700 |
| 250 | 68- 69 | 0.0002 | 0.0160 | 0.0000 | 6.3700 |
| 251 | 68- 69 | 0.0002 | 0.0161 | 0.0000 | 6.3700 |
| 252 | 130-106 | 0.0002 | 0.0160 | 0.0000 | 6.3700 |
| 253 | 119-106 | 0.0002 | 0.0160 | 0.0000 | 6.3700 |
| 254 | 141-140 | 0.0014 | 0.0513 | 0.0000 | 42.5000 |
| 255 | 141-107 | 0.0092 | 0.0701 | 0.0000 | 42.5000 |
| 256 | 141-108 | 0.0013 | 0.1045 | 0.0000 | 42.5000 |
| 257 | 49-120 | 0.0012 | 0.0166 | 0.0000 | 18.7000 |
| 258 | 49-120 | 0.0012 | 0.0166 | 0.0000 | 18.7000 |
| 259 | 137- 76 | 0.0005 | 0.0045 | 0.0001 | 42.5000 |
| 260 | 137-143 | 0.0010 | 0.0096 | 0.0000 | 42.5000 |
| 261 | 72- 9 | 0.0002 | 0.0160 | 0.0000 | 6.3700 |
| 262 | 144- 20 | 0.0004 | 0.0118 | 0.0000 | 42.5000 |
| 263 | 144-132 | 0.0005 | 0.0226 | 0.0000 | 42.5000 |
| 264 | 144- 18 | 0.0021 | 0.0411 | 0.0000 | 42.5000 |
| 265 | 144-145 | 0.0005 | 0.0121 | 0.0000 | 6.3700 |
| 266 | 62-113 | 0.0003 | 0.0241 | 0.0000 | 4.2500 |
| 267 | 62-113 | 0.0003 | 0.0242 | 0.0000 | 4.2500 |
| 268 | 145- 73 | 0.0003 | 0.0150 | 0.0000 | 42.5000 |
| 269 | 145-131 | 0.0004 | 0.0034 | 0.0001 | 42.5000 |
| 270 | 143-132 | 0.0004 | 0.0284 | 0.0000 | 6.3700 |
| 271 | 143-132 | 0.0004 | 0.0285 | 0.0000 | 6.3700 |
| 272 | 19- 40 | 0.0008 | 0.0114 | 0.0000 | 42.5000 |
| 273 | 142- 77 | 0.0020 | 0.0270 | 0.0000 | 18.7000 |
| 274 | 142- 77 | 0.0020 | 0.0270 | 0.0000 | 18.7000 |
| 275 | 142- 29 | 0.0003 | 0.0043 | 0.0001 | 23.0300 |

**** THERE IS NO SHUNT ELEMENT IN THE NETWORK *****

## F.4 ESTIMATED NODAL LOADINGS & LOSSES AT TARGET TIME

Note: Loss is initially estimated as a percentage of the total load demand at target time and shared equally among all node to simulated approximated the transmission losses.

| NODE | LOAD(pu) | LOSS (pu) | TOTAL |
|---|---|---|---|
| 1 | 1.2202 | 0.0401 | 1.2604 |
| 2 | 2.1531 | 0.0401 | 2.1932 |
| 3 | 2.1356 | 0.0401 | 2.1757 |
| 4 | 1.2202 | 0.0401 | 1.2604 |
| 5 | 3.1061 | 0.0401 | 3.1462 |
| 6 | 5.3008 | 0.0401 | 5.3409 |
| 7 | 0.7625 | 0.0401 | 0.8026 |
| 8 | 5.4972 | 0.0401 | 5.5373 |
| 9 | 6.1501 | 0.0401 | 6.1902 |
| 10 | 1.3448 | 0.0401 | 1.3849 |
| 11 | 0.7698 | 0.0401 | 0.8099 |
| 12 | 0.5081 | 0.0401 | 0.5482 |
| 13 | 2.3796 | 0.0401 | 2.4197 |
| 14 | 0.5081 | 0.0401 | 0.5482 |
| 15 | 1.6175 | 0.0401 | 1.6576 |
| 16 | 0.0000 | 0.0401 | 0.0401 |
| 17 | 6.4551 | 0.0401 | 6.4952 |
| 18 | 0.0000 | 0.0401 | 0.0401 |
| 19 | 3.8348 | 0.0401 | 3.8749 |
| 20 | 0.5596 | 0.0401 | 0.5998 |
| 21 | 0.0000 | 0.0401 | 0.0401 |
| 22 | 7.1053 | 0.0401 | 7.1454 |
| 23 | 2.9594 | 0.0401 | 2.9995 |
| 24 | 2.7906 | 0.0401 | 2.8307 |
| 25 | 6.3235 | 0.0401 | 6.3636 |
| 26 | 5.9097 | 0.0401 | 5.9498 |
| 27 | 3.1971 | 0.0401 | 3.2372 |
| 28 | 0.8446 | 0.0401 | 0.8847 |
| 29 | 1.9279 | 0.0401 | 1.9680 |
| 30 | 3.5946 | 0.0401 | 3.6347 |
| 31 | 0.8446 | 0.0401 | 0.8847 |
| 32 | 6.1217 | 0.0401 | 6.1618 |
| 33 | 5.0580 | 0.0401 | 5.0981 |
| 34 | 3.3478 | 0.0401 | 3.3879 |
| 35 | 1.5899 | 0.0401 | 1.6300 |
| 36 | 3.6092 | 0.0401 | 3.6493 |
| 37 | 1.9054 | 0.0401 | 1.9455 |
| 38 | 4.7264 | 0.0401 | 4.7665 |
| 39 | 4.0914 | 0.0401 | 4.1315 |
| 40 | 2.0680 | 0.0401 | 2.1081 |
| 41 | 0.0000 | 0.0401 | 0.0401 |

| | | | |
|---|---|---|---|
| 42 | 8.8522 | 0.0401 | 8.8923 |
| 43 | 3.1088 | 0.0401 | 3.1489 |
| 44 | 0.8775 | 0.0401 | 0.9176 |
| 45 | 0.0000 | 0.0401 | 0.0401 |
| 46 | 0.8775 | 0.0401 | 0.9176 |
| 47 | 0.0000 | 0.0401 | 0.0401 |
| 48 | 2.1789 | 0.0401 | 2.2190 |
| 49 | 4.6324 | 0.0401 | 4.6725 |
| 50 | 8.3078 | 0.0401 | 8.3479 |
| 51 | 3.3037 | 0.0401 | 3.3438 |
| 52 | 6.0793 | 0.0401 | 6.1194 |
| 53 | 10.2785 | 0.0401 | 10.3186 |
| 54 | 4.4315 | 0.0401 | 4.4716 |
| 55 | 2.1000 | 0.0401 | 2.1401 |
| 56 | 0.0000 | 0.0401 | 0.0401 |
| 57 | 2.2352 | 0.0401 | 2.2753 |
| 58 | 1.2282 | 0.0401 | 1.2683 |
| 59 | 5.3596 | 0.0401 | 5.3997 |
| 60 | 0.0000 | 0.0401 | 0.0401 |
| 61 | 0.6648 | 0.0401 | 0.7049 |
| 62 | 0.0000 | 0.0401 | 0.0401 |
| 63 | 0.8810 | 0.0401 | 0.9211 |
| 64 | 0.0000 | 0.0401 | 0.0401 |
| 65 | 8.4357 | 0.0401 | 8.4759 |
| 66 | 0.9597 | 0.0401 | 0.9999 |
| 67 | 5.0027 | 0.0401 | 5.0428 |
| 68 | 6.1878 | 0.0401 | 6.2279 |
| 69 | 0.6955 | 0.0401 | 0.7356 |
| 70 | 1.1473 | 0.0401 | 1.1874 |
| 71 | 4.4008 | 0.0401 | 4.4409 |
| 72 | 0.0000 | 0.0401 | 0.0401 |
| 73 | 4.4345 | 0.0401 | 4.4747 |
| 74 | 1.2558 | 0.0401 | 1.2959 |
| 75 | 1.2558 | 0.0401 | 1.2959 |
| 76 | 10.7436 | 0.0401 | 10.7837 |
| 77 | 0.0000 | 0.0401 | 0.0401 |
| 78 | 6.5940 | 0.0401 | 6.6341 |
| 79 | 1.7755 | 0.0401 | 1.8156 |
| 80 | 4.6271 | 0.0401 | 4.6672 |
| 81 | 2.1440 | 0.0401 | 2.1841 |
| 82 | 4.6271 | 0.0401 | 4.6673 |
| 83 | 4.1887 | 0.0401 | 4.2288 |
| 84 | 4.5391 | 0.0401 | 4.5792 |
| 85 | 4.1096 | 0.0401 | 4.1497 |
| 86 | 5.1333 | 0.0401 | 5.1734 |
| 87 | 2.7371 | 0.0401 | 2.7773 |
| 88 | 16.0369 | 0.0401 | 16.0770 |
| 89 | 2.8171 | 0.0401 | 2.8572 |
| 90 | 3.6530 | 0.0401 | 3.6931 |
| 91 | 0.5265 | 0.0401 | 0.5666 |

| | | | |
|---|---|---|---|
| 92 | 0.0000 | 0.0401 | 0.0401 |
| 93 | 3.4911 | 0.0401 | 3.5312 |
| 94 | 0.0000 | 0.0401 | 0.0401 |
| 95 | 0.4784 | 0.0401 | 0.5185 |
| 96 | 0.0000 | 0.0401 | 0.0401 |
| 97 | 0.0000 | 0.0401 | 0.0401 |
| 98 | 0.0000 | 0.0401 | 0.0401 |
| 99 | 0.0000 | 0.0401 | 0.0401 |
| 100 | 0.0000 | 0.0401 | 0.0401 |
| 101 | 2.3254 | 0.0401 | 2.3655 |
| 102 | 7.5107 | 0.0401 | 7.5508 |
| 103 | 6.5935 | 0.0401 | 6.6336 |
| 104 | 8.6385 | 0.0401 | 8.6786 |
| 105 | 3.6057 | 0.0401 | 3.6458 |
| 106 | 3.5009 | 0.0401 | 3.5410 |
| 107 | 12.6099 | 0.0401 | 12.6500 |
| 108 | 9.9599 | 0.0401 | 10.0000 |
| 109 | 5.7888 | 0.0401 | 5.8289 |
| 110 | 0.0000 | 0.0401 | 0.0401 |
| 111 | 2.2854 | 0.0401 | 2.3255 |
| 112 | 3.3098 | 0.0401 | 3.3499 |
| 113 | 7.5305 | 0.0401 | 7.5707 |
| 114 | 0.0000 | 0.0401 | 0.0401 |
| 115 | 1.8941 | 0.0401 | 1.9342 |
| 116 | 5.0650 | 0.0401 | 5.1051 |
| 117 | 0.0000 | 0.0401 | 0.0401 |
| 118 | 1.8552 | 0.0401 | 1.8953 |
| 119 | 0.0000 | 0.0401 | 0.0401 |
| 120 | 1.9402 | 0.0401 | 1.9803 |
| 121 | 2.6900 | 0.0401 | 2.7301 |
| 122 | 7.7330 | 0.0401 | 7.7731 |
| 123 | 3.0680 | 0.0401 | 3.1081 |
| 124 | 0.0000 | 0.0401 | 0.0401 |
| 125 | 0.0000 | 0.0401 | 0.0401 |
| 126 | 0.7698 | 0.0401 | 0.8099 |
| 127 | 7.5529 | 0.0401 | 7.5931 |
| 128 | 0.0000 | 0.0401 | 0.0401 |
| 129 | 1.5212 | 0.0401 | 1.5613 |
| 130 | 0.0000 | 0.0401 | 0.0401 |
| 131 | 2.4510 | 0.0401 | 2.4911 |
| 132 | 3.0381 | 0.0401 | 3.0782 |
| 133 | 0.0000 | 0.0401 | 0.0401 |
| 134 | 1.3327 | 0.0401 | 1.3728 |
| 135 | 0.9165 | 0.0401 | 0.9566 |
| 136 | 0.0000 | 0.0401 | 0.0401 |
| 137 | 6.3542 | 0.0401 | 6.3943 |
| 138 | 0.0000 | 0.0401 | 0.0401 |
| 139 | 2.9407 | 0.0401 | 2.9808 |
| 140 | 7.9509 | 0.0401 | 7.9910 |
| 141 | 20.8898 | 0.0401 | 20.9299 |

| | | | |
|---|---|---|---|
| 142 | 1.5427 | 0.0401 | 1.5828 |
| 143 | 4.5761 | 0.0401 | 4.6162 |
| 144 | 9.8371 | 0.0401 | 9.8772 |
| 145 | 4.7689 | 0.0401 | 4.8090 |
| | ------------- | ----------- | ----------- |
| Total | 465.2800 | 5.8160 | 471.0960 |

## F.5 DPLM Iterations

```
ITERATION=              1
TOTLOAD=   471.0960          LOSSEST=   5.816000
ITERATION=              2
TOTLOAD=   471.6931          LOSSEST=   6.413126
ITERATION=              3
TOTLOAD=   470.2196          LOSSEST=   4.939567
ITERATION=              4
TOTLOAD=   469.9094          LOSSEST=   4.629397
ITERATION=              5
TOTLOAD=   469.8049          LOSSEST=   4.524900
ITERATION=              6
TOTLOAD=   469.8096          LOSSEST=   4.529627
ITERATION=              7
TOTLOAD=   469.8056          LOSSEST=   4.525579
```

## F.6 DISPATCH SOLUTION:

### a) Optimal Generator Loadings:

| GEN | OUTPUT(p.u) | COST(£) | Pen.Fact | Effective-$ | % SPARE C |
|---|---|---|---|---|---|
| 1 | 5.5000 | 15774.00 | 1.0070 | 2888.16 | 0.0000 |
| 2 | 8.6000 | 24664.80 | 1.0070 | 2888.16 | 0.0000 |
| 3 | 0.5100 | 1462.68 | 1.0070 | 2888.16 | 0.0000 |
| 4 | 3.7598 | 10782.97 | 1.0070 | 2888.16 | 0.0001 |
| 5 | 0.0000 | 0.00 | 0.9912 | 3032.02 | 1.0000 |
| 6 | 11.5498 | 4469.79 | 0.9958 | 385.36 | 0.0000 |
| 7 | 0.0238 | 67.65 | 1.0442 | 2965.51 | 0.9947 |
| 8 | 0.0238 | 67.65 | 1.0442 | 2965.51 | 0.9962 |
| 9 | 0.0238 | 67.65 | 1.0442 | 2965.51 | 0.9950 |
| 10 | 0.0000 | 0.00 | 1.0030 | 5461.50 | 1.0000 |
| 11 | 0.0000 | 0.00 | 1.0465 | 3422.97 | 1.0000 |
| 12 | 2.8199 | 7371.27 | 1.0704 | 2798.14 | 0.0000 |
| 13 | 19.3199 | 50502.13 | 1.0704 | 2798.14 | 0.0000 |
| 14 | 0.6800 | 1777.52 | 1.0704 | 2798.14 | 0.0000 |
| 15 | 2.2395 | 6241.42 | 1.0554 | 2941.27 | 0.0002 |
| 16 | 0.0000 | 0.00 | 1.0472 | 3094.57 | 1.0000 |
| 17 | 0.0000 | 0.00 | 1.0120 | 10879.55 | 1.0000 |
| 18 | 18.2000 | 52161.20 | 0.9972 | 2857.94 | 0.0000 |
| 19 | 0.9997 | 2865.20 | 0.9972 | 2857.94 | 0.0003 |
| 20 | 11.4999 | 4450.48 | 1.0070 | 389.71 | 0.0000 |
| 21 | 11.5500 | 4469.85 | 1.0070 | 389.71 | 0.0000 |
| 22 | 0.7000 | 270.90 | 1.0070 | 389.71 | 0.0000 |
| 23 | 3.3600 | 9710.40 | 1.0143 | 2931.37 | 0.0000 |
| 24 | 1.3998 | 4045.35 | 1.0143 | 2931.37 | 0.0002 |
| 25 | 0.0086 | 25.62 | 1.0197 | 3026.55 | 0.9923 |

| | | | | | |
|---|---|---|---|---|---|
| 26 | 0.0086 | 25.62 | 1.0197 | 3026.55 | 0.9974 |
| 27 | 0.0086 | 25.62 | 1.0197 | 3026.55 | 0.9972 |
| 28 | 0.0086 | 25.62 | 1.0197 | 3026.55 | 0.9975 |
| 29 | 0.0000 | 0.00 | 1.0197 | 3026.55 | 1.0000 |
| 30 | 0.0000 | 0.00 | 1.0143 | 3289.49 | 1.0000 |
| 31 | 0.0000 | 0.00 | 1.0143 | 3289.49 | 1.0000 |
| 32 | 0.0000 | 0.00 | 1.0191 | 10955.82 | 1.0000 |
| 33 | 0.0000 | 0.00 | 0.9963 | 5424.63 | 1.0000 |
| 34 | 0.0000 | 0.00 | 0.9963 | 5424.63 | 1.0000 |
| 35 | 0.0238 | 68.84 | 1.0316 | 2981.18 | 0.9894 |
| 36 | 0.0238 | 68.84 | 1.0316 | 2981.18 | 0.9894 |
| 37 | 1.0150 | 2873.58 | 1.0433 | 2953.50 | 0.5469 |
| 38 | 0.0238 | 68.36 | 1.0391 | 2982.15 | 0.9974 |
| 39 | 9.4016 | 26136.49 | 1.0583 | 2942.15 | 0.0020 |
| 40 | 0.4417 | 1227.95 | 1.0583 | 2942.15 | 0.2112 |
| 41 | 0.0000 | 0.00 | 0.9906 | 5243.51 | 1.0000 |
| 42 | 0.0000 | 0.00 | 0.9906 | 5243.51 | 1.0000 |
| 43 | 0.0000 | 0.00 | 0.9854 | 5365.65 | 1.0000 |
| 44 | 0.0000 | 0.00 | 0.9854 | 5365.65 | 1.0000 |
| 45 | 0.9997 | 2633.26 | 1.0428 | 2746.63 | 0.0003 |
| 46 | 18.4000 | 48465.60 | 1.0428 | 2746.63 | 0.0000 |
| 47 | 1.1200 | 3170.72 | 1.0292 | 2913.62 | 0.0000 |
| 48 | 0.0000 | 0.00 | 1.0445 | 3416.51 | 1.0000 |
| 49 | 0.0000 | 0.00 | 1.0441 | 5601.69 | 1.0000 |
| 50 | 0.0000 | 0.00 | 1.0441 | 5601.69 | 1.0000 |
| 51 | 1.6400 | 4598.56 | 1.0465 | 2934.50 | 0.0000 |
| 52 | 0.6800 | 1906.72 | 1.0465 | 2934.50 | 0.0000 |
| 53 | 1.6400 | 4349.28 | 1.0522 | 2790.38 | 0.0000 |
| 54 | 18.8000 | 49857.60 | 1.0522 | 2790.38 | 0.0000 |
| 55 | 0.6800 | 1803.36 | 1.0522 | 2790.38 | 0.0000 |
| 56 | 1.3998 | 3712.20 | 1.0522 | 2790.38 | 0.0002 |
| 57 | 8.4000 | 7560.00 | 1.0466 | 941.90 | 0.0000 |
| 58 | 0.6800 | 1789.08 | 1.0304 | 2711.08 | 0.0000 |
| 59 | 19.3199 | 50830.56 | 1.0304 | 2711.08 | 0.0000 |
| 60 | 0.5000 | 1292.50 | 1.0317 | 2667.03 | 0.0000 |
| 61 | 9.2000 | 23782.00 | 1.0317 | 2667.03 | 0.0000 |
| 62 | 18.7495 | 47905.07 | 1.0625 | 2714.74 | 0.0000 |
| 63 | 1.0494 | 2681.34 | 1.0625 | 2714.74 | 0.0005 |
| 64 | 1.0494 | 2681.34 | 1.0660 | 2723.59 | 0.0005 |
| 65 | 18.7495 | 47905.07 | 1.0660 | 2723.59 | 0.0000 |
| 66 | 8.6000 | 23297.40 | 1.0614 | 2875.34 | 0.0000 |
| 67 | 0.3400 | 921.06 | 1.0614 | 2875.34 | 0.0000 |
| 68 | 0.3400 | 921.06 | 1.0669 | 2890.10 | 0.0000 |
| 69 | 8.6000 | 23297.40 | 1.0669 | 2890.10 | 0.0000 |
| 70 | 9.6000 | 3715.20 | 1.0126 | 391.87 | 0.0000 |
| 71 | 11.5500 | 4469.85 | 1.0126 | 391.87 | 0.0000 |
| 72 | 11.5500 | 4469.85 | 1.0126 | 391.87 | 0.0000 |
| 73 | 0.0000 | 0.00 | 0.9895 | 10638.36 | 1.0000 |
| 74 | 0.0000 | 0.00 | 0.9895 | 10638.36 | 1.0000 |
| 75 | 0.0000 | 0.00 | 0.9890 | 10632.49 | 1.0000 |

| | | | | | |
|---|---|---|---|---|---|
| 76 | 0.0238 | 71.96 | 0.9893 | 2988.79 | 0.9650 |
| 77 | 0.0238 | 71.96 | 0.9893 | 2988.79 | 0.9982 |
| 78 | 6.4200 | 2657.88 | 1.1339 | 469.42 | 0.0000 |
| 79 | 10.4000 | 4305.60 | 1.1339 | 469.42 | 0.0000 |
| 80 | 12.5196 | 32951.52 | 1.0968 | 2886.70 | 0.0000 |
| 81 | 0.0000 | 0.00 | 0.9874 | 5318.00 | 1.0000 |
| 82 | 0.0000 | 0.00 | 0.9874 | 5318.00 | 1.0000 |
| 83 | 0.0000 | 0.00 | 0.9874 | 5318.00 | 1.0000 |
| 84 | 1.0200 | 2907.93 | 1.0323 | 2943.13 | 0.0000 |
| 85 | 2.2395 | 6384.75 | 1.0323 | 2943.13 | 0.0002 |
| 86 | 2.9998 | 8552.43 | 1.0323 | 2943.13 | 0.0001 |
| 87 | 3.9200 | 11175.92 | 1.0323 | 2943.13 | 0.0000 |
| 88 | 3.7598 | 10719.06 | 1.0323 | 2943.13 | 0.0001 |
| 89 | 0.3400 | 940.10 | 1.0273 | 2840.51 | 0.0000 |
| 90 | 9.1900 | 25410.35 | 1.0273 | 2840.51 | 0.0000 |
| 91 | 0.0000 | 0.00 | 0.9895 | 5308.62 | 1.0000 |
| 92 | 0.0000 | 0.00 | 0.9895 | 5308.62 | 1.0000 |
| 93 | 0.3812 | 1142.53 | 0.9889 | 2963.81 | 0.4394 |
| 94 | 7.4181 | 22231.98 | 0.9889 | 2963.81 | 0.6136 |
| 95 | 18.4000 | 49569.60 | 1.0469 | 2820.29 | 0.0000 |
| 96 | 0.6800 | 1831.92 | 1.0469 | 2820.29 | 0.0000 |
| 97 | 9.9999 | 4139.96 | 1.0534 | 436.09 | 0.0000 |
| 98 | 0.7000 | 289.80 | 1.0719 | 443.75 | 0.0000 |
| 99 | 9.9999 | 4139.96 | 1.0719 | 443.75 | 0.0000 |
| 100 | 12.3000 | 5092.20 | 1.0719 | 443.75 | 0.0000 |
| 101 | 0.0000 | 0.00 | 0.9833 | 5329.55 | 1.0000 |
| 102 | 0.0000 | 0.00 | 0.9833 | 5329.55 | 1.0000 |
| 103 | 0.0000 | 0.00 | 0.9946 | 10693.00 | 1.0000 |
| 104 | 11.5500 | 4469.85 | 1.0180 | 393.98 | 0.0000 |
| 105 | 11.5500 | 4469.85 | 1.0180 | 393.98 | 0.0000 |
| 106 | 0.0000 | 0.00 | 1.0531 | 3444.68 | 1.0000 |
| 107 | 0.0000 | 0.00 | 1.0531 | 3444.68 | 1.0000 |
| 108 | 3.7496 | 9820.28 | 1.0587 | 2772.75 | 0.0001 |
| 109 | 6.4200 | 16897.44 | 1.0942 | 2879.85 | 0.0000 |
| 110 | 12.5196 | 32951.52 | 1.0942 | 2879.85 | 0.0000 |
| 111 | 11.5200 | 30320.64 | 1.0942 | 2879.85 | 0.0000 |
| 112 | 0.0000 | 0.00 | 1.0046 | 10800.10 | 1.0000 |
| 113 | 3.7598 | 11012.32 | 0.9924 | 2906.69 | 0.0001 |
| 114 | 8.6399 | 25306.22 | 0.9924 | 2906.69 | 0.0000 |
| 115 | 0.0000 | 0.00 | 0.9767 | 10500.28 | 1.0000 |
| | ---------- | --------------- | | | |
| Total = | 469.806 | £929621.19 | | | |

**b) Line Flow**

| Line | Node | Power Flow Limit | Flow by AC Loadflow | Flow by DPLM Approx. |
|---|---|---|---|---|
| 1 | 1- 2 | 9.3500 | -2.4490 | -2.4534 |
| 2 | 1- 3 | 9.3500 | 1.2288 | 1.2317 |
| 3 | 4- 2 | 9.3500 | -1.7346 | -1.7400 |
| 4 | 4- 5 | 9.3500 | 0.5144 | 0.5177 |
| 5 | 6- 7 | 42.5000 | 3.2730 | 3.2988 |
| 6 | 6- 8 | 42.5000 | 3.3856 | 3.3605 |
| 7 | 6- 9 | 42.5000 | 6.4106 | 6.3427 |
| 8 | 10- 5 | 13.0000 | 1.4204 | 1.4158 |
| 9 | 10- 11 | 13.9400 | -2.7652 | -2.7691 |
| 10 | 12- 13 | 11.5100 | -5.5237 | -5.4764 |
| 11 | 14- 13 | 11.5100 | -5.3659 | -5.3505 |
| 12 | 15- 16 | 18.7000 | -0.6528 | -0.6764 |
| 13 | 17- 18 | 42.5000 | -1.6307 | -1.6827 |
| 14 | 17- 19 | 42.5000 | -3.8101 | -3.7166 |
| 15 | 20- 17 | 6.3700 | 1.0145 | 1.0631 |
| 16 | 20- 21 | 12.0000 | -5.0844 | -5.1365 |
| 17 | 22- 23 | 4.4200 | -1.3652 | -1.3943 |
| 18 | 22- 23 | 4.4200 | -1.3614 | -1.3901 |
| 19 | 22- 24 | 42.5000 | -0.5585 | -0.5395 |
| 20 | 25- 26 | 18.7000 | 2.9318 | 3.0293 |
| 21 | 25- 26 | 18.7000 | 2.9318 | 3.0293 |
| 22 | 25- 27 | 23.0300 | -6.0935 | -6.2075 |
| 23 | 25- 27 | 23.0300 | -6.0935 | -6.2075 |
| 24 | 28- 29 | 23.0300 | -3.9861 | -3.7682 |
| 25 | 28- 30 | 23.0300 | 3.1415 | 2.9187 |
| 26 | 31- 32 | 23.0300 | -8.9926 | -8.8803 |
| 27 | 31- 30 | 23.0300 | 8.1480 | 8.0022 |
| 28 | 33- 34 | 42.5000 | 1.6417 | 1.6471 |
| 29 | 33- 35 | 42.5000 | -2.7540 | -2.7522 |
| 30 | 33- 36 | 42.5000 | -1.3460 | -1.3622 |
| 31 | 33- 37 | 63.7000 | -2.2126 | -2.2222 |
| 32 | 33- 38 | 42.5000 | -0.3871 | -0.3815 |
| 33 | 32- 39 | 11.5600 | -0.6711 | -0.5440 |
| 34 | 40- 41 | 18.7000 | -8.6339 | -8.5575 |
| 35 | 40- 41 | 18.7000 | -8.6339 | -8.5575 |
| 36 | 40- 42 | 18.7000 | 5.0834 | 5.0121 |
| 37 | 40- 42 | 18.7000 | 5.0834 | 5.0121 |
| 38 | 40- 43 | 15.7200 | -1.7778 | -1.7322 |
| 39 | 40- 43 | 15.7200 | -1.7778 | -1.7322 |
| 40 | 44- 45 | 15.8100 | -4.6274 | -4.6020 |
| 41 | 46- 45 | 15.8100 | -4.6274 | -4.6020 |
| 42 | 47- 45 | 42.5000 | 9.2734 | 9.7690 |
| 43 | 48- 29 | 23.0300 | 4.1078 | 4.0395 |
| 44 | 48- 49 | 23.0300 | -6.2867 | -6.2356 |
| 45 | 50- 51 | 42.5000 | -1.8615 | -1.8490 |

| | | | | |
|---|---|---|---|---|
| 46 | 50- 52 | 42.5000 | 0.3913 | 0.3782 |
| 47 | 50- 53 | 42.5000 | 0.1232 | 0.1263 |
| 48 | 50- 54 | 42.5000 | -2.2007 | -2.2074 |
| 49 | 55- 56 | 15.7200 | 6.6458 | 6.5227 |
| 50 | 55- 57 | 15.7200 | 7.0950 | 6.9723 |
| 51 | 55- 58 | 15.7200 | -7.9204 | -7.8279 |
| 52 | 55- 58 | 15.7200 | -7.9204 | -7.8279 |
| 53 | 59- 60 | 11.5100 | 4.0327 | 3.9355 |
| 54 | 59- 61 | 9.3500 | -4.8452 | -4.7591 |
| 55 | 59- 62 | 9.3500 | -0.0088 | -0.0410 |
| 56 | 63- 64 | 18.7000 | -0.1581 | -0.1196 |
| 57 | 63- 16 | 18.7000 | -0.7229 | -0.7651 |
| 58 | 65- 66 | 42.5000 | 0.1687 | -0.2691 |
| 59 | 65- 67 | 42.5000 | -1.0680 | -1.4393 |
| 60 | 65- 68 | 42.5000 | -2.8360 | -3.0446 |
| 61 | 65- 69 | 42.5000 | 2.4730 | -2.6703 |
| 62 | 7- 70 | 23.5500 | 0.3824 | 0.3872 |
| 63 | 7- 70 | 23.5500 | 0.3877 | 0.3924 |
| 64 | 7- 71 | 9.4300 | -1.4133 | -1.3771 |
| 65 | 7- 72 | 23.5500 | 0.9093 | 0.8795 |
| 66 | 73- 74 | 12.2500 | -3.7984 | -3.9309 |
| 67 | 73- 75 | 12.2500 | -3.7830 | -3.9275 |
| 68 | 73- 76 | 42.5000 | 0.2109 | 0.3732 |
| 69 | 77- 78 | 16.1500 | 8.2195 | 8.0859 |
| 70 | 77- 79 | 11.2200 | 4.7089 | 4.7087 |
| 71 | 77- 69 | 20.4800 | -9.3675 | -9.0780 |
| 72 | 80- 41 | 8.7000 | -0.9293 | -1.0137 |
| 73 | 80- 41 | 18.7000 | -0.9293 | -1.0137 |
| 74 | 80- 13 | 18.7000 | -3.0061 | -2.9100 |
| 75 | 80- 81 | 9.4300 | 1.7504 | 1.7675 |
| 76 | 80- 82 | 18.7000 | -2.0621 | -2.0476 |
| 77 | 80- 71 | 9.3500 | 0.5494 | 0.6142 |
| 78 | 83- 84 | 11.5100 | 1.2833 | 1.2405 |
| 79 | 83- 84 | 11.5100 | 1.2833 | 1.2405 |
| 80 | 83- 24 | 23.0300 | -0.6527 | -0.6483 |
| 81 | 85- 59 | 13.0000 | 4.5644 | 4.4969 |
| 82 | 85- 86 | 13.9400 | 0.9594 | 0.9251 |
| 83 | 85- 86 | 3.9400 | 0.9594 | 0.9251 |
| 84 | 85- 87 | 15.8100 | -3.9855 | -3.9141 |
| 85 | 86- 88 | 42.5000 | -7.4524 | -7.4259 |
| 86 | 86- 89 | 13.0000 | 6.4295 | 6.3338 |
| 87 | 86- 90 | 23.0300 | -1.2141 | -1.2190 |
| 88 | 86- 90 | 23.0300 | -1.2141 | -1.2190 |
| 89 | 86- 91 | 13.9400 | 0.2341 | 0.2242 |
| 90 | 51- 53 | 42.5000 | 3.8470 | 3.8335 |
| 91 | 60- 51 | 6.3700 | 4.5268 | 4.4932 |
| 92 | 60- 51 | 6.3700 | 4.4987 | 4.4653 |
| 93 | 60- 92 | 15.7200 | 8.0562 | 7.9728 |
| 94 | 60- 93 | 11.5100 | -5.6215 | -5.5868 |
| 95 | 60- 94 | 13.0000 | -7.4441 | -7.4619 |

| | | | | |
|---|---|---|---|---|
| 96 | 95- 83 | 23.0300 | 6.1244 | 6.0403 |
| 97 | 95- 96 | 17.0000 | -2.8932 | -2.7935 |
| 98 | 95- 97 | 18.7000 | 1.6849 | 1.8952 |
| 99 | 95- 84 | 23.0300 | 14.4056 | 14.1258 |
| 100 | 96- 98 | 21.0680 | -2.1057 | -2.0991 |
| 101 | 96- 99 | 23.0300 | 5.9105 | 5.8217 |
| 102 | 96- 69 | 22.4400 | 13.1005 | 13.2583 |
| 103 | 100- 27 | 23.0300 | 7.7440 | 7.8337 |
| 104 | 100- 27 | 23.0300 | 7.7440 | 7.8337 |
| 105 | 100- 58 | 11.5100 | 8.6060 | 8.4816 |
| 106 | 100- 58 | 11.5100 | 8.6060 | 8.4816 |
| 107 | 13-101 | 15.7200 | -7.3935 | -7.2906 |
| 108 | 13- 93 | 15.7200 | -8.0289 | -7.9508 |
| 109 | 13- 82 | 18.7000 | -0.9055 | -1.0047 |
| 110 | 97-102 | 8.0900 | 3.6938 | 3.7000 |
| 111 | 97- 61 | 23.0300 | 6.9307 | 7.1089 |
| 112 | 98- 35 | 23.0300 | -7.9438 | -7.9946 |
| 113 | 98- 37 | 23.0300 | 5.5453 | 5.4492 |
| 114 | 98- 69 | 18.7000 | 9.2323 | 9.3407 |
| 115 | 34- 35 | 42.5000 | -1.7295 | -1.7189 |
| 116 | 34- 38 | 42.5000 | -1.1644 | -1.1456 |
| 117 | 34-103 | 9.5200 | 1.1857 | 1.1680 |
| 118 | 104-105 | 42.5000 | -0.0500 | -0.0397 |
| 119 | 104- 82 | 6.3700 | -4.3490 | -4.3426 |
| 120 | 104- 82 | 6.3700 | -4.3490 | -4.3426 |
| 121 | 104-106 | 42.5000 | 0.1095 | 0.0693 |
| 122 | 107- 23 | 42.5000 | 4.1090 | 4.0357 |
| 123 | 108- 23 | 42.5000 | 2.7019 | 2.6930 |
| 124 | 64- 15 | 18.7000 | 0.9651 | 0.9402 |
| 125 | 2- 64 | 42.5000 | 1.1250 | 1.3212 |
| 126 | 2- 44 | 20.4800 | -3.7351 | -3.7146 |
| 127 | 2- 46 | 20.4800 | -3.7351 | -3.7146 |
| 128 | 109-110 | 42.5000 | 4.3594 | 4.0802 |
| 129 | 111-110 | 18.7000 | -2.5747 | -2.5384 |
| 130 | 111-112 | 18.7000 | 1.8405 | 1.8308 |
| 131 | 52- 53 | 42.5000 | -0.3954 | -0.3773 |
| 132 | 52- 54 | 42.5000 | -1.1726 | -1.1554 |
| 133 | 52-113 | 42.5000 | -2.6047 | -2.6206 |
| 135 | 114- 92 | 15.8100 | -5.2332 | -5.1751 |
| 136 | 114-115 | 13.0000 | -7.5139 | -7.4579 |
| 137 | 114- 43 | 15.7200 | 5.0042 | 4.8710 |
| 138 | 114- 71 | 15.7200 | 6.2265 | 6.0826 |
| 139 | 88-116 | 42.5000 | -0.9818 | -0.9487 |
| 140 | 42- 26 | 15.8100 | 0.6520 | 0.5814 |
| 141 | 42- 26 | 15.8100 | 0.6520 | 0.5814 |
| 142 | 117- 56 | 18.7000 | -1.3817 | -1.1676 |
| 143 | 117- 57 | 18.7000 | 0.3833 | 0.6033 |
| 144 | 117-118 | 18.7000 | 2.6924 | 1.9470 |
| 145 | 117-119 | 18.7000 | -1.6940 | -1.3999 |
| 146 | 78- 82 | 15.8100 | 4.4743 | 4.4162 |

| | | | | |
|---|---|---|---|---|
| 147 | 78- 82 | 15.8100 | 4.4743 | 4.4162 |
| 148 | 78-120 | 16.1500 | -7.4835 | -7.5163 |
| 149 | 92- 53 | 8.0900 | 2.7633 | 2.7671 |
| 150 | 121- 23 | 4.0000 | -2.6900 | -2.6968 |
| 151 | 122- 22 | 42.5000 | 1.6397 | 1.6011 |
| 152 | 122- 24 | 42.5000 | 0.6273 | 0.6629 |
| 153 | 123- 23 | 42.5000 | 1.8061 | 1.8300 |
| 154 | 123-116 | 42.5000 | 18.1259 | 18.0576 |
| 155 | 66- 68 | 42.5000 | -1.0531 | -1.0911 |
| 156 | 124- 66 | 8.0900 | -0.1781 | 0.0823 |
| 157 | 124- 66 | 4.2500 | -0.0839 | 0.0387 |
| 158 | 124- 93 | 11.5100 | 3.3737 | 3.0258 |
| 159 | 124-120 | 11.5100 | -3.1117 | -3.1702 |
| 160 | 47-125 | 42.5000 | 8.0625 | 8.8643 |
| 161 | 125- 43 | 15.8100 | 4.0243 | 4.0404 |
| 162 | 125- 43 | 15.8100 | 4.0243 | 4.0404 |
| 163 | 47- 11 | 15.8100 | 3.5542 | 3.5517 |
| 164 | 47-126 | 15.8100 | 2.8770 | 2.8722 |
| 165 | 5-126 | 13.0000 | -2.0858 | -2.0921 |
| 166 | 127-128 | 8.5000 | -2.5105 | -2.5411 |
| 167 | 127- 43 | 42.5000 | -3.2471 | -3.2902 |
| 168 | 127- 9 | 42.5000 | -1.7953 | -1.7359 |
| 169 | 128- 7 | 21.2500 | -0.6097 | -0.6109 |
| 170 | 128- 43 | 22.3500 | -1.9018 | -1.9324 |
| 171 | 115- 54 | 8.0900 | 3.9191 | 3.9070 |
| 172 | 115- 54 | 8.0900 | 3.9191 | 3.9070 |
| 173 | 115- 94 | 11.5100 | -2.2229 | -2.2151 |
| 174 | 105- 12 | 42.5000 | -1.8367 | -1.8347 |
| 175 | 105- 14 | 42.5000 | -1.8190 | -1.8119 |
| 176 | 129- 14 | 42.5000 | -3.0231 | -3.0081 |
| 177 | 129- 19 | 6.4600 | 1.5019 | 1.4787 |
| 178 | 84-120 | 23.0300 | 7.3104 | 7.1246 |
| 179 | 84-120 | 18.7000 | 5.0376 | 4.8996 |
| 180 | 87- 37 | 15.8100 | -7.5870 | -7.5402 |
| 181 | 56- 18 | 15.7200 | 5.2401 | 5.3469 |
| 182 | 57- 21 | 15.7200 | 5.2159 | 5.3231 |
| 183 | 118- 74 | 13.9400 | 5.0711 | 5.1899 |
| 184 | 118- 75 | 13.9400 | 5.0620 | 5.1958 |
| 185 | 118-130 | 18.7000 | -3.1322 | -2.5119 |
| 186 | 131- 12 | 4.2500 | -3.1517 | -3.1038 |
| 187 | 131- 40 | 42.5000 | -2.1606 | -2.1113 |
| 188 | 131- 19 | 42.5000 | -0.2224 | -0.2008 |
| 189 | 3- 5 | 9.3500 | -0.9084 | -0.9042 |
| 190 | 89-115 | 13.0000 | 2.8005 | 2.7446 |
| 191 | 89-115 | 13.0000 | 2.8005 | 2.7446 |
| 192 | 89- 91 | 13.0000 | -2.0135 | -1.9980 |
| 193 | 132- 18 | 13.0000 | -1.7339 | -1.7857 |
| 194 | 132- 21 | 13.0000 | -0.0947 | -0.1503 |
| 195 | 26-111 | 9.4300 | 0.7759 | 0.7898 |
| 196 | 26-111 | 9.4300 | 0.7759 | 0.7898 |

| | | | | |
|---|---|---|---|---|
| 197 | 26-110 | 18.8700 | -1.7789 | -1.7510 |
| 198 | 26-112 | 18.7000 | 1.4711 | 1.4750 |
| 199 | 133-109 | 18.7000 | 5.0864 | 5.0506 |
| 200 | 133-109 | 18.7000 | 5.0864 | 5.0506 |
| 201 | 16-133 | 42.5000 | 10.1797 | 11.2972 |
| 202 | 43- 81 | 15.7200 | 0.3944 | 0.3772 |
| 203 | 43- 72 | 21.2500 | 0.7092 | 0.7029 |
| 204 | 53-113 | 42.5000 | -3.9514 | -3.9529 |
| 205 | 67- 61 | 6.3700 | -1.3508 | -1.6501 |
| 206 | 67- 68 | 42.5000 | -4.7210 | -4.8117 |
| 207 | 74- 75 | 42.5000 | -0.0032 | 0.0005 |
| 208 | 24- 99 | 23.0300 | -3.2964 | -3.2057 |
| 209 | 39- 49 | 11.5100 | -1.0899 | -0.9947 |
| 210 | 39- 49 | 11.5100 | -1.0899 | -0.9947 |
| 211 | 36- 35 | 42.5000 | -2.3901 | -2.3962 |
| 212 | 99- 36 | 6.3700 | 2.5848 | 2.6048 |
| 213 | 37-116 | 15.8100 | -3.9935 | -4.0224 |
| 214 | 29-134 | 23.0300 | 8.0854 | 7.9433 |
| 215 | 29-135 | 23.0300 | 7.7952 | 7.6670 |
| 216 | 29- 82 | 23.0300 | -0.4640 | -0.3035 |
| 217 | 29- 49 | 23.0300 | -5.3293 | -5.2760 |
| 218 | 70-136 | 18.7000 | 0.5277 | 0.5209 |
| 219 | 70- 71 | 9.3500 | -0.9055 | -0.8909 |
| 220 | 90- 91 | 9.3500 | 2.3161 | 2.3058 |
| 221 | 116- 85 | 15.8100 | 7.9869 | 7.9299 |
| 222 | 116- 23 | 42.5000 | 0.0052 | 0.0051 |
| 223 | 93-101 | 18.7000 | 9.8495 | 9.6809 |
| 224 | 93- 79 | 11.2200 | -4.0181 | -4.0375 |
| 225 | 93- 62 | 11.5100 | 0.2255 | 0.2452 |
| 226 | 30-130 | 11.5100 | 4.4182 | 4.0982 |
| 227 | 30-119 | 11.5100 | 3.2488 | 3.1967 |
| 228 | 102- 87 | 42.5000 | -3.1743 | -3.1847 |
| 229 | 102-103 | 10.8800 | -0.6640 | -0.6410 |
| 230 | 134-137 | 42.5000 | 3.3667 | 3.2992 |
| 231 | 134-137 | 42.5000 | 3.3667 | 3.2992 |
| 232 | 135-137 | 42.5000 | 3.4307 | 3.3694 |
| 233 | 135-137 | 42.5000 | 3.4307 | 3.3694 |
| 234 | 138- 32 | 26.0100 | 7.2702 | 7.2535 |
| 235 | 138- 32 | 26.0100 | 7.2702 | 7.2535 |
| 236 | 138- 39 | 26.0100 | 2.5911 | 2.6522 |
| 237 | 138- 29 | 23.0300 | 5.8071 | 5.8828 |
| 238 | 38- 35 | 42.5000 | -6.2813 | -6.3037 |
| 239 | 103- 85 | 42.5000 | -1.3062 | -1.3326 |
| 240 | 103- 69 | 8.0900 | -4.7682 | -4.7505 |
| 241 | 139- 22 | 42.5000 | 2.1905 | 2.1344 |
| 242 | 139-140 | 11.5100 | 0.7200 | 0.7224 |
| 243 | 139- 24 | 20.4800 | 0.0529 | 0.0788 |
| 244 | 139- 24 | 42.5000 | 0.0267 | 0.0403 |
| 245 | 139-141 | 11.5100 | -5.9309 | -5.9697 |
| 246 | 82-142 | 23.0300 | -7.8823 | -7.8847 |

| | | | | |
|---|---|---|---|---|
| 247 | 136- 7 | 18.7000 | -1.6237 | -1.6352 |
| 248 | 136- 8 | 6.3700 | 1.0755 | 1.0780 |
| 249 | 136- 8 | 6.3700 | 1.0755 | 1.0780 |
| 250 | 68- 69 | 6.3700 | -2.4390 | -2.6649 |
| 251 | 68- 69 | 6.3700 | -2.4331 | -2.6586 |
| 252 | 130-106 | 6.3700 | 1.2745 | 1.5893 |
| 253 | 119-106 | 6.3700 | 1.5486 | 1.7954 |
| 254 | 141-140 | 42.5000 | 3.4979 | 3.4889 |
| 255 | 141-107 | 42.5000 | -0.1010 | -0.1022 |
| 256 | 141-108 | 42.5000 | 0.1419 | 0.1524 |
| 257 | 49-120 | 18.7000 | -9.2439 | -9.1334 |
| 258 | 49-120 | 18.7000 | -9.2439 | -9.1334 |
| 259 | 137- 76 | 42.5000 | 10.5834 | 10.3921 |
| 260 | 137-143 | 42.5000 | -3.3523 | -3.4479 |
| 261 | 72- 9 | 6.3700 | 1.6177 | 1.5836 |
| 262 | 144- 20 | 42.5000 | -3.5055 | -3.5085 |
| 263 | 144-132 | 42.5000 | -3.2421 | -3.2667 |
| 264 | 144- 18 | 42.5000 | -1.8456 | -1.8600 |
| 265 | 144-145 | 6.3700 | -1.2440 | -1.2067 |
| 266 | 62-113 | 4.2500 | 0.1084 | 0.1010 |
| 267 | 62-113 | 4.2500 | 0.1083 | 0.1009 |
| 268 | 145- 73 | 42.5000 | -2.9339 | -3.0341 |
| 269 | 145-131 | 2.5000 | -3.0798 | -2.9479 |
| 270 | 143-132 | 6.3700 | 2.2344 | 2.1869 |
| 271 | 143-132 | 6.3700 | 2.2265 | 2.1792 |
| 272 | 19- 40 | 42.5000 | -6.3799 | -6.2986 |
| 273 | 142- 77 | 18.7000 | -7.7997 | -7.7143 |
| 274 | 142- 77 | 18.7000 | -7.7997 | -7.7143 |
| 275 | 142- 29 | 23.0300 | 6.1610 | 5.8701 |

APPENDIX G

EXAMPLE TEST SYSTEM FOR CCDP UNIT COMMITMENT TECHNIQUE

## G.1 Forecast Load Data

Creation Time: 07/02/1989.23:30:58

| Interval | Time | Load(pu) |
|---|---|---|
| 1 | 07/02/1989.23:30:58 | 2.78763 |
| 2 | 08/02/1989.00:30:58 | 2.41584 |
| 3 | 08/02/1989.01:30:58 | 3.35704 |
| 4 | 08/02/1989.02:30:58 | 2.37357 |
| 5 | 08/02/1989.03:30:58 | 2.41881 |
| 6 | 08/02/1989.04:30:58 | 2.46977 |
| 7 | 08/02/1989.05:30:58 | 2.59380 |
| 8 | 08/02/1989.06:30:58 | 3.13985 |
| 9 | 08/02/1989.07:30:58 | 4.00743 |
| 10 | 08/02/1989.08:30:58 | 4.34407 |
| 11 | 08/02/1989.09:30:58 | 4.32895 |
| 12 | 08/02/1989.10:30:58 | 4.27594 |
| 13 | 08/02/1989.11:30:58 | 4.31826 |
| 14 | 08/02/1989.12:30:58 | 4.19189 |
| 15 | 08/02/1989.13:30:58 | 4.22599 |
| 16 | 08/02/1989.14:30:58 | 4.23733 |
| 17 | 08/02/1989.15:30:58 | 4.26424 |
| 18 | 08/02/1989.16:30:58 | 4.56830 |
| 19 | 08/02/1989.17:30:58 | 4.59079 |
| 20 | 08/02/1989.18:30:58 | 4.45107 |
| 21 | 08/02/1989.19:30:58 | 4.33508 |
| 22 | 08/02/1989.20:30:58 | 4.21500 |
| 23 | 08/02/1989.21:30:58 | 4.09217 |
| 24 | 08/02/1989.22:30:58 | 3.81805 |

## G.2 Thermal Generator Data

No. of Unit = 12

| Name | Capacity(pu) Min | Max. | Cost a | b | c | Cold | a | Sh.down |
|---|---|---|---|---|---|---|---|---|
| Thermal-01 | 0.50 | 2.00 | 29.0 | 190.0 | 100.0 | 113.0 | 2.0 | 13.50 |
| Thermal-02 | 0.50 | 1.50 | 29.0 | 200.0 | 150.0 | 113.0 | 1.5 | 11.50 |
| Thermal-03 | 0.20 | 0.70 | 25.0 | 210.0 | 170.0 | 101.0 | 1.0 | 10.00 |
| Thermal-04 | 0.10 | 0.50 | 15.0 | 210.0 | 170.0 | 85.00 | 0.5 | 8.50 |
| Thermal-05 | 0.10 | 0.50 | 15.0 | 210.0 | 170.0 | 85.00 | 0.5 | 8.50 |
| Thermal-06 | 0.10 | 0.50 | 15.0 | 210.0 | 170.0 | 85.00 | 0.5 | 8.50 |
| Thermal-07 | 0.50 | 2.00 | 29.0 | 190.0 | 100.0 | 113.00 | 2.0 | 13.50 |
| Thermal-08 | 0.50 | 1.50 | 29.0 | 200.0 | 150.0 | 113.00 | 1.5 | 11.50 |
| Thermal-09 | 0.20 | 0.70 | 25.0 | 210.0 | 170.0 | 101.00 | 1.0 | 10.00 |
| Thermal-10 | 0.10 | 0.50 | 15.0 | 210.0 | 170.0 | 85.00 | 0.5 | 8.50 |
| Thermal-11 | 0.10 | 0.50 | 15.0 | 210.0 | 170.0 | 85.00 | 0.5 | 8.50 |
| Thermal-12 | 0.10 | 0.50 | 15.0 | 210.0 | 170.0 | 85.00 | 0.5 | 8.50 |

Total Capacity = 11.40 p.u. ( 1p.u.=100MW.)

| Name | Min. Time Up | Dwn | Status | Time Changed Last | Ramp Rate (per min.) |
|---|---|---|---|---|---|
| Thermal-01 | 3.00 | 3.00 | 1 | 06/02/1989.23:00:00 | 0.040 |
| Thermal-02 | 3.00 | 3.00 | 0 | 06/02/1989.23:00:00 | 0.030 |
| Thermal-03 | 2.00 | 2.00 | 1 | 06/02/1989.23:00:00 | 0.014 |
| Thermal-04 | 1.00 | 1.00 | 1 | 06/02/1989.23:00:00 | 0.010 |
| Thermal-05 | 1.00 | 1.00 | 1 | 06/02/1989.23:00:00 | 0.010 |
| Thermal-06 | 1.00 | 1.00 | 1 | 06/02/1989.23:00:00 | 0.010 |
| Thermal-07 | 3.00 | 3.00 | 1 | 06/02/1989.23:00:00 | 0.040 |
| Thermal-08 | 3.00 | 3.00 | 1 | 06/02/1989.23:00:00 | 0.030 |
| Thermal-09 | 2.00 | 2.00 | 1 | 06/02/1989.23:00:00 | 0.014 |
| Thermal-10 | 1.00 | 1.00 | 1 | 06/02/1989.23:00:00 | 0.010 |
| Thermal-11 | 1.00 | 1.00 | 1 | 06/02/1989.23:00:00 | 0.010 |
| Thermal-12 | 1.00 | 1.00 | 0 | 06/02/1989.23:00:00 | 0.010 |

Note: *No unit must be 'on', or must be 'off', fixed generation, scheduled for maintenance or forced outage during the study period.*

## G.3 Generation Schedule

| Time | Load | Cap | Spin | Thermal Generators 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 07/02.23:30. | 56 | OK | OK | 13 | 0 | 0 | 6 | 6 | 6 | 13 | 0 | 0 | 6 | 6 | 0 |
| 08/02.00:30 | 48 | OK | OK | 11 | 0 | 0 | 6 | 5 | 5 | 11 | 0 | 0 | 5 | 5 | 0 |
| 08/02.01:30 | 67 | OK | OK | 15 | 0 | 0 | 8 | 8 | 7 | 15 | 0 | 0 | 7 | 7 | 0 |
| 08/02.02:30 | 47 | OK | OK | 11 | 0 | 0 | 5 | 5 | 5 | 11 | 0 | 0 | 5 | 5 | 0 |
| 08/02.03:30 | 48 | OK | OK | 11 | 0 | 0 | 6 | 5 | 5 | 11 | 0 | 0 | 5 | 5 | 0 |
| 08/02.04:30 | 49 | OK | OK | 11 | 0 | 0 | 6 | 6 | 5 | 11 | 0 | 0 | 5 | 5 | 0 |
| 08/02.05:30 | 52 | OK | OK | 11 | 0 | 0 | 6 | 6 | 6 | 11 | 0 | 0 | 6 | 6 | 0 |
| 08/02.06:30 | 63 | OK | OK | 14 | 0 | 0 | 7 | 7 | 7 | 14 | 0 | 0 | 7 | 7 | 0 |
| 08/02.07:30 | 80 | OK | OK | 16 | 0 | 0 | 8 | 8 | 8 | 16 | 0 | 0 | 8 | 8 | 8 |
| 08/02.08:30 | 87 | OK | OK | 17 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 9 |
| 08/02.09:30 | 87 | OK | OK | 17 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 9 |
| 08/02.10:30 | 86 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 9 |
| 08/02.11:30 | 86 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 9 |
| 08/02.12:30 | 84 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 8 | 8 |
| 08/02.13:30 | 85 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 8 |
| 08/02.14:30 | 85 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 8 |
| 08/02.15:30 | 85 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 8 |
| 08/02.16:30 | 91 | OK | OK | 16 | 0 | 0 | 9 | 8 | 8 | 16 | 10 | 0 | 8 | 8 | 8 |
| 08/02.17:30 | 92 | OK | OK | 16 | 0 | 0 | 9 | 9 | 8 | 16 | 10 | 0 | 8 | 8 | 8 |
| 08/02.18:30 | 89 | OK | OK | 16 | 0 | 0 | 8 | 8 | 8 | 15 | 10 | 0 | 8 | 8 | 8 |
| 08/02.19:30 | 87 | OK | OK | 15 | 0 | 0 | 8 | 8 | 8 | 15 | 10 | 0 | 8 | 8 | 7 |
| 08/02.20:30 | 84 | OK | OK | 15 | 0 | 0 | 8 | 8 | 7 | 15 | 10 | 0 | 7 | 7 | 7 |
| 08/02.21:30 | 82 | OK | OK | 15 | 0 | 0 | 7 | 7 | 7 | 15 | 10 | 0 | 7 | 7 | 7 |
| 08/02.22:30 | 76 | OK | OK | 13 | 0 | 0 | 7 | 7 | 7 | 13 | 10 | 0 | 7 | 6 | 6 |

*Note:*

*1) Generation: 1 = generation level 1;*
*0 = Generator is off.*

*2) OK = generation capacity or spinning reserve requirenent satisfied*

## G.4 Operating Costs of Each Interval

| Intval | Load | Fuel(£) | Startup(£) | ShutDwn(£) | Subtotal |
|---|---|---|---|---|---|
| 1 | 56 | 856.00 | 0.00 | 31.00 | 887.00 |
| 2 | 48 | 733.30 | 0.00 | 0.00 | 733.00 |
| 3 | 67 | 1035.87 | 0.00 | 0.00 | 1035.87 |
| 4 | 47 | 718.13 | 0.00 | 0.00 | 718.13 |
| 5 | 48 | 733.30 | 0.00 | 0.00 | 733.30 |
| 6 | 49 | 748.48 | 0.00 | 0.00 | 748.48 |
| 7 | 52 | 794.00 | 0.00 | 0.00 | 794.00 |
| 8 | 63 | 968.62 | 0.00 | 0.00 | 968.62 |
| 9 | 80 | 1247.20 | 85.00 | 0.00 | 1332.20 |
| 10 | 87 | 1371.30 | 0.00 | 0.00 | 1371.30 |
| 11 | 87 | 1371.30 | 0.00 | 0.00 | 1371.30 |
| 12 | 86 | 1353.55 | 0.00 | 0.00 | 1353.55 |
| 13 | 86 | 1353.55 | 0.00 | 0.00 | 1353.55 |
| 14 | 84 | 1318.10 | 0.00 | 0.00 | 1318.10 |
| 15 | 85 | 1335.83 | 0.00 | 0.00 | 1335.83 |
| 16 | 85 | 1335.83 | 0.00 | 0.00 | 1335.83 |
| 17 | 85 | 1335.83 | 0.00 | 0.00 | 1335.83 |
| 18 | 91 | 1431.42 | 108.74 | 0.00 | 1540.16 |
| 19 | 92 | 1449.15 | 0.00 | 0.00 | 1449.15 |
| 20 | 89 | 1396.45 | 0.00 | 0.00 | 1396.45 |
| 21 | 87 | 1362.32 | 0.00 | 0.00 | 1362.32 |
| 22 | 84 | 1311.70 | 0.00 | 0.00 | 1311.70 |
| 23 | 82 | 1277.95 | 0.00 | 0.00 | 1277.95 |
| 24 | 76 | 1179.90 | 0.00 | 0.00 | 1179.90 |
| Total Cost: | | 28019.07 | 193.73 | 31.00 | 28243.81 |

## G.5 Operating Cost of Each Thermal Unit

| No. | Name | Fuel (£) | StartUp(£) | ShutDn(£) | Subtotal |
|---|---|---|---|---|---|
| 1 | Thermal-01 | 5305.75 | 0.00 | 0.00 | 5305.75 |
| 2 | Thermal-02 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | Thermal-03 | 0.00 | 0.00 | 10.00 | 10.00 |
| 4 | Thermal-04 | 2942.85 | 0.00 | 0.00 | 2942.85 |
| 5 | Thermal-05 | 2894.78 | 0.00 | 0.00 | 2894.78 |
| 6 | Thermal-06 | 2828.13 | 0.00 | 0.00 | 2828.13 |
| 7 | Thermal-07 | 5253.00 | 0.00 | 0.00 | 5253.00 |
| 8 | Thermal-08 | 1165.50 | 108.74 | 11.00 | 1285.24 |
| 9 | Thermal-09 | 0.00 | 0.00 | 10.00 | 10.00 |
| 10 | Thermal-10 | 2828.13 | 0.00 | 0.00 | 2828.13 |
| 11 | Thermal-11 | 2794.38 | 0.00 | 0.00 | 2794.38 |
| 12 | Thermal-12 | 2006.57 | 85.00 | 0.00 | 2091.57 |
| Total Cost: | | 28019.07 | 193.73 | 31.00 | 28243.81 |

## APPENDIX H

## TECHNICAL PAPERS PUBLISHED


1. "*Large Scale Unit Commitment using a Composite Thermal Generator Operating Cost Function*", IEE Second Int. Conf. on Power System Monitoring and Control, July 1986, Publication No. 266, pp.332-337.

2. "*Large Scale Dynamic Programming based Dispatch including Transmission Losses*", IFAC Symposium Power System, Modelling and Control Applications", Brussels, 5-8 September 1988, paper 15.2.

3. "*Security Constrained Dispatch with Post-contingency Corrective Rescheduling using Linear Programming*", IFAC Symposium on Power Systems and Power Plant Control, Seoul, Korea, August 22-25, 1989.

# LARGE SCALE UNIT COMMITMENT USING A COMPOSITE THERMAL GENERATOR OPERATING COST FUNCTION

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## ABSTRACT

Operating cost saving can be achieved by proper scheduling of the startup and shutdown of available generating units. This paper describes a composite operating cost model which is used with a new computational algorithm based on the dynamic programming (DP) principle for selecting and assigning loading levels of generators to obtain the optimal commitment schedule. The method proposed brings the dimensionality problem normally associated with the DP technique under control by storing only an appropriate range of stages and states necessary to allow the computation to proceed. Experience of the algorithm show that the computer time required to obtain the optimal unit combination is independent of the number of generators in the system but depends on the total generating capacity and required accuracy. An approximation formula is presented for estimating the computer time requirement. A test system which has 224 units and 51,750 MW installed capacity is used to demonstrate the potential practicability of the technique to a large real system.

## INTRODUCTION

The daily load pattern of a power system may exhibit large differences between minimum and maximum demand despite tariff adjustment which attempts to produce a more uniform profile. Faced with this situation, electric utilities usually have fewer generating units running at lighter load periods to save cost. Due to the non-linearity and time variant characteristics of various constraints governing the operation of the units and the system, this unit commitment problem is highly complex. Generally, rigorous mathematical optimization methods are impractical for solution of realistically sized systems (1). With the advance of computer aided control in power systems, many different methods of solution with varying degree of simplification have been reported in the literature. Among these, the dynamic programming technique has attracted considerable interest because of its ability to recognise the non-linearity and time dependent nature of the constraints (2). There is, however, a major disadvantage in the DP technique since it requires excess computer storage and processing time when the number of generators in a system increases.

This paper presents a composite cost dynamic programming (CCDP) method which it is proposed overcomes the dimensionality problem and can produce the unit commitment schedule within a reasonable time.

## OPERATING CONSTRAINTS

The proposed unit commitment method schedules the units with the following constraints taken into consideration:

1. Unit minimum and maximum output limits: These output limits define the allowable output power of the generating units.

2. Fuel cost non-linearity: The fuel cost to output power relation is often non-linear. A generally accepted approximation of this fuel cost function is in quadratic form, ie.

$$F_i(y) = A_i+B_i.y+C_i.y^2 \qquad i=1,2,3,...N \quad ...... (1)$$

where $N$ = total number of units in the system
$F_i(y)$ = fuel cost when unit i generates y MW
$A_i$, $B_i$, $C_i$ = fuel cost coefficients

The commitment method proposed is applicable to any form of fuel cost function. It may be non-differentiable, non-convex or emperical.

3. Startup cost: The startup cost of a unit depends on the length of time the unit has been shutdown prior to starting up. Without loss of generality, the following startup cost function can be implemented:

$$S_i(t) = U_i.R_i.t/(1.+R_i.t) \qquad i=1,2,3,...N \quad .... (2)$$

where $S_i(t)$ = Startup cost of unit i
$U_i$ = Cold startup cost
$R_i$ = Cooling rate
$t$ = Time passed since the unit shutdown

4. Shutdown cost: The shutdown cost of a thermal unit is normally small compare with its startup cost. The proposed technique uses a fixed shutdown cost for each unit.

5. Minimum up/down time: In daily operation there is generally a requirement that an unit runs or stays shutdown for a certain minimum period of time before it changes status again.

6. Fixed generation units: These are the pre-scheduled units. Any unit may be pre-scheduled to must be "on", must be "off" or fixed generation for certain intervals of the study period. Specification of such requirements is feed to the commitment program as input data. Units scheduled for maintenance or in forced outage can therefore be treated as must be "off" units.

7. Derated capacity: Partial outages of the units leading to derated capacity, or changes from derated state to full capacity state or to another derated state, in certain intervals of the stuty period can be specified and treated as input data to the commitment program.

8. Spinning reserve: Spinning reserve requirement is one of the major considerations in unit commitment. It can be defined as the extra generation available on demand from the on-line generators within a time period short enough to maintain acceptable frequency for possible operating contingencies (3). In the technique proposed, two conditions must be satisfied by the unit combination selected. Firstly the total capacity of the committed units at any time interval must exceed the forecast load of that interval by a certain percentage. Secondly the loss of generation of any loaded unit must be picked up by the remaining on-line units within a specified short time period.

Figure 1 depicts the various input data required by the unit commitment program. The program produces two important results, namely, the commitment schedule and the estimated production cost for the forecast load. The commitment schedule feeds the economic dispatch program for finer tuning of the load sharing between the committed units.

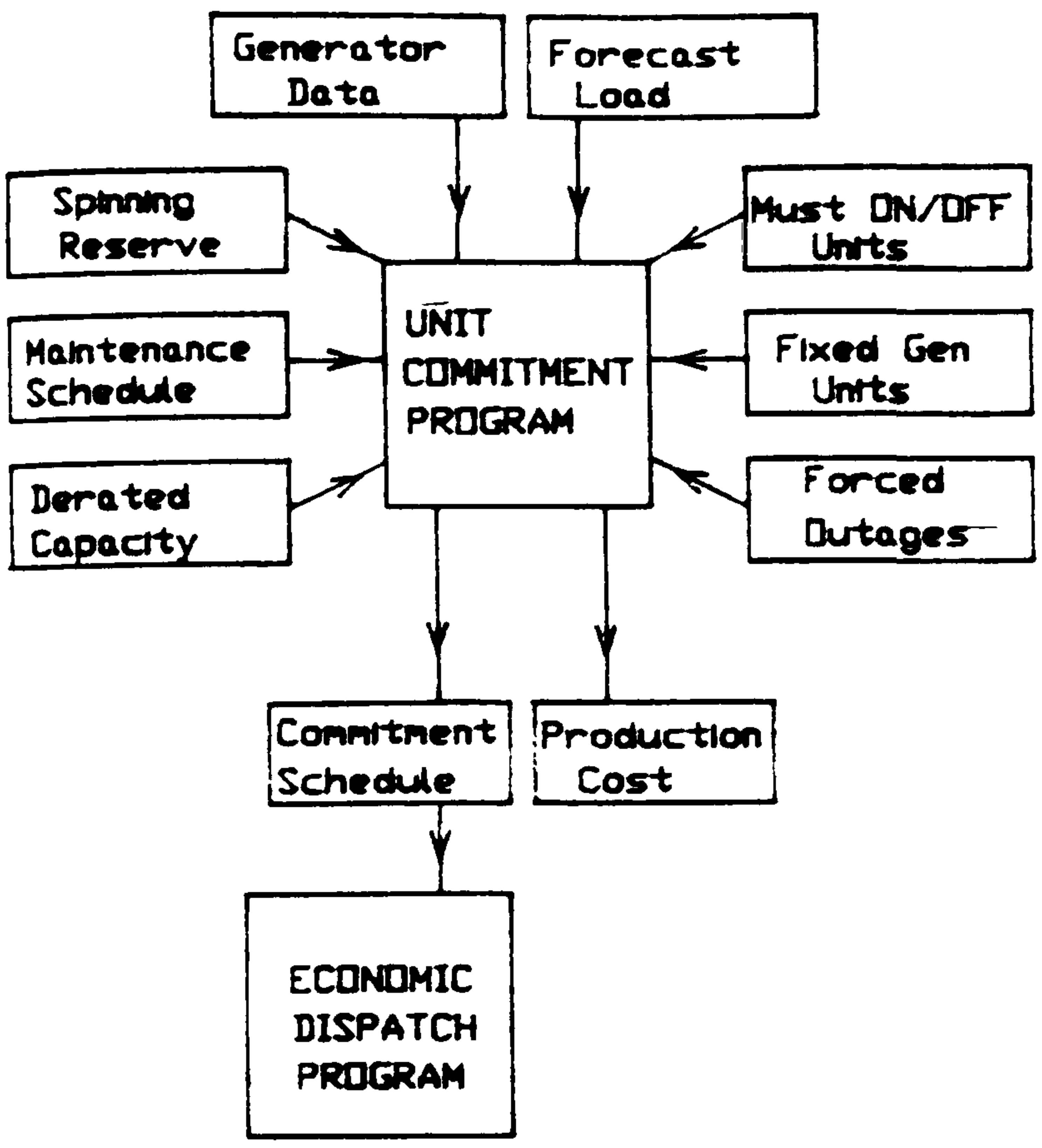


Figure 1 Input and output data of unit commitment program

## DYNAMIC PROGRAMMING COMPUTATIONAL ALGORITHM

The use of the dynamic programming technique in the unit commitment problem has been reported in the literature (4-8) and has received great attention because of its flexibility and ability to recognize non-linear and time dependent constraints. In the following, a new calculation algorithm based on the dynamic programming principle (9-10) will be outlined.

Given that there are N units in a system with each unit having a known fuel cost function as described by equation (1). It will be assumed that the generating capacity of all units can be discretised to a multiple of x MW steps such that its fuel cost function may now be described as costs at different output levels. To find the optimal combination of the N generating units to give a minimum fuel cost at a certain load level L, the following equation may be used:

$$G(L) = MIN\ \{G(L-j.x)+\Delta F_i(j.x)\} \qquad i=1,2,3...N \quad j=1,2,3...M \quad ..(3)$$

where

$G(L)$ = Optimal total fuel cost for load L

L = 0, x, 2x, 3x, ... Tx where Tx equals total generating capacity

$\Delta F_i(j.x)$ = Additional operating cost for unit i to generate further j.x MW from its optimal loading at $G(L-j.x)$

N = Number of generating units

M = Highest generation level of the largest unit

$G(L)$ = 0.0 for $L \leq 0.0$

Since $G(L)$ is known for L=0.0, optimal operating cost and the corresponding optimal unit combination at load levels x, 2x, 3x ... Tx can be calculated with the given unit cost function $F_i(y)$ by applying equation (3) iteratively: hence the recursive nature of dynamic programming.

From equation (3), it can be observed that in finding G(L), M optimal unit combinations at optimal cost G(L-x), G(L-2x), G(L-3x), ... G(L-M.x) are needed. In other words, besides storing the cost functions of each unit and other necessary data, the computer memory requirement for the algorithm is only N.M words. For the U.K. which has approximately 90 plants with the largest station of 4000 MW, for an accuracy of 10 MW step, the computer storage requirement will be 36K words.

It is obvious that further memory reduction can be achieved by breaking up the largest plant into several smaller plants so that the maximum generation level, M, needed to represent the largest plant is smaller. For example, if the 4000 MW station is represented by two equal size but smaller plants, then the memory requirement will then be 18.2k words. Trials with the algorithm indicate that the number of generators in the system does not contribute directly to the computer time required to determine the optimal generator combination. As Figure 2 below shows, the computer time requirement is a function of system capacity and desirable accuracy. In the figure, the CPU time is the computer processing time to obtain the optimal generator combinations and costs for generation levels from 0 MW .to total system capacity in a step size chosen. The number of stages is the total system capacity divided by the step size. The curve in the figure may be approximated by:

$$Log(t) = 1.843\ Log(\text{total capacity/step size}) - 4.465 \quad ...\ (4)$$

and can be used to estimate the computer time required to execute the unit commitment program; the validity of which is illustrated in the results section of this paper.

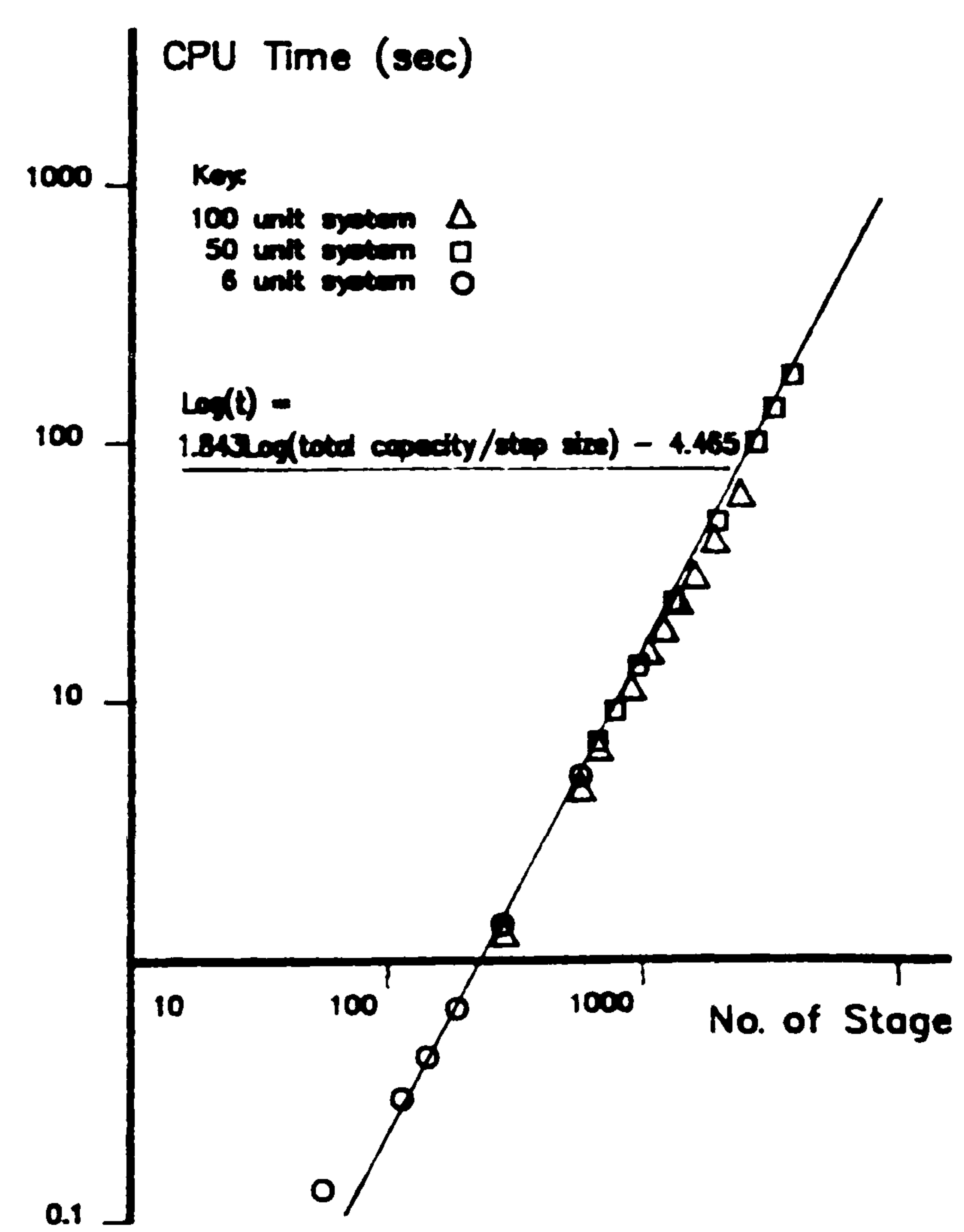


Figure 2 Graph: Computer processing time versus number of stages

## THE SCHEDULING METHOD

The CCDP approach proposed may be summarized by the following steps:

1. Input data including forecast load, generator parameters, must on/off and fixed generation requirements, capacity changes requirements etc.
2. Starting with the first interval, consider the commitment problem interval by interval. Check unit availability and form composite cost functions of each unit for the interval.
3. With the composite cost functions obtained in Step 2, find the optimal unit combination using the dynamic progamming calculation algorithm described in the last section so that load for that interval is satisfied at minimum cost.
4. Check that the optimal unit combination obtained in Step 3 satisfies spinning reserve requirments. If not satisfied, go back to Step 3 to find another combination. If spining reserves are satisfied, go to Step 2 for the next interval.
5. Calculate the overall production cost and output commitment schedule.

## COMPOSITE COST FUNCTION

In Step 2 of the CCDP approach, the composite cost of each unit at the each interval under consideration must be formed. The composite cost function of unit i at interval j is defined as follows:

1. If the unit was "off" in interval (j-1) :

$$W_i(y) = F_i(y) + S_i(t_i)/h_i \quad .......... \quad (5.a)$$

where
$W_i(y)$ = composite cost of unit i at output y MW
$F_i(y)$ = fuel cost at output y MW described by equation (1)
$S_i(t_i)$ = startup cost calculated using equation (2)
$h_i$ = estimated no. of intervals the unit will be up if it were started up in interval j.

2. If the unit was "on" in interval (j-1) :

$$W_i(y) = F_i(y) - D_i \quad ................ \quad (5.b)$$

where $D_i$ is the shutdown cost of unit i.

The composite cost of an unit is an artificial operating cost function which combines fuel cost, startup cost and shutdown cost in a single cost description. Functions (5.a) and (5.b) may be derived by using intiative reasonings outlined as follows.

Assume that there are K units which were in "off" status during interval (j-1). The unit commitment problem is to determine which of these units, if any, should be started up in interval j so that the overall cost will be minimum. Assuming that any unit started up at interval j will be shut down at a later interval when load returns to the same level as in interval (j-1) then $h_i$ for all units will be equal to h. The total operating cost of any of these units supplying energy in the h intervals will be:

$$W_i^{total} = F_i^j + F_i^{j+1} + F_i^{j+2} + ... + F_i^{j+h-1} + S_i(t_i)$$

where
$i$ = index of "off" unit. i=1,2,3,....,K
$F_i^j$ = Fuel cost of unit i at interval j
j+h = interval at which load resumes to level as in interval (j-1)
$S_i(t_i)$ = startup cost after unit i has been shutdown for $t_i$ hours.

The effective operating cost of unit i at each interval between j and (j+h-1) is therefore the fuel cost plus an average startup cost as described by equation (5.a).

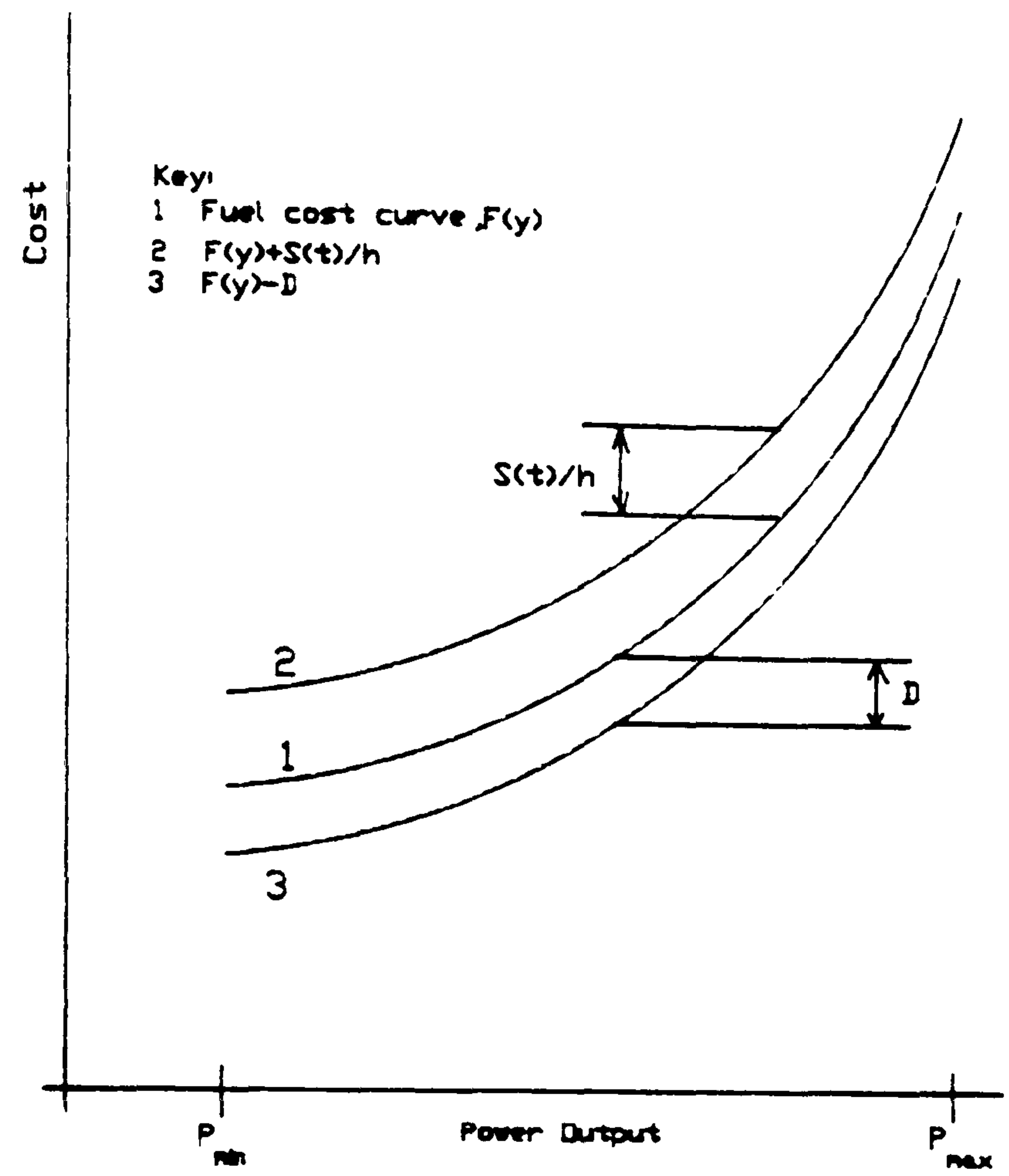


Figure 3 Composite cost functions for units was "on" and was "off" conditions

In the above description, the estimated up time of all units started up in interval j is h. Those units which are pre-scheduled to shut down before (j+h) will have a smaller expected up time and the contribution from their startup costs to the resultant composite costs will be larger than it would have been should these pre-scheduled shutdown constraints not have been there.

For a unit which was already "on" in interval (j-1), there is no startup cost involved for it to continue to operate in interval j. The unit, however, could be shut down in interval j and incur a shutdown cost to the system. Therefore, if the unit is to continue to operate in interval j, the cost to the system is effectively the fuel cost minus the shut down cost i.e. equation (5.b).

The composite cost of an available unit in interval j is depicted in Figure 3 and is essentially the fuel cost function plus a constant component. The shape of the fuel cost curve remains the same. The constant component added to the fuel cost will therefore affect the selection of units but not the optimal loading level of the selected units. With the composite cost functions, the CCDP algorithm will automatically select those units which optimize the fuel cost, startup cost and shutdown cost to the system.

## SPINNING RESERVE

Spinning reserve is the excess capacity of synchronized generators above the load. Spinning reserve is costly as it implies that some units will be partially loaded at which fuel efficiency is usually less than at higher loading points. Hence it is desirable, from an economic point of view, to have a minimum amount of spinning reserve subject to acceptable risk.

The unit commitment algorithm proposed commits thermal units to satisfy two spinning reserve criteria. These are described as follow:

1. Fixed percentage over forecast load : The total on-line capacity committed at any interval is equal to or exceeds the expected load of that interval plus a certain percentage. This can be easily assured by carrying out the dynamic programming calculation for a load equal to or exceeding the expected load while checking the total committed capacity. The CCDP technique will automatically select the unit combination which minimizes the overall cost to the system.

2. Loss of generation : The second requirement is to pick up the loss of generation of any loaded generator within a pre-specified short time period. The spinning reserve available from a unit is the spare capacity available from the unit or the ramping capacity of this unit within the specified time whichever is smaller, ie.

   Unit spinning reserve = MIN ((capacity - loading), (ramping rate x time))

   To satisfy this criterion, the total spinning reserve available from the remaining units must be greater than or equal to the loading of the unit under consideration. A special technique is developed to ensure that the spinning reserve of a unit combination is adequate to cover the loss of any units. The method used is best illustrated by an example as given below.

Example: Assume that for a fixed percentage reserve criterion, three generators depicted in Table 1 are selected to supply the load. Assuming a response requirement which ensures picking up the loss of any generator within 10 seconds is specified, Table 2 may be constructed.

TABLE 1 - Generator Data

| | Unit 1 | Unit 2 | Unit 3 |
|---|---|---|---|
| Capacity (MW) min | 20.00 | 30.00 | 40.00 |
| max | 100.00 | 150.00 | 150.00 |
| Ramping rate (MW/s) | 2.50 | 4.00 | 4.50 |

TABLE 2 - Calculation of maximum load a given unit combination can supply

| | Unit 1 | Unit 2 | Unit 3 |
|---|---|---|---|
| Capacity[1] | 100.00 | 150.00 | 150.00 |
| Ramp cap[2] | 25.00 | 40.00 | 45.00 |
| Avail Spin[3] | 85.00 | 70.00 | 65.00 |
| Output[4] | 75.00 | 70.00 | 65.00 |
| Difference[5] | 10.00 | 0.00 | 0.00 |

Note:
1 - Capacity of the unit.
2 - Ramping capacity of the unit within specified time.( = Ramping rate x 10 sec )
3 - Total spinning reserve availbale to cover the loss of generation of the unit
4 - Maximum output of the unit without reducing its contribution to overall spinning reserve available to the other units
5 - Difference between (3) and (4).

The maximum load the units can supply without violating the 10 seconds pick up time requirement is (a)+(b) where (a) is the summation of row 4 and (b) is the minimum of row 5. In this example (a)=75+70+65=210MW, (b)=0.0. The maximum load these three units can supply is therefore 210MW. Should this maximum load be less than the forecast load of the interval, the CCDP algorithm is used to find a new unit combination and the response time requirement then rechecked.

RESULTS

The method proposed has been programmed in FORTRAN 77 for use on a Perkin Elmer 3230 computer. As an illustrative example the commitment of 12 thermal units is shown in Figure 4. The computer processing time required for the above study with a step size 5MW is less than 10 seconds. A comparison of the proposed method with a priority order scheme is shown in Table 3. All on-line units, in the two methods, are optimally loaded satisfying both spinning reserve requirements. Various studies have shown that the proposed method has an overall cost improvement over the priority order scheme ranging from 0 to 2 % depending on system data, generating unit characteristic etc.

TABLE 3 - Comparison of operating cost using commitment schedules obtained by CCDP technique and Priority Order technique

| Method | Fuel Cost | Startup | Shutdown | Total Cost | Time |
|---|---|---|---|---|---|
| Priority | 28452.30 | 0.00 | 11.00 | 28463.30 | 5.9 s |
| CCDP | 28019.07 | 193.73 | 31.00 | 28243.80 | 9.6 s |
| Diff % | 1.55 | - | - | 0.78 | - |

The CCDP technique is robust with respect to the step size chosen. As Table 4 shows, the effect of a 5 fold step size difference has only a marginal effect on the overall operating cost. Closer examination of the two commitment schedules for the two step sizes reveals that the two schedules are in fact identical as far as generator startup and shutdown time are concerned.

TABLE 4 - Effect of step size on operating cost

| Step size | Fuel cost | Startup | Shutdown | Tot Cost |
|---|---|---|---|---|
| 2.0 MW | 28019.88 | 193.73 | 31.00 | 28244.61 |
| 10.0 MW | 28042.80 | 193.73 | 31.00 | 28267.53 |
| Diff % | 0.082 | - | - | 0.081 |

To investigate the practicality of the method for a large scale system, the commitment program has been applied to the EPRI Scenario System A (11). In this test system, there is 224 thermal generating units with total capacity 51,750 MW. Production cost results for one of the tests carried out are given in Table 5. The computer time required for this study is 20.9 minutes. Using equation (4), the estimated CPU time is:

| | |
|---|---|
| System capacity | = 51750 MW |
| Generation step size chosen | = 25 MW |
| Number of stages = 51750/25 | = 2070 |
| Using equation (4), CPU time/interval | = 44 sec |
| 24 hour study period, total CPU time | = 44x24 = 17.6min. |

The actual computer time used is greater than that estimation because of the additional processing time for data input/output, spinning reserve calculation etc.

TABLE 5 - Sample operating cost result for a 224 unit system

| Method | Fuel cost | Startup | Shutdown | Total Cost |
|---|---|---|---|---|
| Priority | 1181971.00 | 7451.00 | 3069.00 | 1192491.00 |
| CCDP | 1179229.00 | 7875.00 | 3381.00 | 1190491.00 |
| Diff % | 0.23 | - | - | 0.17 |

FORECAST LOAD DATA

CREATION TIME: 07/02/1985.23:30:58

| INTERVAL | TIME | LOAD(PU) |
|---|---|---|
| 1 | 07/02/1985.23:30:58 | 2.78763 |
| 2 | 08/02/1985.00:30:58 | 2.41584 |
| 3 | 08/02/1985.01:30:58 | 3.35704 |
| 4 | 08/02/1985.02:30:58 | 2.37357 |
| 5 | 08/02/1985.03:30:58 | 2.41881 |
| 6 | 08/02/1985.04:30:58 | 2.46977 |
| 7 | 08/02/1985.05:30:58 | 2.59380 |
| 8 | 08/02/1985.06:30:58 | 3.13985 |
| 9 | 08/02/1985.07:30:58 | 4.00743 |
| 10 | 08/02/1985.08:30:58 | 4.34407 |
| 11 | 08/02/1985.09:30:58 | 4.32895 |
| 12 | 08/02/1985.10:30:58 | 4.27594 |
| 13 | 08/02/1985.11:30:58 | 4.31826 |
| 14 | 08/02/1985.12:30:58 | 4.19189 |
| 15 | 08/02/1985.13:30:58 | 4.22599 |
| 16 | 08/02/1985.14:30:58 | 4.23733 |
| 17 | 08/02/1985.15:30:58 | 4.26424 |
| 18 | 08/02/1985.16:30:58 | 4.56830 |
| 19 | 08/02/1985.17:30:58 | 4.59079 |
| 20 | 08/02/1985.18:30:58 | 4.45107 |
| 21 | 08/02/1985.19:30:58 | 4.33508 |
| 22 | 08/02/1985.20:30:58 | 4.21500 |
| 23 | 08/02/1985.21:30:58 | 4.09217 |
| 24 | 08/02/1985.22:30:58 | 3.81805 |

THERMAL GENERATOR DATA

NUMBER OF UNITS = 12

| ITEM | NAME | CAP(PU) | | COST | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | MIN | MAX | A | B | C | COLD | ALPHA | SH.DWN |
| 1 | THERMAL-01 | 0.50 | 2.00 | 29.00 | 190.00 | 100.00 | 113.00 | 2.0 | 13.50 |
| 2 | THERMAL-02 | 0.50 | 1.50 | 29.00 | 200.00 | 150.00 | 113.00 | 1.5 | 11.00 |
| 3 | THERMAL-03 | 0.20 | 0.70 | 25.00 | 210.00 | 170.00 | 101.00 | 1.0 | 10.00 |
| 4 | THERMAL-04 | 0.10 | 0.50 | 15.00 | 210.00 | 170.00 | 85.00 | 0.5 | 8.50 |
| 5 | THERMAL-05 | 0.10 | 0.50 | 15.00 | 210.00 | 170.00 | 85.00 | 0.5 | 8.50 |
| 6 | THERMAL-06 | 0.10 | 0.50 | 15.00 | 210.00 | 170.00 | 85.00 | 0.5 | 8.50 |
| 7 | THERMAL-07 | 0.50 | 2.00 | 29.00 | 190.00 | 100.00 | 113.00 | 2.0 | 13.50 |
| 8 | THERMAL-08 | 0.50 | 1.50 | 29.00 | 200.00 | 150.00 | 113.00 | 1.5 | 11.00 |
| 9 | THERMAL-09 | 0.20 | 0.70 | 25.00 | 210.00 | 170.00 | 101.00 | 1.0 | 10.00 |
| 10 | THERMAL-10 | 0.10 | 0.50 | 15.00 | 210.00 | 170.00 | 85.00 | 0.5 | 8.50 |
| 11 | THERMAL-11 | 0.10 | 0.50 | 15.00 | 210.00 | 170.00 | 85.00 | 0.5 | 8.50 |
| 12 | THERMAL-12 | 0.10 | 0.50 | 15.00 | 210.00 | 170.00 | 85.00 | 0.5 | 8.50 |

TOTAL CAP INSTALLED = 11.40 P.U.

| NO. | NAME | MIN-UP | MIN-DWN | STATUS | T-CHANGE | RAMP |
|---|---|---|---|---|---|---|
| 1 | THERMAL-01 | 3.00 | 3.00 | 1 | 06/02/1985.23:00:00 | 0.040 |
| 2 | THERMAL-02 | 3.00 | 3.00 | 0 | 06/02/1985.23:00:00 | 0.030 |
| 3 | THERMAL-03 | 2.00 | 2.00 | 1 | 06/02/1985.23:00:00 | 0.014 |
| 4 | THERMAL-04 | 1.00 | 1.00 | 1 | 06/02/1985.23:00:00 | 0.010 |
| 5 | THERMAL-05 | 1.00 | 1.00 | 1 | 06/02/1985.23:00:00 | 0.010 |
| 6 | THERMAL-06 | 1.00 | 1.00 | 1 | 06/02/1985.23:00:00 | 0.010 |
| 7 | THERMAL-07 | 3.00 | 3.00 | 1 | 06/02/1985.23:00:00 | 0.040 |
| 8 | THERMAL-08 | 3.00 | 3.00 | 1 | 06/02/1985.23:00:00 | 0.030 |
| 9 | THERMAL-09 | 2.00 | 2.00 | 1 | 06/02/1985.23:00:00 | 0.014 |
| 10 | THERMAL-10 | 1.00 | 1.00 | 1 | 06/02/1985.23:00:00 | 0.010 |
| 11 | THERMAL-11 | 1.00 | 1.00 | 1 | 06/02/1985.23:00:00 | 0.010 |
| 12 | THERMAL-12 | 1.00 | 1.00 | 0 | 06/02/1985.23:00:00 | 0.010 |

Note:

1. No unit must be 'on', must be 'off', fixed generation, scheduled for maintenance or forced outages during the study period.

2. Per unit base = 100 MW

GENERATION SCHEDULE

STEPSIZE = 5.0 MW

| NO. | TIME | LOAD | CAP | SPIN | THERMAL GENERATOR | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 07/02/1985.23:30:58 | 56 | OK | OK | 13 | 0 | 0 | 6 | 6 | 6 | 13 | 0 | 0 | 6 | 6 | 0 |
| 2 | 08/02/1985.00:30:58 | 48 | OK | OK | 11 | 0 | 0 | 6 | 5 | 5 | 11 | 0 | 0 | 5 | 5 | 0 |
| 3 | 08/02/1985.01:30:58 | 67 | OK | OK | 15 | 0 | 0 | 8 | 8 | 7 | 15 | 0 | 0 | 7 | 7 | 0 |
| 4 | 08/02/1985.02:30:58 | 47 | OK | OK | 11 | 0 | 0 | 5 | 5 | 5 | 11 | 0 | 0 | 5 | 5 | 0 |
| 5 | 08/02/1985.03:30:58 | 48 | OK | OK | 11 | 0 | 0 | 6 | 5 | 5 | 11 | 0 | 0 | 5 | 5 | 0 |
| 6 | 08/02/1985.04:30:58 | 49 | OK | OK | 11 | 0 | 0 | 6 | 6 | 5 | 11 | 0 | 0 | 5 | 5 | 0 |
| 7 | 08/02/1985.05:30:58 | 52 | OK | OK | 11 | 0 | 0 | 6 | 6 | 6 | 11 | 0 | 0 | 6 | 6 | 0 |
| 8 | 08/02/1985.06:30:58 | 63 | OK | OK | 14 | 0 | 0 | 7 | 7 | 7 | 14 | 0 | 0 | 7 | 7 | 0 |
| 9 | 08/02/1985.07:30:58 | 80 | OK | OK | 16 | 0 | 0 | 8 | 8 | 8 | 16 | 0 | 0 | 8 | 8 | 8 |
| 10 | 08/02/1985.08:30:58 | 87 | OK | OK | 17 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 9 |
| 11 | 08/02/1985.09:30:58 | 87 | OK | OK | 17 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 9 |
| 12 | 08/02/1985.10:30:58 | 86 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 9 |
| 13 | 08/02/1985.11:30:58 | 86 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 9 |
| 14 | 08/02/1985.12:30:58 | 84 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 8 | 8 |
| 15 | 08/02/1985.13:30:58 | 85 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 8 |
| 16 | 08/02/1985.14:30:58 | 85 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 8 |
| 17 | 08/02/1985.15:30:58 | 85 | OK | OK | 16 | 0 | 0 | 9 | 9 | 9 | 16 | 0 | 0 | 9 | 9 | 8 |
| 18 | 08/02/1985.16:30:58 | 91 | OK | OK | 16 | 0 | 0 | 9 | 8 | 8 | 16 | 10 | 0 | 8 | 8 | 8 |
| 19 | 08/02/1985.17:30:58 | 92 | OK | OK | 16 | 0 | 0 | 9 | 9 | 8 | 16 | 10 | 0 | 8 | 8 | 8 |
| 20 | 08/02/1985.18:30:58 | 89 | OK | OK | 16 | 0 | 0 | 8 | 8 | 8 | 15 | 10 | 0 | 8 | 8 | 8 |
| 21 | 08/02/1985.19:30:58 | 87 | OK | OK | 15 | 0 | 0 | 8 | 8 | 8 | 15 | 10 | 0 | 8 | 8 | 7 |
| 22 | 08/02/1985.20:30:58 | 84 | OK | OK | 15 | 0 | 0 | 8 | 8 | 7 | 15 | 10 | 0 | 7 | 7 | 7 |
| 23 | 08/02/1985.21:30:58 | 82 | OK | OK | 15 | 0 | 0 | 7 | 7 | 7 | 15 | 10 | 0 | 7 | 7 | 7 |
| 24 | 08/02/1985.22:30:58 | 76 | OK | OK | 13 | 0 | 0 | 7 | 7 | 7 | 13 | 10 | 0 | 7 | 6 | 6 |

NOTE: GENERATION OUTPUT
1 = GENERATION LEVEL 1
0 = GENERATOR IS OFF

OPERATING COSTS OF EACH INTERVAL

| INTVAL | LOAD | FUEL ($) | STARTUP($) | SH-DWN($) | SUB-TOT($) |
|---|---|---|---|---|---|
| 1 | 56 | 856.00 | 0.00 | 31.00 | 887.00 |
| 2 | 48 | 733.30 | 0.00 | 0.00 | 733.30 |
| 3 | 67 | 1035.87 | 0.00 | 0.00 | 1035.87 |
| 4 | 47 | 718.13 | 0.00 | 0.00 | 718.13 |
| 5 | 48 | 733.30 | 0.00 | 0.00 | 733.30 |
| 6 | 49 | 748.48 | 0.00 | 0.00 | 748.48 |
| 7 | 52 | 794.00 | 0.00 | 0.00 | 794.00 |
| 8 | 63 | 968.62 | 0.00 | 0.00 | 968.62 |
| 9 | 80 | 1247.20 | 85.00 | 0.00 | 1332.20 |
| 10 | 87 | 1371.30 | 0.00 | 0.00 | 1371.30 |
| 11 | 87 | 1371.30 | 0.00 | 0.00 | 1371.30 |
| 12 | 86 | 1353.55 | 0.00 | 0.00 | 1353.55 |
| 13 | 86 | 1353.55 | 0.00 | 0.00 | 1353.55 |
| 14 | 84 | 1318.10 | 0.00 | 0.00 | 1318.10 |
| 15 | 85 | 1335.83 | 0.00 | 0.00 | 1335.83 |
| 16 | 85 | 1335.83 | 0.00 | 0.00 | 1335.83 |
| 17 | 85 | 1335.83 | 0.00 | 0.00 | 1335.83 |
| 18 | 91 | 1431.42 | 108.74 | 0.00 | 1540.16 |
| 19 | 92 | 1449.15 | 0.00 | 0.00 | 1449.15 |
| 20 | 89 | 1396.45 | 0.00 | 0.00 | 1396.45 |
| 21 | 87 | 1362.32 | 0.00 | 0.00 | 1362.32 |
| 22 | 84 | 1311.70 | 0.00 | 0.00 | 1311.70 |
| 23 | 82 | 1277.95 | 0.00 | 0.00 | 1277.95 |
| 24 | 76 | 1179.90 | 0.00 | 0.00 | 1179.90 |
| TOTAL COST | | 28019.07 | 193.73 | 31.00 | 28243.81 |

OPERATING COST OF EACH THERMAL UNIT

| NO. | NAME | FUEL ($) | STARTUP($) | SH.DWN($) | SUB-TOT($) |
|---|---|---|---|---|---|
| 1 | THERMAL-0 | 5305.75 | 0.00 | 0.00 | 5305.75 |
| 2 | THERMAL-0 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | THERMAL-0 | 0.00 | 0.00 | 10.00 | 10.00 |
| 4 | THERMAL-0 | 2942.85 | 0.00 | 0.00 | 2942.85 |
| 5 | THERMAL-0 | 2894.78 | 0.00 | 0.00 | 2894.78 |
| 6 | THERMAL-0 | 2828.13 | 0.00 | 0.00 | 2828.13 |
| 7 | THERMAL-0 | 5253.00 | 0.00 | 0.00 | 5253.00 |
| 8 | THERMAL-0 | 1165.50 | 108.74 | 11.00 | 1285.24 |
| 9 | THERMAL-0 | 0.00 | 0.00 | 10.00 | 10.00 |
| 10 | THERMAL-1 | 2828.13 | 0.00 | 0.00 | 2828.13 |
| 11 | THERMAL-1 | 2794.38 | 0.00 | 0.00 | 2794.38 |
| 12 | THERMAL-1 | 2006.57 | 85.00 | 0.00 | 2091.57 |
| TOTAL COST : | | 28019.07 | 193.73 | 31.00 | 28243.81 |

Figure 4 Sample example of unit commitment program for a 12-unit system

## CONCLUSION

A new unit commitment method based on the dynamic programming principle has presented. The computer run time requirement has been found to be largely independent of the number of units but rather a function of total system generating capacity and required accuracy. The spinning reserve constraint considered in the approach has been shown to be capable of handling response time requirements. A technique for inclusion of the ramping rate of on-line units and response time required to pick up load shed by any loaded generator has also been described.

## ACKNOWLEDGEMENT

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# LARGE SCALE DYNAMIC PROGRAMMING BASED DISPATCH INCLUDING TRANSMISSION LOSSES

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Abstract. A new technique for economic dispatch based on the principle of dynamic programming and including both transmission limits and loss representation is presented. Despite the general view that DP is inherently time consumming and requires enormous computer memory, the method described here has neither of these disadvantages. It has many desirable characteristics including numerical stability, high speed, high accuracy and superior capability in handling non-linear, non-convex generation costs. Individual line flow constraints are considered and the costs of transmission losses are included within the overall minimization stategy. Generating units in a station connected to the same busbar need not neccessarily be lumped into a single entity but can be modelled separately. Because of its speed, the approach is potentially suitable for on-line application even for a very large system. The theoretical derivation of the method is presented and numerical results on applications to various networks including a data set from CEGB are reported.

Keywords. Economic dispatch; Dynamic programming; Nonlinear generation cost; Transmission losses minimization; Sensitivity matrix

## INTRODUCTION

The complex optimization problem associated with the economic allocation of generator active outputs to meet a future load demand, typically 5 to 30 minutes ahead, has been the subject of considerable research. A large amount of effort has been directed towards the application of Largrange multipliers, linear and quadratic programming [1-6] and other sophisticated methods to the solution of the dispatch problem. While in general these recorded techniques work satisfactorily for small and medium size networks, for large systems, heuristic approaches, linearization of non-linear fuel cost models, simplification of line flow limits or disregard of network losses have to be introduced in order to reduce the problem size. Any of these approximation will probably introduce sub-optimality to the final solution. In this paper a new method, Dynamic Programming with Loss Minimization (DPLM), is described. Tests indicate that the proposed technique has many desirable characteritics and the solution obtained is a global optimum.

Dynamic programming (DP) has not attracted much attention for economic dispatch application in either on-line or off-line mode because of its inherent large CPU time and memory requirements. However, the method described here has not only overcome this well known "curse of dimensionality" problem, but is fundamentally robust and computationally more efficient than some of the popular approaches. It also has the additional advantages of complete flexibility, the capability of handling practically any form of generator fuel cost function together with individual line flow limits while optimizing the cost of transmission. Furthermore generating units with different fuel cost properties in a power station connected to the same busbar need not be lumped into a single source but can be treated separately. As a result of its flexibility, complete numerical stabilty, simplicity and computational efficiency, the technique is suitable for on-line application for large systems. Theoretical derivation and numerical results for a 22-unit system and a 115-unit system are reported.

## OBJECTIVE FUNCTION

The objective function for DPLM approach is simply the minimization of the total production cost of all on-line units whose total active power generation equals to the forecast demand plus any transmission loss that may occur.

$$\text{Min } T(D_{total}) = \sum_{g}^{N_g} G_g(P_g) \qquad g=1,2,3,\ldots,N_g \qquad (1)$$

where

- $D_{total}$ = total load demand for the system including losses in the network,
- $T$ = total production cost of all on-line units,
- $G_g$ = fuel cost as function of active power output of generator $g$,
- $P_g$ = active power generation of generator $g$,
- $N_g$ = number of available generating plants, including off-line gas turbine and pumped storage units which can be started up rapidly.

The generator fuel cost functions, $G_g(P_g)$, in the above equation are completely general, restricted neither to linear, piecewise linear, convexity, nor differentiability requirement. Any analytical or empirical cost to generation output relationship may be used as long as the generation cost at any active power output level of a unit can be readily calculated.

## CONSTRAINTS

The cost optimization is subject to a large number of constraints derived from operational restrictions for a power system. In this section, the most frequently referenced constraints are treated and the method to incorporate these in DPLM approach is described.

### a) Network Limitations

These constaints are essentially the current carrying limits of the transmission lines. Line flows in a network depend on both the distribution of the load and of generation in a non-linear fashion. However an approximate linear relationship between line flows, load distribution, transmission losses and generation injections at different buses of the network can be derived as shown in equation (2).

$$[F] = [H][A]^{-1}([K_g][P] - [K_d][D] - [U][M])$$

$$= [AA][P] - [C] \qquad (2)$$

where

$[F]$ - line real current flows, a column vector having $N_L$ elements,

$[P]$ - generator injection, a column vector having $N_g$ elements,

$[D]$ - load demands, a column vector having $N_d$ elements,

$[M]$ - nodal load, converted from transmission line losses, (details in Transmission Losses section)

$[H]$ - $N_L$ x $N_n$ connectivity matrix, for row $L$ corrresponding to line $L$ having sending and receiving nodes $i$ and $j$,

$H(L,i)=X_L/(R_L^2+X_L^2)$; $H(L,j)=-X_L/(R_L^2+X_L^2)$

$H(L,k)=0.0$ for $k \neq i$ or $j$,

$[A]$ - a $N_n$ x $N_n$ admittance matrix of the network,

$[AA]$ - $[H][A]^{-1}[K_g]$ - sensitivity matrix,

$[C]$ - $[H][A]^{-1}[K_d][D] + [H][A]^{-1}[U][M]$,

$[K_g]$ - a $N_n$ x $N_g$ connection matrix between nodes and generators,

$[K_d]$ - a $N_n$ x $N_d$ connection matrix between nodes and loads,

$[U]$ - a unity matrix of $N_n$ order,

$N_L$ - number of lines in the network,

$N_n$ - number of nodes in the network,

$N_d$ - number of loads in the network.

In the above equation, $[A]$ is a $N_n$ square matrix as against ($N_n$-1) square that normally used in a DC load flow. In here, there is no slack or swing bus. Singularity of $[A]$ is avoided by the fact that shunts or line charging exists in the system. Since $[AA]$ is a constant for any particular network topology, and $[C]$ is a constant for a forecast load distribution and a given set of line loss values, $[F]$ can be calculated directly for a given $[P]$. DPLM is used to determine $[P]$ so that line flows monitored using equation (2) do not violate any current rating limit while the total fuel cost described by the objective function is a minimum. It should be noted that the line flow calcualted using equation (2) is the active current flow only. Since the current rating of a line is the magnitude of a complex current value, an inequality constraint in the form of equation (3) may be used to reflect this.

$$[\mathrm{SQRT}(F^2+E^2)] \leq [\text{current limit}] \qquad (3)$$

where

$[E]$ - estimated reactive current in the lines.

It is generally recognised that the re-distribution of active power generation does not significantly affect the reactive current flow in a line [1,2,3]. $[E]$ can be treated as a constant in the active power dispatch process and can therefore be calculated from an on-line state estimator.

The power transfer stability limits of the lines may be added to the analysis by imposing additional flow limits to the lines :

$$[F] \leq [\text{Active Power Transfer Limits}] \qquad (4)$$

Matrix $[A]$ is sparse and to save memory space and increase speed, $[A]^{-1}$ may by obtained using a sparse matrix factorizing technique such as Zollenkopf's algorithm [7].

### b) Generator Output Constraints

In general, a generating unit has lower and higher output limits, such that

$$[P_{min}] \leq [P] \leq [P_{max}] \qquad (5)$$

For dispatching the generators for a future load, the ramping rates for the generators from their present outputs must also be included. Further limitations on generator outputs therefore apply.

$$[P^o]-[R_d]*t \leq [P] \leq [P^o]+[R_i]*t \qquad (6)$$

where

$[P^o]$ - generator output at the time of executing the active power dispatch program obtained from an on-line State Estimator,

$[R_d]$, $[R_i]$ - ramping rate to decrease and increase respectively which can be a constant or $[P^o]$ dependent,

$t$ - look ahead time, typically *5* to *30* minutes.

To improve system security or to represent approximately the station or boiler limitations, lower and upper limits can be imposed on a group of generators which may or may not be in the same station. Thus,

$$[P_{min}^{group\_i}] \leq [\sum_{g \in group\_i} P_g] \leq [P_{max}^{group\_i}]$$

As for unit ramping rate limits, group ramping limits can also be imposed on a group of generators which will affect the group capacity.

### c) Area Import/Export Constraints and Tie-line Constraints

To further enhance the security of the system, import and export limitations can be applied to certain areas in the system. These constraints can be written in the form of group line flow limits as depicted in the equation which follows and in Fig.1.

$$[F_{min}^{group\_i}] \leq [\sum_{L \in group\_i} F_L] \leq [F_{max}^{group\_i}]$$

Tie line power transfer constraints between systems can also be represented accurately in a similar fashion.

### d) Power Balance

In DPLM, transmission loss in each line is distributed equally at the two end nodes of the line as additional loads to the system. The summation of geneator outputs is therefore equal to the summation of loads including losses at each node, or,

$$\sum_{g}^{Ng} P_g = \sum_{i}^{Nd}(\text{Forescast load demand at node } i) + \sum_{i}^{Nn}(\text{Estimated nodal load due to line losses})$$

### e) Tranmission Losses

To facilitate the DPLM algorithm and for improved accuracy, a new formula for evaluating transmission losses is proposed here. It has been

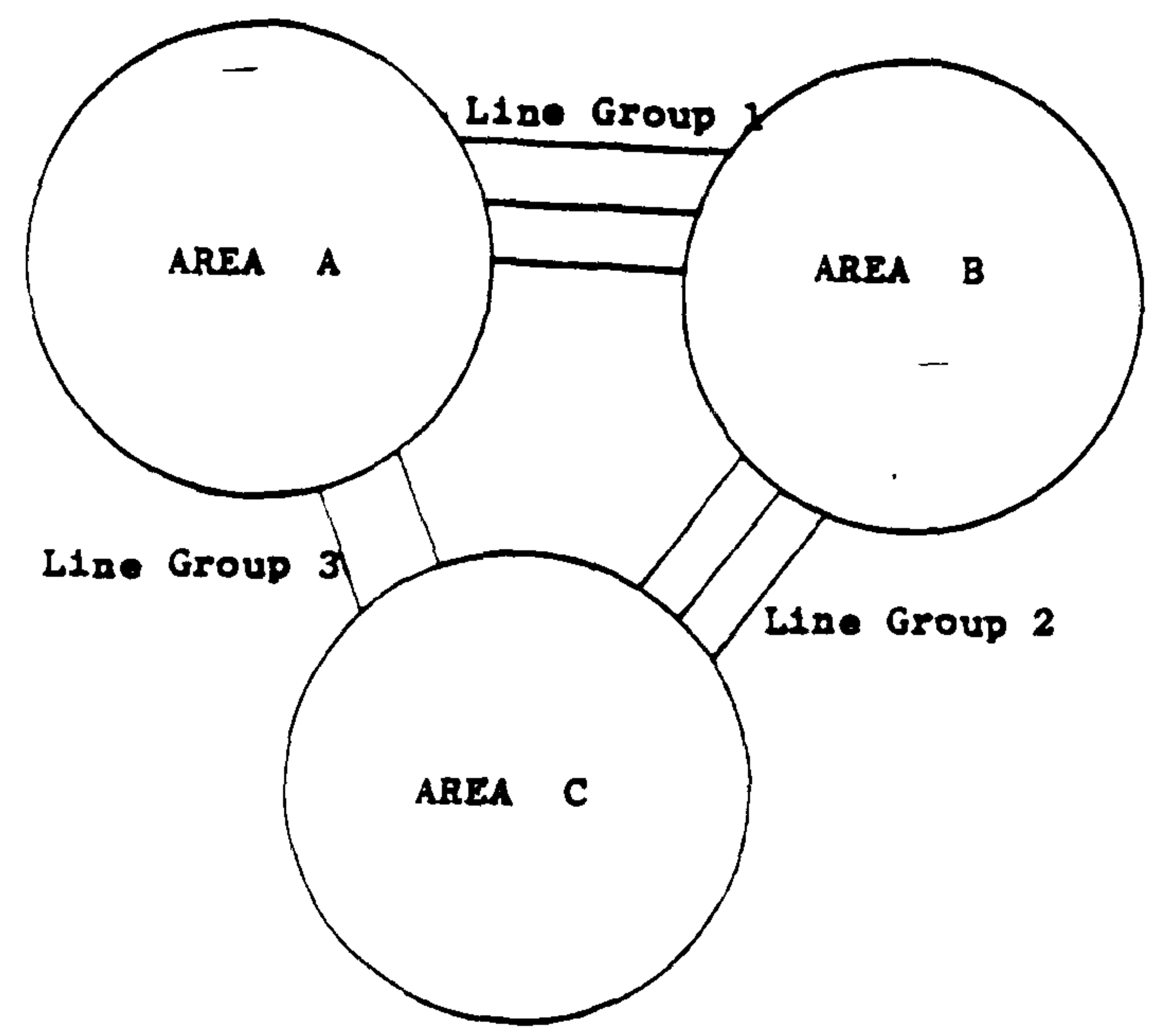


Fig. 1. Area Import/Export Constraints (line Group Constraints)

shown [8] that for minimum total operation cost, the incremental cost of all contributing units including losses should be equal. Thus,

$$dG_g(P_g)/dP_g \cdot pf_g = \lambda = \text{net incremental cost} \qquad (7)$$

where

$pf_g$ = penalty factor of unit $g$

$$= 1/[1 - d(\text{Loss}_{total})/d(P_g)]$$

Now, the total active power losses in the system is

$$\text{Loss}_{total} = \sum_{L}^{NL} \text{Loss}_L = \sum_{L}^{NL}(F_L^{\,2} + E_L^{\,2}) \cdot R_L \qquad (8)$$

Hence,

$$d(\text{Loss}_{total})/dF_L = 2 \cdot F_L \cdot R_L$$

From equation (2),

$dF_L/dP_g = AA(L,g)$ where $AA(L,g)$ is the $(L,g)$ element of $[AA]$

Since

$$d(\text{Loss}_{total})/dP_g = \sum_{L}^{NL}(d\text{Loss}_{total}/dF_L \cdot dF_L/dP_g)$$

Therefore, the penalty factor for unit $g$ is

$$pf_g = 1 / [\ 1 - \sum^{NL} 2 \cdot F_L \cdot R_L \cdot AA(L,g)\ ] \qquad (9)$$

The proposed DPLM technique dispatches the generator outputs iteratively. In each iteration the optimal generator outputs to meet the forcast load and losses are calculated. Using equation (2) the active current flow in each line for the estimated optimal generation pattern can be determined. Revised loss in each line is found by substituting the line flows in equation (8) and is then distributed equally at the two end nodes of the line as additioanl nodal loads to the system. Penalty factors of the generating units can Therefore be updated using eqaution (9) before starting a new iteration. At the first iteration, the penalty factors of the available units may be initialized to unity or the values given by the last dispatch can be used. Line losses may be initially set to a certain percentage of the forecast load or utilizing a more comprehensive formula linked to the losses of the system at its present conditions.

The advantage of using the above formulae for the calculation of losses and penalty factors over the conventional "B" coeffiecients approach is three fold:

1. There is no need for pre-calculation of any penalty factors before executing the dispatch program.
2. There is no need for a "base" case which can only give an approximation to system losses. The base case approach cannot readily reflect the rapid changes in system topology, load distribution and generation pattern.
3. The sensitivity coefficients $AA(L,g)$ and line flow $F_L$ in equations (8) & (9) are an integrated part of the DPLM algorithm. There is no significant additional computation involved to update system losses and unit penalty factors at each iteration. It is particularly useful that the technique is not only responsive to the rapid system topology changes, but also to the predicted load level, its distribution, and optimal distribution of generation for the predicted load.

## COMPUTATIONAL ALGORITHM

It has been shown by Cheung and Sterling [9] that a DP technique can be successfully applied to the unit commitment problem. In this paper, a Successive Dynamic Progamming (SDP) approach is used. The SDP method represents an extension of the previous work designed to further reduce the computation time, storage requirements and improve accuracy. In essence, the proposed SDP calculation mechanism retains the same basic recursive formula of [9], but is applied iteratively so that the number of stages in each DP iteration is reduced. Accuracy of the solution is improved by progressively approaching the exact optimal generation outputs (within tolerance) utilising the estimated optimal generation outputs obtained in the previous iteration.

### a) Generation Production Cost Model

The generation fuel cost model used in DPLM is designed to have three important characteristics:

1. The cost to generation output relation, $G_g(P_g)$, is not restricted to any particular type of analytical function. Much research on economic dispatch uses linearized or piecewise linearized fuel cost function, but such representations are a poor approximation for many types of turbine/generator plant.
2. It is used to minimize the transmission losses in the system. In each iteration the cost function is modified by the penalty factors calculated with the latest results obtained in the previous iteration to reflect the effective production cost of each available unit for the given load distribution.
3. In each iteration, the optimal operating point of a unit is estimated. With this estimated operating point available, the capacity of a unit can be artificially reduced to a pseudo maximum and a pseudo minimum limits. This capacity range is then further reduced in each

successive iteration. The production cost model of a unit is therefore also used to progressively improve the accuracy of the dispatch solution. Any convergence criterion can be set on the generator outputs but 0.1 MW might be a typical figure. Closer tolerances have little effect on the overall solution time.

b) Successive Dynamic Programming (SDP)

Equation (10) is the fundamental recursive relationship in the proposed DP approach. It describes how an optimal total operation cost for a load, D, can be obtained by using the known generator fuel cost functions.

$$T(D) = \text{Min} \{ T(D-\Delta P_g) + \Delta G_g(\Delta P_g) \} \qquad (10)$$

where

$T(D)$ = optimal total fuel cost for a total generation $D$ of all units,

$\Delta P_g$ = additional output for generator $g$ from its optimal loading point at $(D - \Delta P_g)$ level,

$\Delta G_g(\Delta P_g)$ = additional fuel cost. To minimize transmission losses, the effective fuel cost function should be used.

To start the recursive process, $T(D_o)$, is determined first.

$$T(D_o) = \sum_g^{Ng} G_g[(P_{min})_g] \qquad \text{where } D_o = \sum_g^{Ng} (P_{min})_g$$

The optimal total production cost at any total load level $D$ can then be obtained by repetitive use of equation $(10)$. It is important to check that the increased output of a unit $\Delta P_g$ shall not lead to an overloading of any line in the system. Equation (2) is used to determine the incremental change in line flows due to an incremental change in the generation output. If any optimal generation output combination causes violation of any line flow limit, a sub-optimal $[P]$ satisfying all line flow limits will be stored to allow the DP process to continue.

When $D$ equals the total forecast load plus losses, one dispatch iteration is completed. The loading of the generators corresponding to $T(D_{total})$ is the estimated optimal generator outputs. With these generator loading points calculated, the pseudo_max and pseudo_min of the units may then be adjusted to a shorter range. The fuel cost function between these pseudo limits is then discretized with a new step size. The line flows and transmission losses corresponding to the newly estimated generation pattern can then be calculated and penalty factors also updated. A second iteration to give a more accurate operating point of the units may then proceed. The number of iterations required depends on the size of the units in the system and the desired accuracy but typically 5 to 6 iterations are sufficient for a large network with unit output accuracy set to 0.1 MW tolerance. A flow chart of the optimization scheme is shown in Fig.2.

TEST RESULTS : ACCURACY AND SPEED

The economic dispatch models and algorithm described in the previous sections have been implemented in FORTRAN 77 on a VAX-8600 computer and the results of its application to a small test network of 22 generators and to a large system whose data was provided by CEGB are given below. Single precision 32 bit floating point storage and arithmetic was found sufficient for all the test cases considered and no convergence difficulties were encountered.

a) A 22-unit System

A 22-unit system in reference [10] has been used for comparison. The reference describes results of a quadratic programming technique (QP). Table 1 shows the results from using QP and DPLM. The generator outputs dispatched by either methods are similar, but DPLM approach requires less than half of the CPU time needed for QP technique. The table also shows another characteristic of DPLM. It tends not to schedule the units to their maximum output limits if there is another unit of similar cost which can share the load. This has the advantage of providing more spinning reserve.

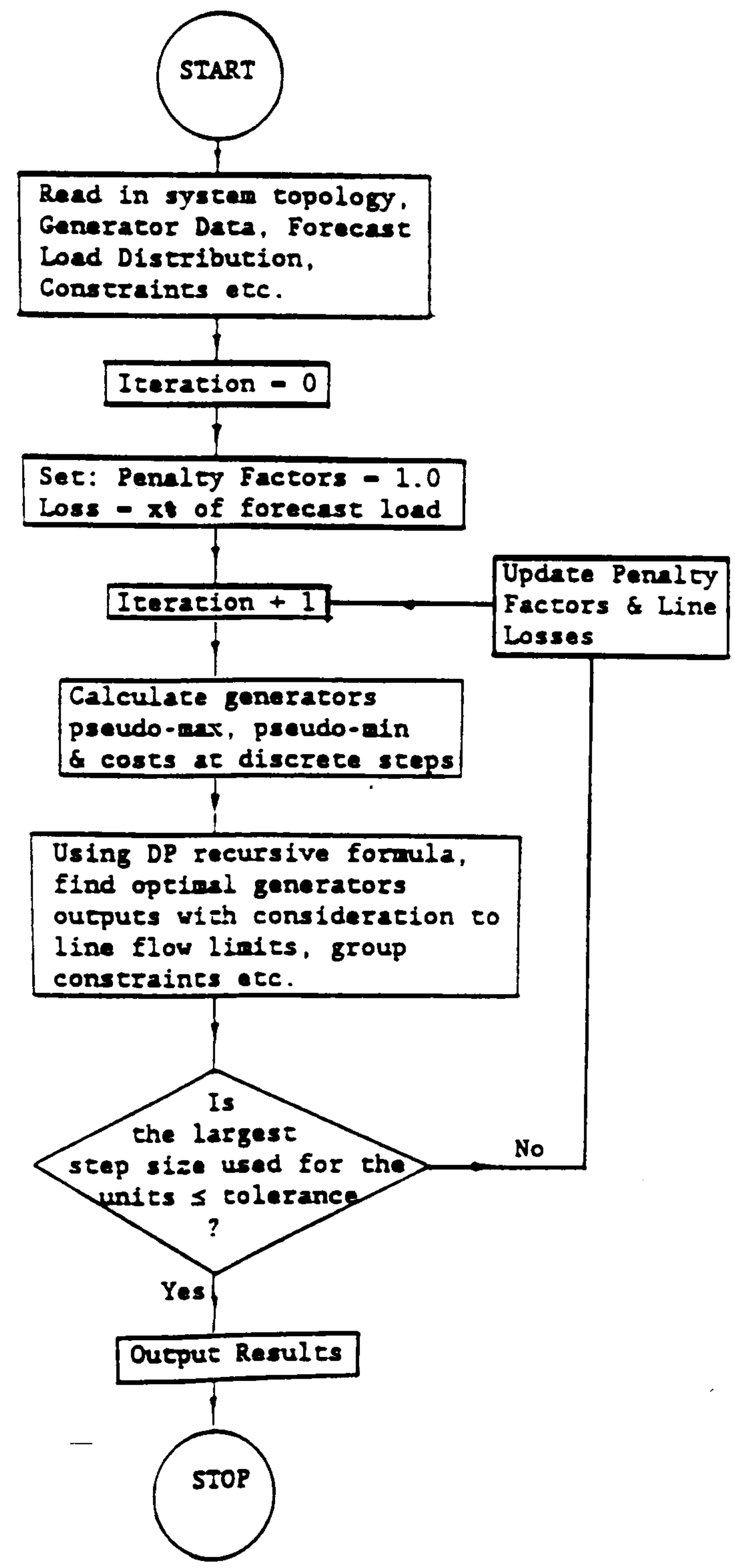


Fig. 2. Flow Chart of DPLM Dispatch Algorithm

b) A CEGB Test Network

A 145 node, 115 generating unit, 275 branch test network with four loading patterns was provided by the Central Electricity Research Laboratory of CEGB. The four loading conditions were dispatched with the proposed technique and a comparison of the results with those obtained using Linear Programming (LP) is shown in Table 2 (without loss optimization) and in Table 3 (with loss optimization). A selected set of line flows for the winter plateau load conditions using an accurate A.C. load flow are presented in Table 4 for comparison with the DPLM derived values. The CPU time required by the DPLM approach is different for the four loading conditions. Generally, in the summer time, the loads in southern England supplied by the relatively economic generators in the north activate more line overload constraints than the evenly spreaded heavy load conditions in the winter and hence requires slightly more computer time to converge to the required accuracy. One useful application of the DPLM technique is therefore to identify the small number of lines which restrict the flow of power preventing the system from operating more economically.

From the results depicted, it is apparent that :

1. The optimal total production cost calculated by DPLM and those of QP/LP, as shown in Tables 1 and Table 2, are approximately equal. This indicates that DPLM is as accurate as QP/LP and the solution is a global optimal for the two test systems.
2. DPLM is applicable to both small and large systems and its computing time requirement is much better than the QP technique. For the test network provided by CEGB, computing time in the region of 25 seconds is achieved but it is believed that further improvement in efficiency is possible.
3. Table 4 shows the close match of line flows and total losses calculated using an A.C. load flow and those using equations (2) & (8). This demonstrates the validity of the approximate linear relationship between line flows and active power injections assumed.
4. Comparison of results in Table 2 and Table 3 clearly shows a substantial economic benefit by including transmission losses in the optimization. Table 3 indicates a 0.44% average cost saving for the CEGB system. Since the system generally spends more time in the medium load range i.e. winter trough and summer plateau than the extremity of winter plateau and summer trough conditions, an even greater percentage saving is realizable.

## CONCLUSION

The paper has described a new algorithm, DPLM, for active power dispatch. The advantages of the proposed method include the following.

1. Nonlinear representation of generator fuel cost models.
2. Capable of on/off decisions as well as economic loading of pumped storage stations, gas turbines and other plants which can be started/stoped rapidly.
3. Units connected to the system at the same bus bar can be modelled separately. This is a very desirable feature particular when the units in the same station have different production cost characteristics.
4. Uses a new transmission losses formula which is responsive to any change in system topology, load and generation distributions.
5. Minimizes transmission losses while monitoring individual line flow limits which is generally not achievable in most existing algorithms for large systems.
6. Robust. It gives an optimal generation pattern in each iteration. It produces a best relaxed solution when there is no feasible solution.
7. Precise. Resolution of 0.1 MW for generator outputs is easily obtainable.
8. Speed. To dispatch a test network with 145-nodes, 115-generators and 275-lines requires only 25 seconds on average using a VAX 8600.
9. Unlike most dynamic programming approaches, the technique does not incur large computational penalties as the system size grows.
10. The algorithm has been tested on a large system but has yet to be validated with n-1 security constraints and consequently it remains to be shown that the technique will achieve an acceptable degree of optimality in the very heavily constrained case.


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**TABLE 1 Generator Optimal dispatch by DPLM and Quadratic Programming**

| Gen No. | Initial (MW) | QP Dispatch (MW) | DPLM Dispatch (MW) |
|---|---|---|---|
| 1 | 60. | 30. | 29.5 |
| 2 | 60. | 30. | 29.5 |
| 3 | 60. | 25. | 24.5 |
| 4 | 60. | 25. | 24.5 |
| 5 | 60. | 20. | 19.6 |
| 6 | 60. | 20. | 19.7 |
| 7 | 80. | 100. | 71.3 |
| 8 | 80. | 100. | 71.8 |
| 9 | 80. | 100. | 78.4 |
| 10 | 80. | 100. | 99.3 |
| 11 | 80. | 50. | 87.3 |
| 12 | 80. | 50. | 92.0 |
| 13 | 30. | 24. | 23.7 |
| 14 | 30. | 24. | 23.7 |
| 15 | 20. | 50. | 50.0 |
| 16 | 20. | 50. | 50.0 |
| 17 | 20. | 50. | 50.0 |
| 18 | 10. | 18. | 17.8 |
| 19 | 10. | 18. | 17.8 |
| 20 | 10. | 18. | 17.8 |
| 21 | 30. | 56. | 55.6 |
| 22 | 30. | 56. | 55.5 |

| Total Load | QP Dispatch: | DPLM Dispatch: |
|---|---|---|
| =1000.0MW | Loss=10.1MW | Loss=10.1MW |
| | Cost=2135unit/hr | Cost=2135unit/hr |
| | CPU time=2.0s | CPU time=0.9s |

**TABLE 2 Generator Optimal Dispatch by DPLM and LP with Transmission Losses Neglected**

| Load Case | LP Cost Units | DPLM Cost Units | (DP Cost-LP Cost) / DP Cost |
|---|---|---|---|
| Winter Plateau | 914292 | 914269 | negligible |
| Winter Trough | 479269 | 479240 | negligible |
| Summer Plateau | 417334 | 417313 | negligible |
| Summer Trough | 124240 | 124229 | negligible |

**TABLE 3 Generator Optimal Dispatch by DPLM and LP with Transmission Losses Included**

| Load Case | LP Cost Units (Loss) | DPLM Cost Units (Loss) | (DP Cost-LP Cost) / DP Cost |
|---|---|---|---|
| Winter Plateau | 932171 (604.MW) | 929225 (446.MW) | -0.32 % |
| Winter Trough | 491841 (449.MW) | 489070 (324.MW) | -0.57% |
| Summer Plateau | 482537 (407.MW) | 480353 (308.MW) | -0.45 % |
| Summer Trough | 131601 (298.MW) | 131074 (238.MW) | -0.40 % |
| | | Average | -0.44 % |

Notes:
1. The LP costs (without losses) are provided by algorithm.
2. The LP costs (with losses) have been estimated: = ((LP cost without loss) + (Loss calculated by AC load flow)*(marginal cost))
3. DPLM costs (with losses) have been estimated: = ((DPLM cost with loss optimized) + [(losses calculated by AC load flow) - (losses estimated by DPLM)]*(marginal cost))

**TABLE 4 Line Current Flow by DPLM approximation and accurate A.C. Load Flow (Winter Plateau Load Condition of CEGB Test Network)**

| Line No. | Send Node | Recv Node | Flow Limit | Line Flow by AC LF | Line Flow by DPLM |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 9.35 | -2.4490 | -2.4488 |
| 2 | 1 | 3 | 9.35 | 1.2288 | 1.2270 |
| 3 | 4 | 2 | 9.35 | -1.7346 | -1.7354 |
| 4 | 4 | 5 | 9.35 | 0.5144 | 0.5130 |
| 5 | 6 | 7 | 42.50 | 3.2730 | 3.2944 |
| 6 | 6 | 8 | 42.50 | 3.3856 | 3.3577 |
| 7 | 6 | 9 | 42.50 | 6.4106 | 6.3500 |
| 8 | 10 | 5 | 13.00 | 1.4204 | 1.4205 |
| 9 | 10 | 11 | 13.94 | -2.7652 | -2.7738 |
| 10 | 12 | 13 | 11.51 | -5.5237 | -5.5761 |
| \| | \| | \| | \| | \| | \| |
| \| | \| | \| | \| | \| | \| |
| 266 | 62 | 113 | 4.25 | 0.1084 | 0.1069 |
| 267 | 62 | 113 | 4.25 | 0.1083 | 0.1069 |
| 268 | 145 | 73 | 42.50 | -2.9339 | -2.9197 |
| 269 | 145 | 131 | 42.50 | -3.0798 | -3.0992 |
| 270 | 143 | 132 | 6.37 | 2.2344 | 2.2363 |
| 271 | 143 | 132 | 6.37 | 2.2265 | 2.2285 |
| 272 | 19 | 40 | 42.50 | -6.3799 | -6.3896 |
| 273 | 142 | 77 | 18.70 | -7.7997 | -7.8358 |
| 274 | 142 | 77 | 18.70 | -7.7997 | -7.8358 |
| 275 | 142 | 29 | 23.03 | 6.1610 | 6.0864 |
| Total Loss (per unit) | | | = | 4.4597 | 4.6031 |

All line limits and current flows are in P.U.
( 1 P.U. = 100 MVA.)

# SECURITY CONSTRAINED DISPATCH WITH POST-CONTINGENCY CORRECTIVE RESCHEDULING USING LINEAR PROGRAMMING

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Abstract. The objective of a security constrained dispatch is to minimize the operational cost of an electric power network to meet immediate future load demand satisfying various physical and operational constraints for the normal and post-contingency system state. It has been recognised for some years that the solution obtained for such a dispatch is pessimistic as it does not take into account the post-contingency corrective capability of the system. This paper describes a solution method, using a Sparse Dual Revised Simplex algorithm, which can efficiently include the corrective rescheduling capability of the generating units. Because the solution technique is based on linear programming, the method is inherently reliable, fast and robust. A simple formula to obtain the post-disturbance system sensitivity matrix in terms of pre-outage sensitivity matrix is also derived. Results included for a 115-unit system show that very significant saving can be achieved when the rescheduling capability of the system is included in the dispatch algorithm and that the proposed approach is practical for real-time large system applications.

Keywords. power system control, load dispatching, security, linear programming, output power constraints, corrective rescheduling, sensitivity analysis, current injection compensation.

## 1. INTRODUCTION

Security is one of the essential considerations in the operational control of an electric power system. Not only is it a statutory requirement but it also makes economic sense since without secure system operation any economic gain obtained by breaching the physical and operational limitations of the plant will soon be negated by expensive plant failure, loss of supply and consequent loss of revenue.

The economic benefit of optimal sharing of the system demand among the synchronised generating units has long been recognised. Since the introduction of the classical equal incremental cost concept in the late 50's (Kirchmayer,1958), the economic dispatch solution has gone through many significant stages of improvement. From the security point of view, the equal incremental cost approach normally includes only the generator output limits in the problem formulation and neglects the transmission system limitations (Shoults, 1977). By applying more sophisticated mathematical optimization techniques such as linear programming (Irving, Sterling, 1983), quadratic programming (Irving, Sterling, 1985) and dynamic programming (Cheung, Irving, Sterling, 1988), the transmission network may be modelled and incorporated in the optimization process. With this enhancement, the dispatch ensures that the transmission line thermal capacities and the line power transfer agreements between the utilities are not violated. However, the continuously changing conditions in which a power system operates mean that sudden failure of a vital plant is probable. This consideration has lead to the implementation of many security constrained dispatch methods (Cheung, Sterling, Irving, 1988; Li, 1987) since the 70's and this work is still growing in volume. Such algorithms minimize the operational cost of an electric power network subject to the various limitations of both the normal and post contingency system [illegible] is a big forward step towards more secure system operation. It has been apparent, however, for some years that the solution obtained by such a dispatch is pessimistic (IEEE Working Group, 1988); the possible post-contingency corrective capability of the system initiated either automatically by the automatic generator controllers or manually by the operators have not been taken into consideration. With a strict application of the security constained dispatch, the system probably is operated in an unnecessary expensive region to prevent system insecurity which might never happen or could be easily rectifed. Furthermore, for some weakly connected networks, such a practice may even lead to an inoperable system, as a result forcing the operators to adopt a less stringent security requirement, primarily based on their knowlege of the system concerned, instead of a well defined and consistent security criterion. In the last few years, various possible post-disturbance system response capabilities such as network switching (Schnyder, Glavitsch, 1988) and generation rescheduling (Monticelli, Pereira, Granville, 1987) have been proposed to further improve the economic dispatch methodology, aiming to assist the system operator in determining a more realistic and economical solution without sacrificing system security.

This paper investigates the problem of including post-contingency generation rescheduling capability in an economic dispatch solution using a linear programming(LP) approach. The paper uses an Iterative Constraint Selection (Stott, Marinho, Alsac, 1979) process to reduce the dimensionality problem. A simple formula which can efficiently deduce the sensitivity matrix of the post-contingency system state from the sensitivity matrix of the intact system is also derived. Tests on a 115-unit system indicate that the solution scheme is computationally effective and compatable with on-line applications for large electric power systems.

## 2. PROBLEM FORMULATION

The LP formulation of a dispatch problem to include the post-disturbance generation shift ability is straight forward. One of the main difficulties in achieving a computationally effective solution is to devise implementable and efficient means to deal with the enormous dimensionality problem arising from the number of possible contingencies for a large power system and their associated post-contingency generation rescheduling possibilites. Monticelli *el. al.* (1987) offered an exellent proposal utilizing a Benders decomposition technique, such that generation rescheduling for each contingency may be optimized separately from the master problem of minimizing the operational cost of the intact system. One of the major disadvantages of the method is that convergence of the optimization process to a global or even a local optimum is not guaranteed. In this paper all intact and post-contingency generation schedules are expressed in one large LP problem preserving the inherent advantages, such as simplicity, robustness and speed of the LP approaches.

Without loss of generality, the security constrained dispatch with corrective generation rescheduling capabilities may be represented as follows:

$$\text{Minimize} \quad z = \sum_{g=1}^{N_g} f(P_g{}^0) \tag{1}$$

Subject to :

(a) Intact system :

$$\sum_{j=1}^{N_n} D_j = \sum_{g=1}^{N_g} P_g{}^o \tag{2}$$

$$[P]^{min} \leq [P^0] \leq [P]^{max} \tag{3}$$

$$-[F^0]^{max} \leq [S^0][P^0] + [C^0] \leq [F^0]^{max} \tag{4}$$

(b) For each line/generator outage contingency k and response time allowance t for those generators which participate in the state correction process

$$\sum_{j=1}^{N_n} Dj = \sum_{g=1}^{N_g} Pg^k \tag{5}$$

$$[P]^{min} \leq [P^k] \leq [P]^{max} \tag{6}$$

$$-[F^k]^{max} \leq [S^k][P^k] + [C^k] \leq [F^k]^{max} \tag{7}$$

$$-[R^{down}] * t \leq [P^k - P^0] \leq [R^{up}] * t \tag{8}$$

where

$f(P)$ = operating cost functions of the generators
$D_j$ = nodal system load demand including any transmission losses
$[P^0]$, $[P^k]$ = generator outputs for the intact and post-contingency system
$[P]^{min}$, $[P]^{max}$ = generator stable lower and upper output limits
$[F^0]^{max}$, $[F^k]^{max}$ = circuit rating for normal and emergency system operation
$[R^{down}]$, $[R^{up}]$ = ramping down and ramping up rate of the generator outputs
$[S^0]$, $[S^k]$ = sensitivity matrix for intact and emergency system state which relates the line current to generation injections
$[C^0]$, $[C^k]$ = line flow for intact and contingency cases due to nodal load demand
$t$ = time allowance for the generators to react to the line/generator outage condition to bring the system to a tolerable state as defined by Eq. 7
$N_g$ = number of on-line generators
$N_n$ = number of nodes.

Eq. 1 states that the objective of the dispatch is to minimize the total generation production cost of the intact system. This is subject to the power balance, unit capacity limits, line flow limits and unit generation shift limits for the intact (Eqs. 2,3,4) and contingency conditions (Eqs. 5,6,7,8). For generator outage consideration, Eqs. 5-8 define the regulating margin requirement since they ensure that the load will be pickup by the remaining units upon failure of a unit within the specified time t. For any given contingency, a series of Eqs.5-8 corresponding to different response times may also be utilised. Coupled with the associated generator shift and temporary line flow limitations, these may then be used to reflect the dynamic limitations of the system. The results of such a multistage dispatch would define a time sequence of controls to return a disturbed system to the normal state.

It is apparent from the above formulation that for a large system, the number of variables and constraints in the LP problem can be very large indeed. For example, the test system used in this paper has 115 generating units and 275 transmission lines. Assuming that single line failures are considered and that all generators participate in the correction process, for a single stage problem, there would be over 150,000 variables (including constraints) in the LP formulation. A LP solution is generally efficient only when the number of variables is reasonable, i.e. under a few thousands. Since the CPU time of LP execution increases quadratically with the number of variables (Irving, Sterling, 1983), the CPU time requirement for a large scale problem with hundreds of thousands of variables would be impractical from both the execution time and computer storage points of view. To overcome the dimensionality problem, an Iterative Constraint Selection (Stott, Marinho, Alsac, 1979) process is implemented with details described as follows.

## 3. CONSTRAINTS RELAXATION

The success of the iterative constraint selection process is based on the exploitation of the special feature of the economic dispatch problem in which, although the potential number of constraints is large, the number of active constraints is normally small. By relaxing the economic dispatch (EDP) problem to include a small set of known active constraints initially, resulting in a much smaller LP problem, the EDP may be solved very quickly. When such an initial EDP solution is obtained, the full set of constriants is checked for violations. Any violated constraint detected is then added to the original set and a second LP iteration is performed. The final EDP solution is obtained when there is no constraint violation detected in the checking phase. Generally, it is found that the economic dispatch problem is solved more efficiently utilizing the iterative constraint selection scheme rather than solving the complete LP problem with all possible contraints in the formulation.

In the present implementation, constraint relaxation is explored in two areas, line flow constraint and generator limits, based on the following observations. For a well designed power system, the number of line outage contingencies which may lead to insecurity is relatively small, say 0.5%. For each contingency, there may be 0.5% lines in the remaining transmission network near or over their rated limits. Associated with each contingency, a number of generators, say 10%, may need to reschedule to their ramping or capacity limits. For the 115-unit and 275-line example system, the number of active line constraints is roughly about 225 (15x15), the number of active generator constraints is about 150 (15x10) and the number of generator variables is about 1720 (115x15). Therefore the number of variables in the relaxed LP formulation is in the region of two thousands instead of hundreds of thousands as estimated above. In the tests carried out, the largest number of variables is under three thousand as shown in Table 2.

## 4. OUTAGE SIMULATION

Having resolved the dimensionality problem, another hurdle to overcome is to obtain the line flow sensitivity coefficients of the outage cases efficiently. This is crucial for the overall solution scheme to be practical, because in addition to the sensitivity coefficients for the limited number of constraints in Eq. 7, sensitivity matrices for all other contingencies will be needed. These are required in other parts of the solution scheme, such as the constraint checking phase. AC load flow for constraint checking of all contingencies in every iteration is regarded as too CPU intensive. Furthermore, the sensitivity coefficients must be calculated as and when they are required. It is impractical to store the sensivity matrices of all possible contingency cases. There are various derivations reported in the literature (Stott, Marinho, Alsac, 1979; Wood, Wollenburg, 1984) to analyse line outages. The best known concept is perhaps the line flow transfer participation factors. Other approaches modify the Jacobian or admittance matrices of the intact system based on the Householder (1953) inversion lemma. These techniques do not match conveniently with other parts of the present implementation. A different approach for outage simulation is therefore proposed. The proposed technique is discussed into two parts. The first part gives a simple example of the current injection compensation concept for outage simulation. The second part applies the concept to derive a simple expression for the post-contingency sensitivity matrices in terms of the pre-contingency sensitivity matrix.

### 4.1 Current Injection Method (CIM) - A Simple Example

Consider a linear network which has one current source, one sink and two resistive branches connected in parallel as depicted in Fig. 1(a). By the current divider theorem, currents of 8A and 2A are flowing in branches 1 and 2 respectively. When branch 1 is taken out of the network then branch 2 will be carrying the full load of the system as shown in Fig. 1(b). By applying the superposition theorem, the solution in Fig.1(b) can be obtained in two steps.

First, line 1 and all active sources are disconnected from the network. Inject the pre-outage current of the outage line into the system, but with opposite direction, at the two ports of line 1 and calculate the line flow in all parts of the system. The resultant current flow in the system is then superpose on the original network to obtained the final solution. These are shown in Fig. 2(a),(b) and (c). Note that in the process described, line 1 is taken out of the network in Fig. 2(b) and then the currents in the remaining lines are calculated. This is an undesirable procedure because when line 1 is taken out, the topology of the system is changed and so are the admittance and impendance matrices of the system. The published techniques based on the inverse matrix lemma utilize the admittance matrix and its inverse of the original network to obtain the required matrices for the line outage cases. Although such techniques avoid a direct matrix inversion for the modified network, substaintial computation is still required. It would be ideal if it were not necessary to modify the network, its admittance or its inverse in any way and yet arrive at the same solution.

For the above example network, the problem is, without changing the network, to find out the required injections into the two nodes of branch 1 which would result in currents in the remaining branches of the network, as if branch 1 had been removed. The solution is achieved in two stages and is depicted in Fig. 3(a),(b) and (c).

**Stage 1:** Beause an electrical network is linear with respect to current, a sensitivity matrix for the example network can be formed which relates the current flow in all lines to the injections at different nodes of the system. By inspection,

$$F_1 = 0.4I_1 + (-0.4)I_2 \qquad (9)$$
$$F_2 = 0.1I_1 + (-0.1)I_2 \qquad (10)$$

where

$F_1$ = current in branch 1,
$I_1$ = current injection into node 1.

**Stage 2:** Let the required current injections into the two ends of line 1, namely node 1 and node 2, be X and -X ( X is positive) respectively in Fig. 3(b) to simulate the condition of Fig. 2(b) but with line 1 remaining in the system. Since the net current injection into the system external to branch 1 must be 8A and -8A as in Fig. 2(a), then at node 1,

$$X - F_1' = X - ( 0.4X + (-0.4)(-X)) = 8$$

where $F_1'$ is the current in line 1 due to X and -X at nodes 1 and 2.

This implies :

$$X - 0.8X = 8$$
$$X = 40$$

Checking the solution, substitute X = 40A in Eq.10,

$$F_2' = 0.1 * 40 - 0.1 * (-40) = 8A$$

This is identical to Fig. 2(b) although for this case, $F_1'$ is now 32A. This, however, is not of any consequence because branch 1 is switched out in reality.

Likewise, the solution of Fig. 3(b) is superposed on the orignal network state, Fig. 3(a), in which $F_2$ = 2A. The resultant current in branch 2 for the outage of line 1 is therefore 10A as shown in Fig. 3(c) which is identical to Fig.1(b) and Fig.2(c). The importance of the example is clear. It demonstrates that the current flow in the remaining lines of a network following a line outage can be calculated without resort to any topological change of the network.

### 4.2 Application of Current Injection Method in Security Constrained Economic Dispatch with Corrective Rescheduling

Consider the linear relationship between line flow and nodal power injection described in Eq. 4, i.e.

$$[F] = [S] [P] \qquad (11)$$
$$[\Delta F] = [S][\Delta P] \qquad (12)$$

The superscript '0' and the current flow constant [C] due to load demand, estimated losses and inaccuracy correction factor are omitted for clarity. This does not affect the development of the algorithm that follows.

Let a line $l$ which has its sending and receiving ends at nodes m and k respectively carries a pre-outage current of F. Using the same principle as in the above example, let the current injections to nodes m and k be X and -X to simulate line $l$ outage, then

$$\begin{aligned} & X - ( S(l,m)X + S(l,k)(-X)) = F \\ \Rightarrow\ & X \{ 1. - [ S(l,m)-S(l,k) ] \} = F \\ \Rightarrow\ & X = F / \{ 1.-[ S(l,m)-S(l,k) ] \} \end{aligned} \tag{13}$$

Substituting $P_m = X$, $P_k = -X$ and other P equal to zero in Eq.11, the effect of line $l$ outage on the remaining part of the sytem can be determined, i.e.

$$[\Delta F] = [S]\ [0\ 0\ 0\ 0\ 0..\underset{m}{X}...0...\underset{k}{-X}...0....]^T \tag{14}$$

$[\Delta F]$ is then added to the pre-outage line flow [F] to obtained the power flow in the remaining lines of the system after the outage of line $l$.

That is,

$$[F\ contingency] = [F'] = [F] + [\Delta F] \tag{15}$$

for any line which is not tripped out.

$F' = 0.0$ for the outage line.

The current flow for all lines following the outage of line $l$ is therefore available. Any line that becomes overloaded as a result of the contingency for a given power generation pattern can therefore be determined.

Substitute (11) in (13) for pre-outage current of line L,

$$F = S(l,1)P_1+S(l,2)P_2+S(l,3)P_3+....+S(l,n)P_{Ng}$$

$$X = \{S(l,1)P_1+S(l,2)P_2+S(l,3)P_3+...+S(l,n)P_{Ng})\} / \{1.-[S(l,m)-S(l,k)]\} \tag{16}$$

Let $S(l,1)/\{1.-[S(l,m)-S(l,k)]\}$ be B(1)
$S(l,2)/\{1.-[S(l,m)-S(l,k)]\}$ be B(2)
.
.
$S(l,n)/\{1.-[S(l,m)-S(l,k)]\}$ be B(Ng)

Eq. 16 becomes,

$$X = B(1)P_1 + B(2)P_2 + B(3)P_3 + ... + B(Ng)P_{Ng} \tag{17}$$

then substitute X in Eq. 14. For a monitored line $j$,

$$\Delta F_j = [S(j,m)-S(j,k)]*[B(1)*P_1+B(2)*P_2+ ...+B(Ng)*P_{Ng}]$$

Substituing this in Eq. 15, we have

$$\begin{aligned} F_j\ contingency = & \{S(j,1)+[S(j,m)-S(j,k)]B(1)\}P_1 + \\ & \{S(j,2)+[S(j,m)-S(j,k)]B(2)\}P_2 + ... + \\ & \{S(j,Ng)+[S(j,m)-S(j,k)]B(Ng)\}P_{Ng} \end{aligned} \tag{18}$$

$$F_j' = S'(j,1)P_1+S'(j,2)P_2+ ...+ S'(j,Ng)P_{Ng} \tag{19}$$

In Eq. 19, [S'] is the sensitivity matrix for the line outage condition. The [P] for the outage case can be the same as the pre-outage case as normally assumed in a security constained dispatch or it may have changed to a new value if post-contingency rescheduling is permitted. By substituting the B's of Eq. 17 in Eq. 18, the sensitivity coefficients for the outage case may therefore be expressed in terms of the sensitivity coefficients of the intact system.

$$S'(j,h) = S(j,h) + [S(j,m)-S(j,k)]S(l,h)/\{1.-[S(l,m)-S(l,k)]\} \tag{20}$$

The simple expression of Eq. 20, allows rapid calculation of the post-contingency sensitivity coefficient as and when they are needed. The sensitivity matrix of the intact case can be stored in sparse form avoiding a very large storage overhead.

### 4.3 System Split

The proposed technique has a natural way of identifying any line outage which causes a system split. In Eq. 13, for any line outage $l$ causing a sytem spit, the factor

$$\{ S(l,m)-S(l,k) \}$$

will be equal to unity making $F/\{1.-(S(l,m)-S(l,k))\}$ infinite. There is a logical physical interpretation for this condition. Any injections of opposite signs at the two nodes of a line whose failure would cause a system split will cause current flow in that line only. Therefore an infinite current injection would be needed to supply any current external to this line.

## 5. COMPUTATIONAL EXAMPLES

Example system: A 115 unit, 275 line and 145 node system based on a data set provided by the Central Eletricity Research Laboratory of the Central Electricity Generating Board is used to investigate the effect of including post-contingency rescheduling ability on the operating cost and CPU time requirement.

Table 1 compares the dispatch results for a 'pure' economic dispatch in which line constraints for the intact system are considered, a secure economic dispatch which allows transmission lines loaded to their emergency rating immediately after the occurance of a line outage, and a secure economic dispatch allowing 8 minutes for the generators to shift output level to bring the transmission lines to or below their emergency rating after a line outage occurs.

TABLE 1 Comparsion of dispatch results
1. Pure Economic Dispatch
2. (N-1) security constrained
3. (N-1) security with post-contingency corrective rescheduling

| Load Condition | Pure EDP | (N-1) Security | Security with Rescheduling |
|---|---|---|---|
| Winter Plateau | £914279 | £916255 (+0.2 %) | £914279 (+0.0 %) |
| Winter Trough | £479244 | £495842 (+3.5 %) | £479244 (+0.0 %) |
| Summer Plateau | £471317 | £491187 (+4.2 %) | £471317 (+0.0 %) |
| Summer Trough | £124230 | £153764 (+23.8 %) | £124230 (+0.0 %) |

In Table 1, the solution of the 'pure' economic dispatch is used as the reference. For the four load conditions studied, the operating costs when considering rescheduling capability are the same as the 'pure' economic dispatch results. Comparing the conventional security constrained dispatch, the economic saving achieved by taking into account the effect of the generation shift is apparent. The economic saving realizable in practice may be much less than the maximum 23.8% postulated in the table because of other limitations such as practical generator response rate and maximum overloading immediately after a line outage, but the potential is evident.

Table 2 below depicts the CPU time requirements for the four load conditions.. It also provides further information regarding number of active contingencies, number of line overloads, number of active generator ramping limits and number of variables.

TABLE 2 CPU time for Security Constrained Dispatch with Post-contingency Corrective Rescheduling

| Load Condition | No. of Variables | No. of Cont'ncy | Active Constraints Line | Gen_Ramp | CPU Min:Sec |
|---|---|---|---|---|---|
| Winter Plateau | 783 | 5 | 5 | 80 | 0:31 |
| Winter Trough | 1467 | 10 | 17 | 174 | 1:29 |
| Summer Plateau | 2973 | 20 | 63 | 474 | 6:33 |
| Summer Trough | 1441 | 10 | 14 | 151 | 1:27 |

For each of the four load cases, the pure dispatch and security constrained dispatch require only 5 and 8 CPU seconds respectively. The execution time including rescheduling is considerably longer but is still tolerable for real time applications. By inspection of Table 2, the CPU time requirement bares a close relationship to the number of variables of the LP problem. This is a well known LP solution characteristic. However, there seems to be no simple relationship between the computational time and the corresponding economic benefits. The economic saving depends more on the design and operation of the system. When the economic operation of the system is seriously affected by the limitations of a small number of lines, as indicated by a large increase in operation cost for a security constrained dispatch in comparison to a pure economic dispatch, there will be a good chance that dispatch with corrective rescheduling has a significant impact on operational costs. Given that a system exhibits the property of having great potential economic saving when considering corrective capability, it is likely that an experienced system operator will have already instigated an *ad hoc* scheme similar to the corrective dispatch. This is another factor which will affect the theorectical maximum economic saving achieved by implementing the rigorous approach. The present methodology, however, offers a basic framework for further progress in analysing and maximizing the possible security and economic potential inherent in the dynamic capacity rating of the plant.

## 6. CONCLUSION

The paper has presented a LP formulation of the security constrained dispatch which considers the post-contingency corrective rescheduling capability of the generators. Two major technical problems in dealing with such a formulation are discussed. A constraint selection scheme employed to reduce the problem size is described and a simple formula for the determination of the power flow sensitivity coefficients for line outage conditions is derived. The implimentation was tested on a 115-unit system. Study results indicate that the potential economic saving achieved by considering post-contingency rescheduling capacity is very significant and further work in pursuing the concept will prove to be worthwhile.

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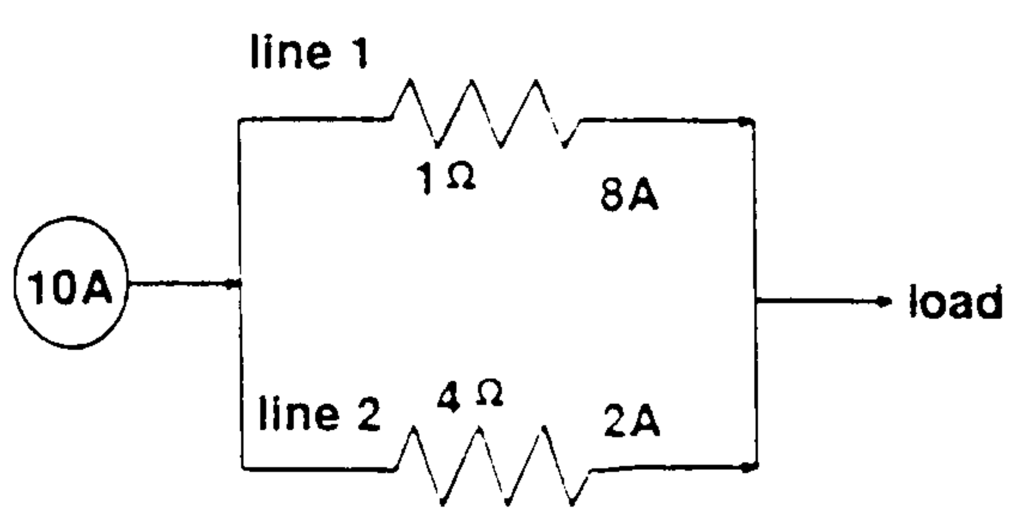


Fig.1(a) Intact system

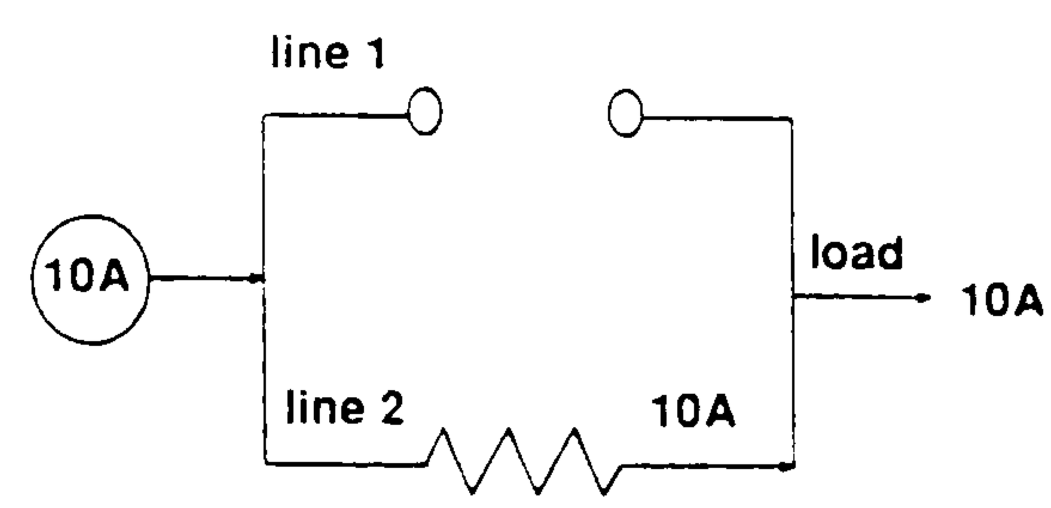


Fig.1(b) System condition after line 1 switched out

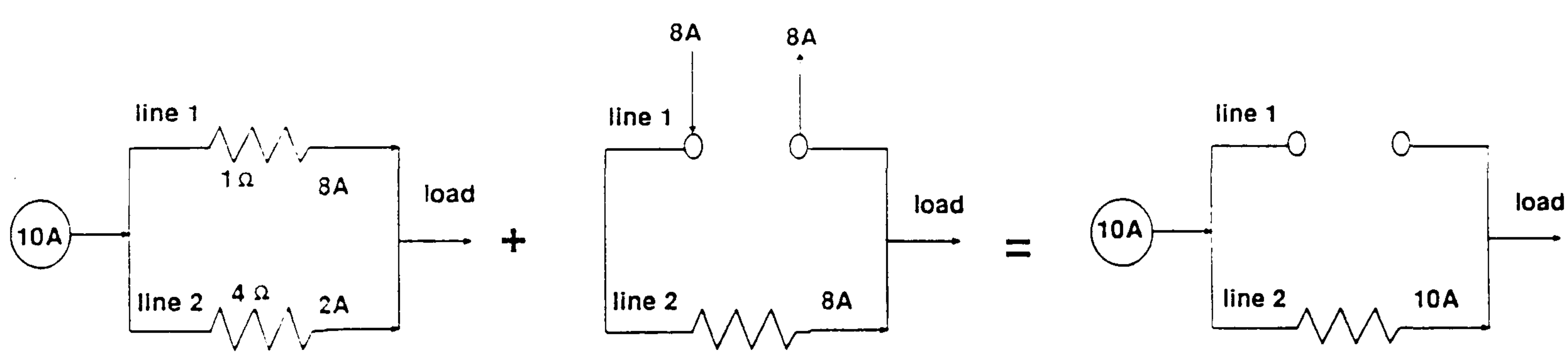


Fig.2(a) Intact system

Fig.2(b) Line 1 switched out with all active sources eliminated from system

Fig.2(c) Post-contingency system conditon using Superposition theorem

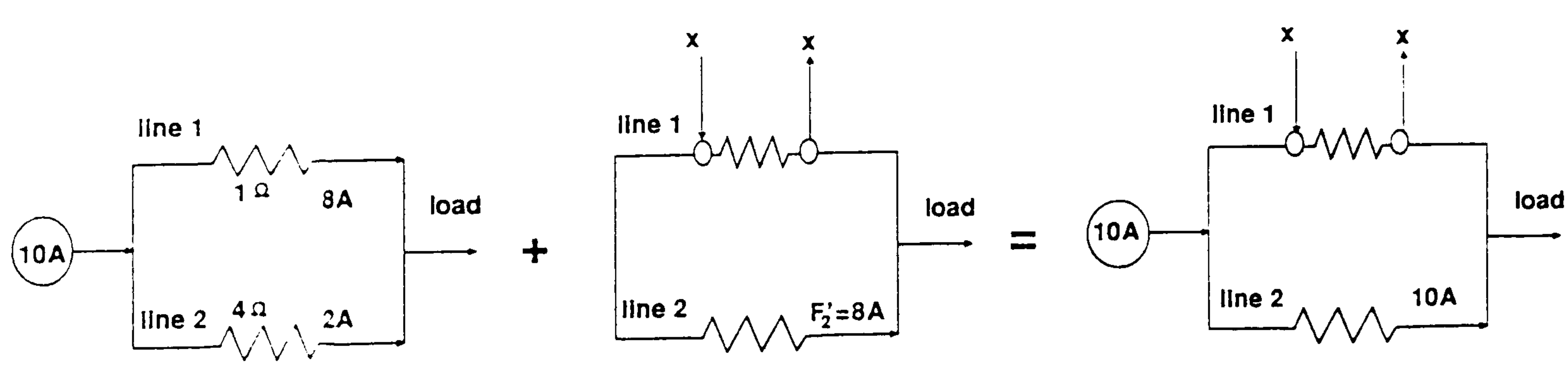


Fig.3(a) Intact system

Fig.3(b) Line 1 outage simulated by external injection

Fig.3(c) Post-contingency system conditon using CIM technique